

AIML 231/DATA 302 — Week 6

# **Dimensionality Reduction and Feature Selection**

Dr Bach Hoai Nguyen

School of Engineering and Computer Science

Victoria University of Wellington

[Bach.Nguyen@vuw.ac.nz](mailto:Bach.Nguyen@vuw.ac.nz)

---

# Week Overview

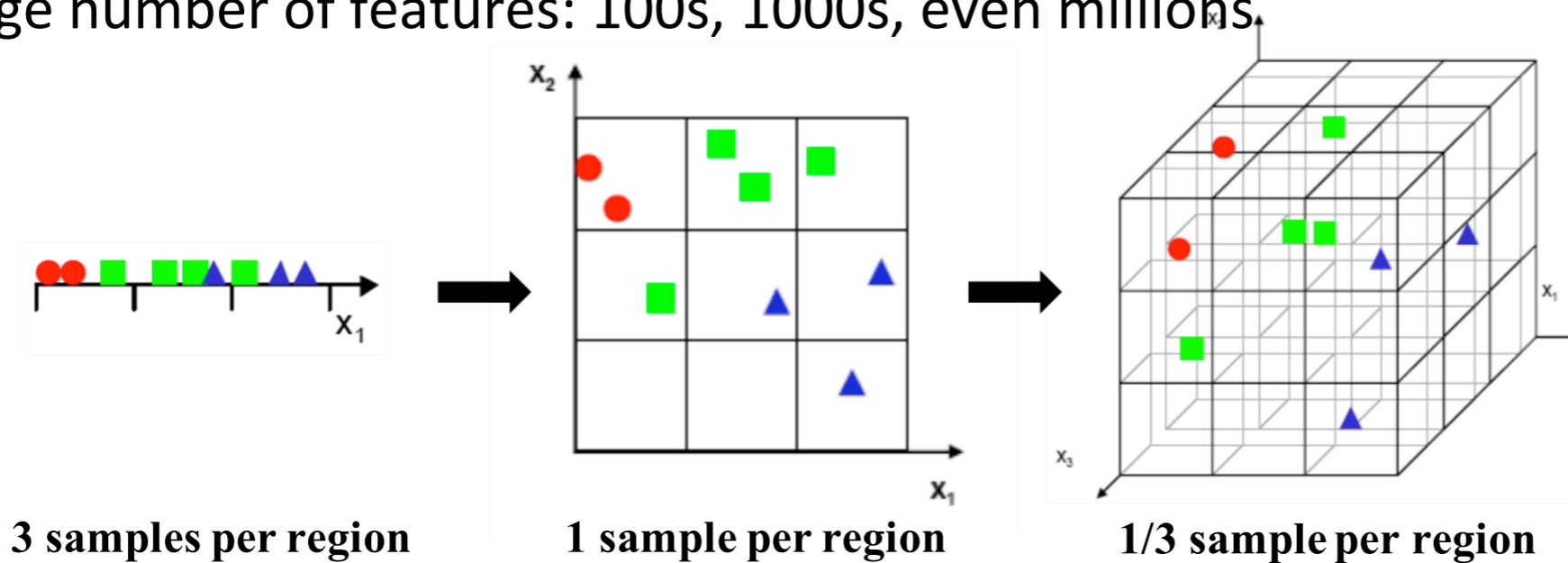
---

- Introduction of Data Preparation/Preprocessing
  - What data preparation include
  - Why data preparation
  - Types of data preparation techniques
- Data Preparation Techniques
  - Training vs Testing, k-fold cross validation
  - Categorical Data Encoding
  - Normalisation
  - Discretisation
- Dimensionality Reduction
  - Feature Selection
  - Feature Construction

# Why Dimensionality Reduction?

- “Curse of dimensionality”

- Large number of features: 100s, 1000s, even millions.



- Data density decreases **exponentially** with dimensionality ☹️

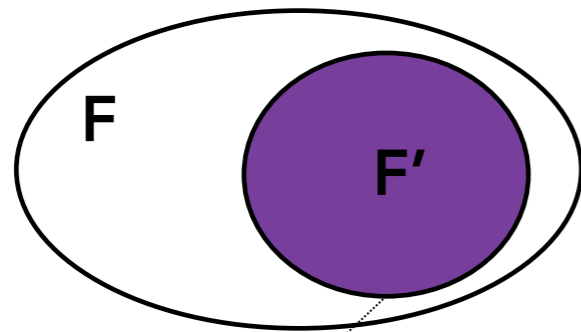
- Irrelevant features: no information for learning task
- Redundant features: same information as other features
- time, memory, and money

# Feature Selection and Feature Construction

## Feature selection (FS)

- Select a **subset of relevant features** to achieve similar or better performance than using all features

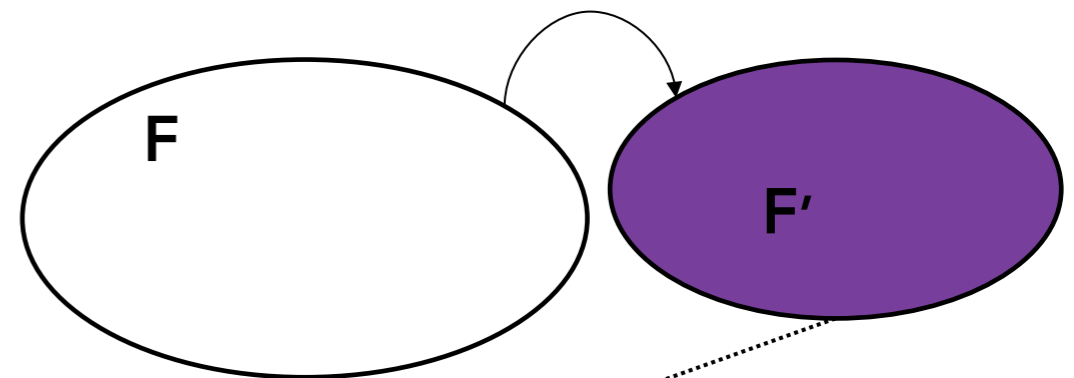
$$F = \{f_1, f_2, \dots, f_n\}$$



$$F' = \{f_{i_1}, f_{i_2}, \dots, f_{i_m}\} \\ (m < n)$$

## Feature construction (FC)

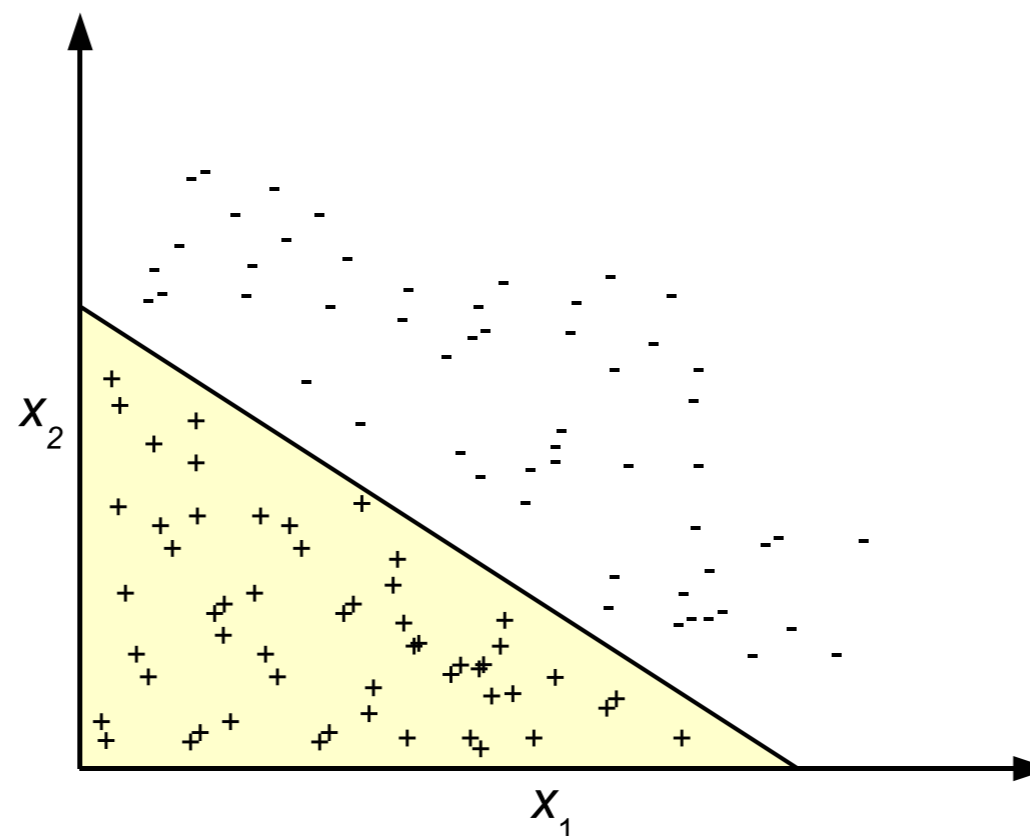
- Build **new high-level features** using the original features to improve the classification performance



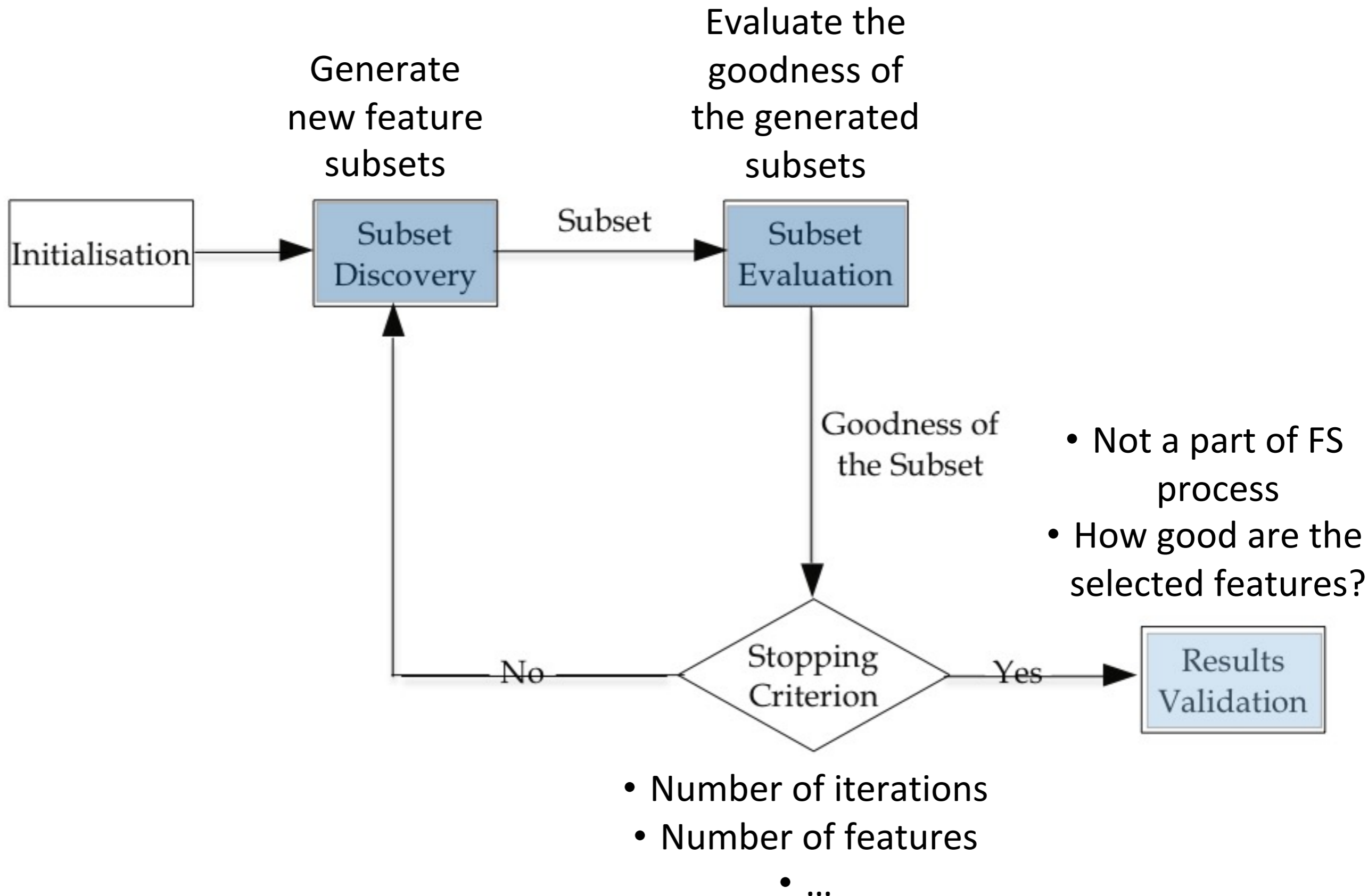
$$F' = \{g_1(f_1, \dots, f_n), g_2(f_1, \dots, f_n), \dots, \\ g_m(f_1, \dots, f_n)\}$$

# Feature Selection Challenges

- Large (**exponential**) search space ( $2^n$  possible feature subsets –  $n$  is the number of features)
- Complex **feature interactions**:
  - **Top relevant features** can be **redundant** as they provide the same information about the class label
  - **Weakly relevant features** can be strongly relevant together – complementary features



# Overall FS System

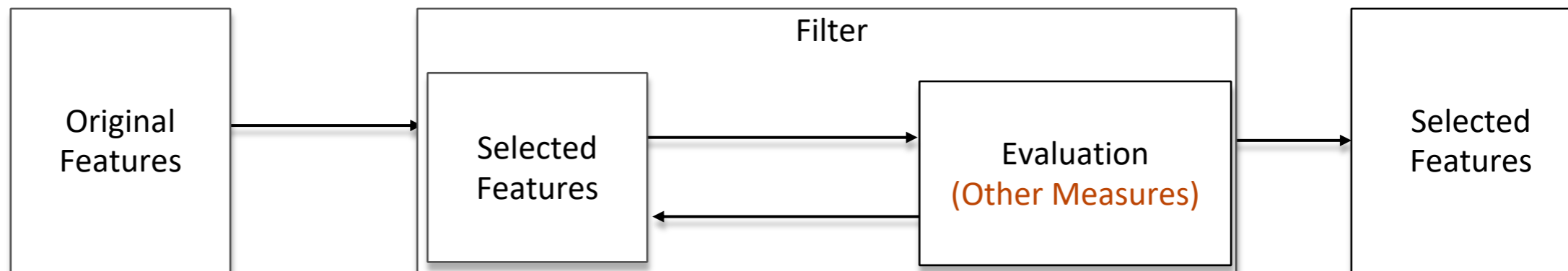


# Filter, Wrapper and Embedded FS (1)

- Based on the **evaluation component**

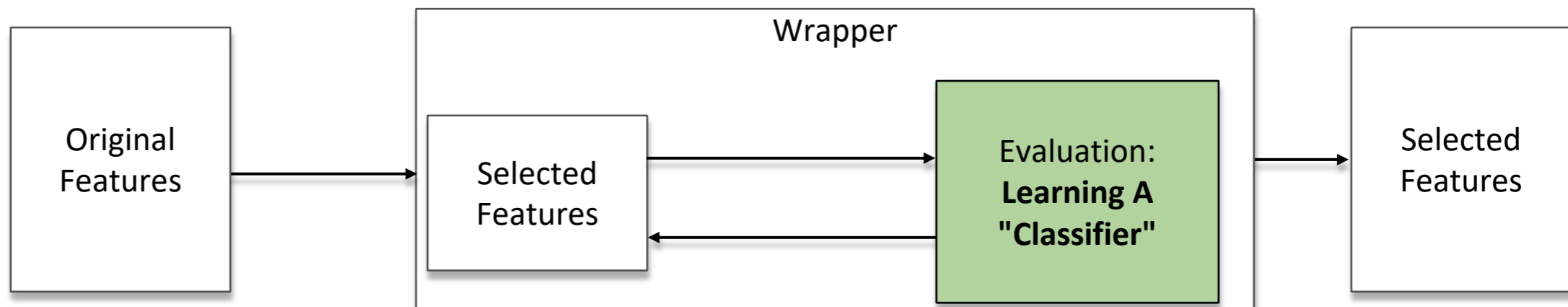
## Filter

Uses existing measures, no learning algorithm



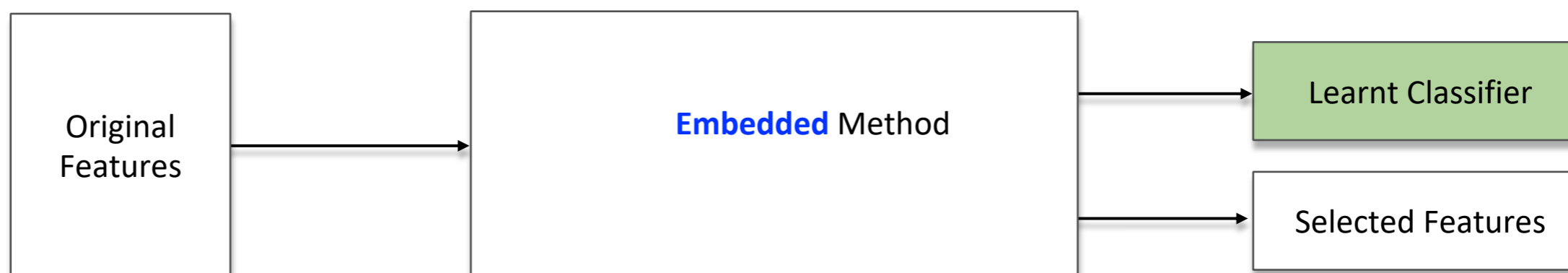
## Wrapper

Uses learning performance, train ML models **many times**



## Embedded

Train ML model **once**  
Select features based on the learned model



# Filter, Wrapper and Embedded FS (2)

---

- Filter FS: can use the following metrics to calculate the **relevance** between features and labels
  - Mutual information
  - Pearson correlation
  - Relief score
- Embedded FS examples:
  - Decision tree: features used **in the tree** are considered selected features
  - Linear SVM: features with **larger weights** are considered important features



# Filter, Wrapper and Embedded FS (3)

---

- Wrapper FS: can use any classification algorithm to evaluate the feature subset
  - Divide the training set into **sub-training** set and **sub-test set**
  - Train the “**wrapped**” **classification algorithm** on the **sub-training set**
  - Examine the obtained classifier on the **sub-test set**
  - The **performance on the sub-test set** can be used as the **quality of the feature subsets**
  - Can apply **K-fold cross validation** to split the training set but it is **more time-consuming**

# Feature Ranking vs Subset Selection

---

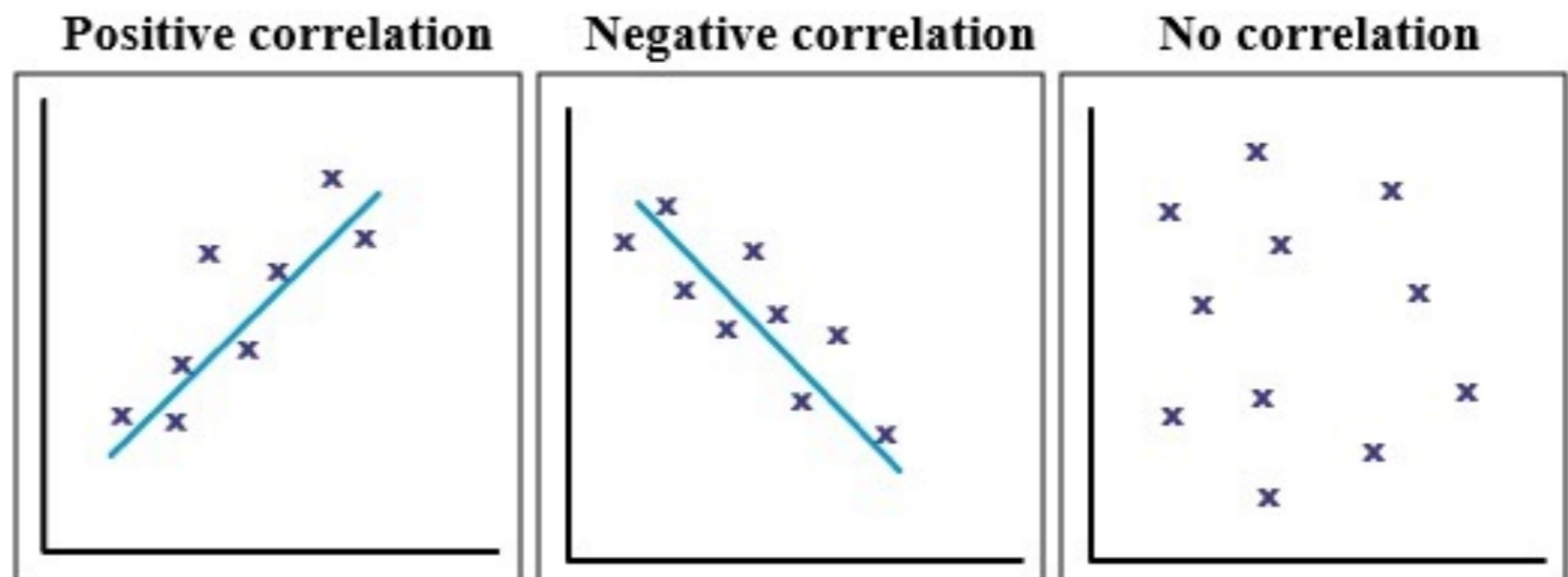
- Based on the Search Mechanism
- **Feature ranking:**
  - Evaluate features **individually**
  - Rank features and select top-ranked features
  - **Simple, efficient**
  - **Ignore feature interactions** (can select redundant features)
- **Feature subset selection:**
  - Evaluate the **whole** feature subset
  - Often an **iterative** process to improve the feature subset
  - Sequential feature selection is an example
  - **Consider feature interactions, usually better performance**
  - **More complicated search, usually more expensive than ranking**

# Univariate FS - Correlation based methods

Univariate methods measure **correlation** between each input feature and the target variable/class label

- **Pearson correlation:** between -1 and 1
- Two variables **move in the same direction/opposite directions**, then have a **positive correlation/negative correlation**
- Rank features based on the **absolute** values of feature correlation
- The higher the correlation, the better the feature

$$r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}$$



- [sklearn.feature\\_selection.r\\_regression](#)

# Some Other Filter Measures for FS (1)

---

- **Mutual information** - measures the **reduction in uncertainty** for one variable given a known value of the other variable, measures **mutual dependency**
- $I(X; Y)$  measures the common information between two  $X$  and  $Y$ .
- Rank features based on the **mutual information** between each feature and the class label
- The higher the mutual information, the more relevant the feature
- Use [sklearn.feature\\_selection.mutual\\_info\\_classif](#)  
[sklearn.feature\\_selection.mutual\\_info\\_regression](#)

## Some Other Filter Measures for FS (2)

---

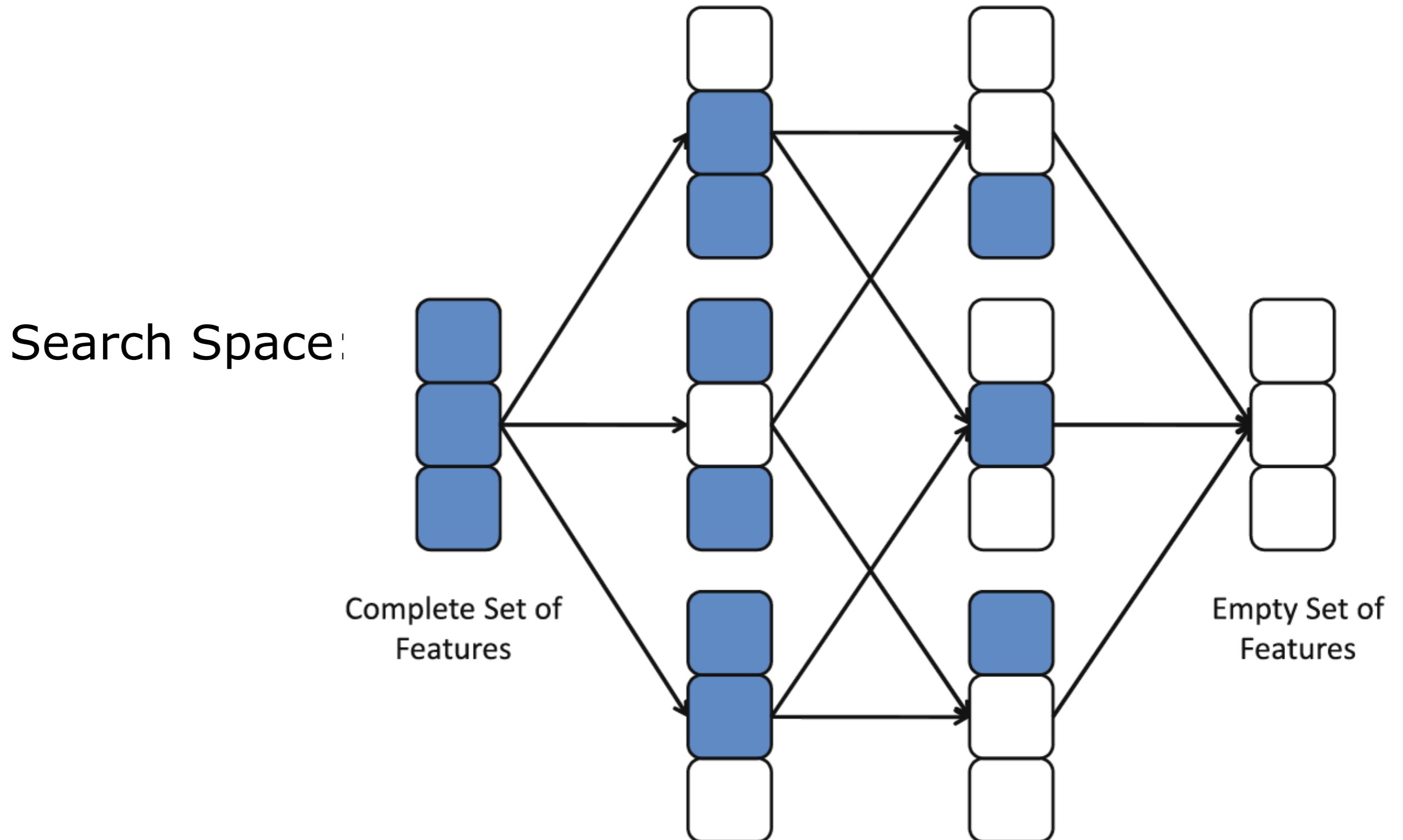
- **Spearman**: for continuous features/variable, nonlinear correlation, use [scipy.stats.spearmanr](#)
- **ANOVA**: between continuous feature and discrete label  
use [sklearn.feature\\_selection.f\\_classif](#)

# Subset Selection: Sequential Search

---

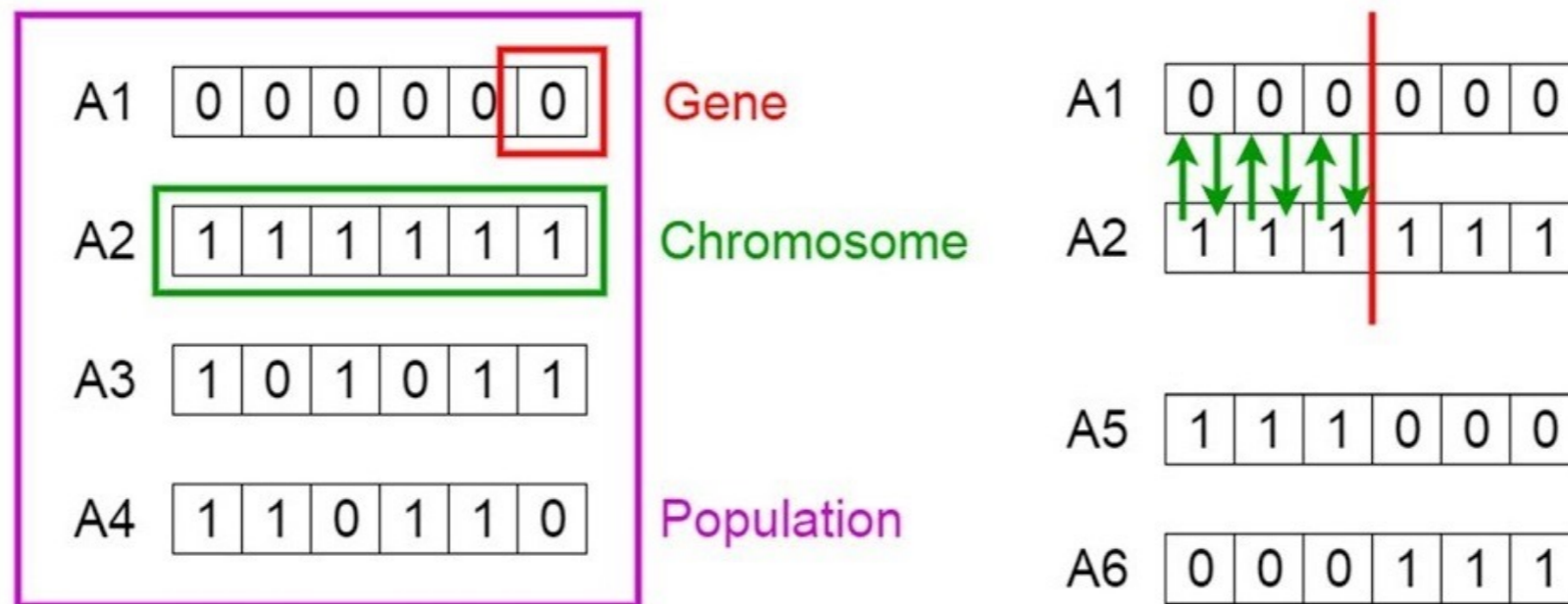
- **Sequential Forward Feature Selection (SFFS):**
  - starting from an **empty** set of features
  - sequentially **add the feature X** that results in the **highest objective value** when combined with the **current set**
  - stop when a pre-defined number of features is selected
  - works best when the optimal subset has **a small number of features**
  
- **Sequential Backward Feature Selection (SBFS):**
  - starting from the **full** set
  - sequentially **remove the feature X** that results in the highest objective value
  - stop when a pre-defined number of features is selected
  - works best when the optimal subset has **a large number of features**

# Subset Selection Illustration



# More advanced FS Methods

- Genetic Algorithm for FS



- Particle Swarm Optimization for feature selection

