## Fundamentals of Artificial Intelligence

VICTORIA UNIVERSITY OF WELLINGTON TE HERENGA WAKA

## COMP307/AIML420

## Search 1

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## Information

- Assignment 1 (due on week 5-27 March 2024)
- Extension requests (use the Submission system)
- Teaching evaluation (Heitor)
- Helpdesks starting from 2pm until 4pm (Thursday until next Wednesday)


## Search in AI

- Why searching is relevant in AI?
- An agent in an environment
- We are looking for a solution to a problem and we would like to know the steps (path) to reach such solution.


Generated with DALLE-2

- Several complex real-world problems rely on search
- Robot navigation; University timetabling; Job shop scheduling; ...
- Search is a critical step in several other AI techniques, such as machine learning and evolutionary computation.


## Towers of Hanoi

- Puzzle that consists of three pegs and a set of disks of different sizes

- Disks are initially stacked on one peg in decreasing order of size, with the largest at the bottom and the smallest at the top
- The goal is to move the entire stack to another peg, one disk at a time, without placing a larger disk on top of a smaller one


## Maze

- Find a path from the Start position (S) to the End position (E)

- Can't go through walls, can only move one position at a time
- The goal is to move the initial position final position, one step at a time


## Abstracting the problem

- Agent: entity that perceives the environment and acts upon that environment
- State: A configuration of the agent in its environment



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- State: A configuration of the agent in its environment
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- Actions: Choices that can be made in a state
- Action(s): Given a state $s$, returns all possible actions from $s$



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- Transition state: a description of the resulting state after action $\boldsymbol{a}$ is applied in state $\boldsymbol{s}$
- Result(s, a) returns the state s' after action a is performed on s


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## State space

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Often we are interested in the "optimal" solution

## Path cost \& Optimal solution

- Path cost: numerical cost associated with a path
- Optimal solution: a solution that has the lowest path cost among all solutions



## The "Frontier"

- Frontier: All the different options that we can explore next
- Simple algorithm:
- Starts by adding the initial state to the Frontier
- Repeat
- If the Frontier is empty, there is no solution
- Remove a node from the Frontier
- Goal test: Node is goal? Done!
- Expand node: add resulting nodes to the Frontier


## Example

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## What can go wrong?

- $S$ is removed and $A$ is added
- $A$ is removed and $S$ is added
- $S$ is removed and $A$ is added
- $A$ is removed and $S$ is added


## What can go wrong?

## What about now?



## What can go wrong?

- $A$ is removed and $B$ is added
- $B$ is removed and $C$ is added
- $C$ is removed and $A$ and $D$ are added
- $A$ is removed and $B$ is added
- $B$ is removed and $C$ is added


## What can go wrong?

- $A$ is removed and $B$ is added
- $B$ is removed and $C$ is added
- $C$ is removed and $A$ and $D$ are added
- $A$ is removed and $B$ is added
- $B$ is removed and $C$ is added

We would like to avoid infinite loops!


## Explored set

- Explored set: maintains a list of already explored nodes
- This allow us to avoid cycles in our basic algorithm


## Example with Explored Set

- Starts by adding the initial state to the Frontier
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- Add removed to the Explored set
- Expand node: add nodes to the Frontier (if not in Explored)


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???

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## Frontier

## Search algorithms

- How we explore the frontier matters!
- There are classical approaches and their variations:
- Depth-first Search (DFS): Stack (Last in, First out)
- Breadth-first Search (BFS): Queue (First in, First out)
- DFS variants
- Depth Limited Search (DLS)
- Iterative Deepening Search (IDS)
- BFS variants
- Uniform Cost Search
- Bidirectional Search


## DFS x BFS - Back to the Maze

- Find a path from the Start position (S) to the End position (E)

- Can't go through walls, can only move one position at a time
- The goal is to move the initial position final position


## DFS x BFS - Graph abstraction



## DFS example



[^0]
## DFS example



## Adding $\mathbf{k}$ to the Frontier

## Frontier

k

[^1]
## DFS example



Adding $z$ to the Frontier
Frontier
z k

Top of the stack

## DFS example



## Removing $z$ to the Frontier

Adding $\mathbf{p}$ to the Frontier

## Frontier

## $\mathrm{p} \neq \mathrm{k}$

Top of the stack

## DFS example



We explore everything here

before exploring from here

## DFS example (graph view)



## DFS example (graph view)



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## DFS example



## DFS example



## DFS example



## BFS example



[^2]
## BFS example



## Adding $\mathbf{k}$ to the Frontier

## Frontier

k

[^3]
## BFS example



Adding $z$ to the Frontier

## Frontier

k

## Z

Added to the back of the Queue

## BFS example



Adding $z$ to the Frontier

## Frontier

k

## Z

Added to the back of the Queue

## BFS example



Adding h to the Frontier

Frontier
K z

Added to the back of the Queue

## BFS example



## BFS example



## BFS example



## BFS example



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## BFS example



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## BFS example



We did it! We removed a node from the Frontier and it was the Goal

## Assessing Search Strategies

- Completeness: Whether the strategy is guaranteed to find a solution when one exists. A complete search strategy will always find a solution if one exists


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- Time complexity: the time it takes to find a solution
- Space complexity: the memory the strategy needs to perform the search


## Depth-First Search Analysis

- DFS is complete if the state space is finite
- DFS is not optimal
- In general, DFS is more efficient than BFS
- Time Complexity: In the worst case*, DFS can explore every node, resulting in a time complexity of $O\left(b^{d}\right)$, where $b$ is the branching factor and $d$ is the maximum depth.
- Space Complexity: depends on the maximum depth of the state space. In the worst case, uses space proportional to the maximum depth $m$, so it is $O(b m)$


## Breadth First Search Analysis

- BFS is complete even if the state space is infinite*
- BFS is optimal if not weighted, i.e. shallowest solution
- In general, BFS can be very expensive
- Time Complexity: Similar to DFS*, in the worst case BFS will explore all nodes, so $O\left(b^{d}\right)$.
- Space Complexity: The space complexity of BFS is also $O\left(b^{d}\right)$ because it needs to keep track of all the nodes on the current level in memory, and the number of nodes at each level grows exponentially with the depth.


## BFS vs DFS

- Important! The time complexity of BFS is typically higher than that of DFS, especially if the branching factor is high and the goal node is located near the bottom of the state space.


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- Important! The time complexity of BFS is typically higher than that of DFS, especially if the branching factor is high and the goal node is located near the bottom of the state space.
- Why? BFS needs to explore all nodes at a given depth before moving on to the next depth, whereas DFS can quickly move down a path until it reaches a dead end (leaf)


## Depth Limited Search (DLS)

- Variant of DFS that limit the maximum depth during exploration
- DLS is complete if the limit is greater than or equal to the depth of the shallowest solution node
- DLS is not optimal


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- Variant of DFS that limit the maximum depth during exploration
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- DLS is not optimal
- Disadvantage 1: it may not be able to find solutions that are deeper than the maximum depth limit, even if they exist
- Disadvantage 2: it may repeat the same path if the limit is not adequate (too low), leading to inefficiency


## Iterative Deepening Search (IDS)

- Combines the benefits of BFS and DFS
- Repeatedly performs DFS with increasing depth limits
- IDS has the same time complexity as BFS $\left(O\left(b^{d}\right)\right)$, but its space complexity is closer to DFS $(O(b d))$ only stores current path
- IDS is complete and optimal (if path cost is non-decreasing with depth)
- IDS is useful when space is large and goal depth is unknown


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Try the maze example using IDS!

## Bidirectional Search

- One BFS from the initial state and one BFS from the goal
- The searches proceed until their frontiers meet in the middle.

- Time complexity: $\mathrm{O}\left(\mathrm{b}^{\mathrm{d} / 2}\right)$
- Space complexity: $O\left(b^{d / 2}\right)$
- Issues? We need to know where is the goal state


## Uniform Cost search

- Behaves as BFS if the actions have the same cost (no weights)
- Expand first nodes with lowest cost; Remember Dijkstra's algorithm?

- Time complexity: $O\left(b^{d}\right)$
- Space complexity: $O\left(b^{d}\right)$
- Optimal if all costs are positive


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## Uninformed \& Informed

- Uninformed Search: The algorithm does not consider specific knowledge related to the problem
- DFS, DLS, IDS, BFS, Bidirectional Search, Uniform Cost
- Informed Search: Exploit knowledge specific to the problem (e.g. Greedy BFS, A*)


## Uninformed \& Informed

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- DFS, DLS, IDS, BFS, Bidirectional Search
- Informed Search: Exploit knowledge specific to the problem (e.g. Greedy BFS, A*)

Question: What is the disadvantage of using an Explored Set?

## Informed Search

A heuristic func $h(n)$ estimates the cheapest cost from node $n$ to the goal
$h(n)$ must be admissible, i.e. never overestimates the cost

- $h(n)$ is defined by relaxing the problem
- E.g. Ignoring walls and using the Manhattan distance in a maze

Why/When do we need heuristics?

- When the search space (state space) is too large!
- Example: chess has a branching factor of $35 . .$.


## Greedy (Best First) Search

- Always expand node whose state appears to be closer to the goal state
- The data structure is a priority queue
- Priority is given by $f(n)$, such that $f(n)=h(n)$


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- Always expand node whose state appears to be closer to the goal state
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- Priority is given by $f(n)$, such that $f(n)=h(n)$
- Example: Route finding in a map


At each node, $\mathrm{h}(\mathrm{n})$ gives the estimated dist.

Initial state: Te Papa
Goal state: VuW
h(Te Papa) = straight line distance, e.g. 1km

## Greedy (Best First) Search



At each node, $h(n)$ gives the estimated dist.
Initial state: Te Papa
Goal state: VuW
h(Te Papa) = straight line distance, e.g. 1km

- Actual path is longer (1.6km)
- $h(n)$ informs the algorithm to avoid unnecessary actions:
- Reaching Victoria Street (in Hamilton) during the search (DFS)
- Exploring the whole CBD first (BFS)


## Greedy (Best First) Search - Maze



$$
\begin{aligned}
& h(z)=8 \\
& h(k)=6
\end{aligned}
$$

- Which node should the algorithm explore next?


## Greedy (Best First) Search - Maze

- Not optimal and not complete!
- May explore 'false' paths
- Time and Space Complexity: $O\left(b^{m}\right)$

Crucial issue: Ignores the path cost $g(n)$, which is the cost from the initial state up to node $\mathbf{n}$

## Greedy (Best First) Search - Maze

- Not optimal and not complete!
- May explore 'false' paths
- Time and Space Complexity: $O\left(b^{m}\right)$


## Can you think of a Maze where this algorithm would explore a "false" path?

Crucial issue: Ignores the path cost $g(n)$, which is the cost from the initial state up to node $\mathbf{n}$

## $A^{*}$

## Estimates the total path cost $f(n)$

- $f(n)=g(n)+h(n)$
- $g(n)$ : from the initial node to node $n$
- $h(n)$ : estimated cost of "relaxed" path from $n$ to goal
$f(n)$ represents the estimated cost of the cheapest solution through n
$A^{*}$ - maze

$A^{*}$ - maze


|  | $f(\mathrm{n}) \mathrm{g}(\mathrm{n}) \mathrm{h}(\mathrm{n})$ |  |  |
| :---: | :---: | :---: | :---: |
| a | 4 | 2 | 2 |
| b | 6 | 2 | 4 |

$A^{*}$ - maze

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|  | $\mathrm{f}(\mathrm{n}) \mathrm{g}(\mathrm{n}) \mathrm{h}(\mathrm{n})$ |  |  |
| :---: | :---: | :---: | :---: |
| a | 4 | 2 | 2 |
| b | 6 | 2 | 4 |
| c | 6 | 3 | 3 |
| d | 6 | 3 | 3 |
| g | 6 | 4 | 2 |
| f | 8 | 4 | 4 |

## $A^{*}$ - maze



|  | $\mathrm{f}(\mathrm{n}) \mathrm{g}(\mathrm{n}) \mathrm{h}(\mathrm{n})$ |  |  |
| :---: | :---: | :---: | :---: |
| a | 4 | 2 | 2 |
| b | 6 | 2 | 4 |
| c | 6 | 3 | 3 |
| d | 6 | 3 | 3 |
| g | 6 | 4 | 2 |
| f | 8 | 4 | 4 |
| ... | ... | ... | ... |
| 0 | 10 | 7 | 3 |
| m | 12 | 6 | 6 |
| p | 12 | 8 | 4 |
| q | 12 | 8 | 4 |
| u | 12 | 8 | 4 |

## $A^{*}$ - maze



|  | $f(n)$ |  |  |
| :--- | ---: | ---: | ---: |
|  | $g(n)$ | $h(n)$ |  |
| $a$ | 4 | 2 | 2 |
| $b$ | 6 | 2 | 4 |
| $c$ | 6 | 3 | 3 |
| $d$ | 6 | 3 | 3 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $m$ | 12 | 6 | 6 |
| $p$ | 12 | 8 | 4 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $w$ | 14 | 10 | 4 |
| $\%$ | 14 | 11 | 3 |
| $!$ | 16 | 10 | 6 |
| $r$ | 16 | 10 | 6 |
| $\#$ | 16 | 11 | 5 |

## $A^{*}$ - maze



|  | $f(n)$ | $g(n)$ | $h(n)$ |
| :--- | ---: | ---: | ---: |
| $a$ | 4 | 2 | 2 |
| $b$ | 6 | 2 | 4 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $\%$ | 14 | 11 | 3 |
| $!$ | 16 | 10 | 6 |
| $r$ | 16 | 10 | 6 |
| $\#$ | 16 | 11 | 5 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $*$ | 14 | 13 | 1 |
|  |  |  |  |
| $!$ | 16 | 10 | 6 |
| $r$ | 16 | 10 | 6 |
| $\#$ | 16 | 11 | 5 |

## Summary

- Search is an important part of several other algorithms
- Abstracting the problem is fundamental
- Selecting an appropriate Search algorithm
- Defining $h(n)$ may not always be trivial
- We were focusing on finding the path from $S$ to $E$


## Coming up next...

- Search 2 (next lecture)
- History AI (Friday Tutorial) - Prof Mengjie Zhang
- Tip: Try out the Search algorithms!


[^0]:    * Explored Set not shown for simplicity; assuming nodes are added as follows: right, down, left, top.

[^1]:    * Skipping the first trivial steps (i.e. only one path); assuming nodes are added as follows: right, down, left, top.

[^2]:    * Explored Set not shown for simplicity; assuming nodes are added as follows: right, down, left, top.

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