COMP361 Tutorial on Divide and Conquer

Question 1

1. Divide-and-conquer algorithms deal with subproblems. What *two* properties must these subproblems have for a divide-and-conquer algorithm to be efficient?

2. Give an efficient divide and conquer algorithm for multiplying two n-digit numbers, assuming that the only available operations are multiplication of single digits, and addition of numbers.

3. Give the general structure of the proof of a divide and conquer algorithm, and use it to show your algorithm from previous part is correct.

4. Give a recurrence relation for the number of single-digit multiplications performed by your algorithm.

5. Hence *guess* the asymptotic running time of your algrithm, stating any additional assumptions you make.

Question 2

Assume you are given two sorted lists of length n:

$$\begin{aligned} u &= \langle u_1, u_2, \ldots, u_n \rangle \\ v &= \langle v_1, v_2, \ldots, v_n \rangle \end{aligned}$$

(i) Devise an efficient divide and conquer algorithm for finding the median of the combined lists: that is, to find the nth largest of the $2n$ elements. Do **not** merge the two lists.

Hint The median of a single sorted list can be determined in constant time. Let u_m be the median of u, and v_m be the median of v. What can you say about the median of the combined lists if $u_m = v_m$? if $u_m < v_m$? if $u_m > v_m$?

(ii) Prove that your algorithm is correct.

Question 3

Where possible, use the Master method to find asymptotic solutions for the following recurrences (assume $T(1) \in \Theta(1)$).

1. $T(n) = 4T(\frac{n}{2}) + n$
2. $T(n) = 4T(\frac{n}{2}) + n^2$
3. $T(n) = 4T(\frac{n}{2}) + n^2 \log n$
4. $T(n) = 4T(\frac{n}{2}) + n^3$
5. $T(n) = 4T(\frac{n}{2}) + 2^n$