

School of

Engineering and Computer Science

Te Kura Mātai Pūkaha, Pūrorohiko

CYBR 171 T1 2023

Ngā whakapūtanga o Te Haumaruru rorohiko
Cybersecurity Fundamentals

Modern Cryptography III RSA and Quantum computing

Learning goals

- Apply RSA cryptography to **confidentiality**.
- Apply RSA cryptography to check **authenticity, integrity** and provide **non-repudiation**
- Explain how we can **establish trust** in **public keys** using RSA cryptography.
- Explain the **threat** to cryptography posed by **quantum computing** and alternatives that might still be safe against these new types of computers.

PART I:

RSA Cryptography

RSA

- Ron Rivest, Adi Shamir and Leonard Adleman (MIT, 1977).
- Difficulty of factoring large integers that are the product of two **large prime numbers**.
- Multiplying two numbers is easy, determining the **original** prime numbers from the total (factoring) is **infeasible**.
- Example key lengths are 256, 512, 1024 and 2048 bits.
- **Four steps** – key generation, distribution, encryption and decryption.
- Public and private key generation is **the most complex** and **expensive** part of RSA.

Comparison Between *RSA* and *DHKE*

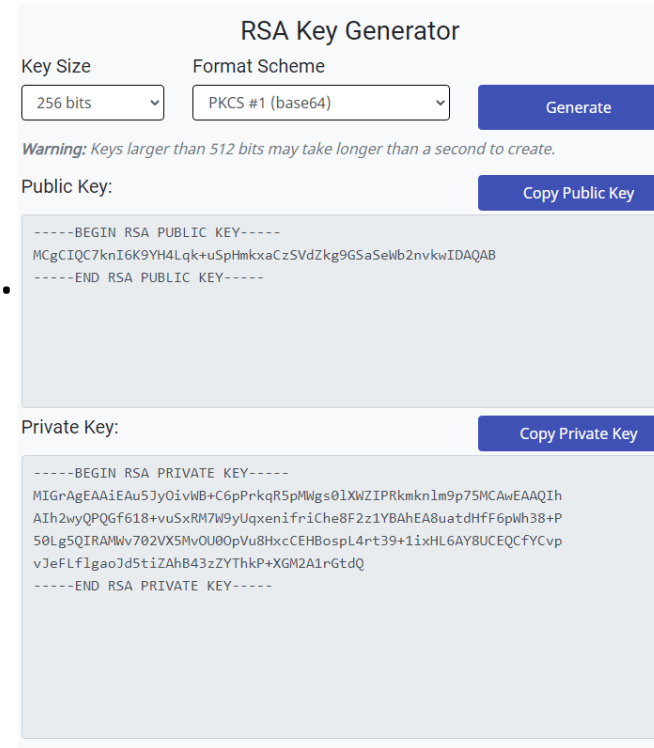
- Diffie-Hellman Key Exchange (DHKE) protocol has two public keys and one symmetric secret key.
- Diffie-Hellman susceptible to MiTM attacks unless fingerprinting is used.
- RSA uses pairs of public and private keys for asymmetric encryption and is resistant to MiTM attacks.
- **RSA supports authentication** of senders and messages **unlike Diffie-Hellman**.

Relationship with **Symmetric** Encryption

- Online commerce because **no need to meet** to set up a key for secure communication.
- Public key systems using **asymmetric** encryption and decryption is much **slower** than symmetric systems.
- Use public key system to send a new symmetric key for **each session**.
E.g. each time I connect to <https://ecs.vuw.ac.nz/>

Generating the Keys

- Imagine Bob wants Alice to communicate securely with him.
- He generates a private and public key pair.
- The **private key** is secret and **never shared**.
- The **public key** is publicly announced.
- Mathematics beyond the scope of this course, illustrate using online tool.
<https://www.csfieldguide.org.nz/en/interactives/rsa-key-generator/>



The screenshot shows the 'RSA Key Generator' interface. It has two dropdown menus: 'Key Size' set to '256 bits' and 'Format Scheme' set to 'PKCS #1 (base64)'. A 'Generate' button is on the right. Below the form, there is a 'Warning' message: 'Warning: Keys larger than 512 bits may take longer than a second to create.' The 'Public Key:' section shows a 'Copy Public Key' button and a text area containing the public key in PEM format: '-----BEGIN RSA PUBLIC KEY-----\nMCgCIQC7knI6K9YH4Lqk+uSpHmkxaCzSVdZkg9GSaSeWb2nvkwIDAQAB\n-----END RSA PUBLIC KEY-----'. The 'Private Key:' section shows a 'Copy Private Key' button and a text area containing the private key in PEM format: '-----BEGIN RSA PRIVATE KEY-----\nMIGrAgEAAiEAu5JyOivWB+C6pPrkqR5pMwgs01XWZIPRkmkn1m9p75MCAwEAAQIh\nAIh2wyQPQGf618+vuSxRM7W9yUqxenifriChe8Fz21YBAHEA8uatdHff6plwh38+P\n50Lg5QIRAMWv702VX5MvOU00pVu8HxcCEHBospL4rt39+1ixHL6AY8UCEQCfYcVp\nvJeFLf1gaoJd5tiZAhB43zZYThkP+XGM2A1rGtdQ\n-----END RSA PRIVATE KEY-----'.

Encrypting Messages

- Alice knows Bob's public key.
- Alice encrypts her message with Bob's public key.
- RSA properties mean that only Bob's private key can decrypt a message encrypted with his public key.

<https://www.csfieldguide.org.nz/en/interactives/rsa-encryption/>

RSA Encryption

Mode: Key Format Scheme: Padding:

Warning: Do not trust this interactive for any real encryption purposes.

Key:

```
-----BEGIN RSA PUBLIC KEY-----  
MCgCIQC7knI6K9YH4Lqk+uSpHmkxaCzSVdZkg9G5aSeWb2nvkwIDAQAB  
-----END RSA PUBLIC KEY-----
```

Plain Message:

BOB MEET ME AT CIVIC SQUARE AT 2PM TODAY.

Encryption successful! Result displayed in base64.

Encrypted Message:

```
cHcTmNqos//fbEZdmp2XxzdJzZ0LIGWfqfBH3NzncDskfd5HdfqXfHfocL5GifGkV/tIt4xdvUw  
b1Z+It006g==
```


Decrypting Messages

- Bob receives the encrypted message.

```
cHcTmNqos // fbEZdmp2Xxz dJz
Z0LIGWfqfBH3NzncDskfd5Hdf
qXfHFocL5GitfGkV/tIt4xdvU
wb1Z+It006g==
```

- Bob uses his private key to decrypt it.

<https://www.csfieldguide.org.nz/en/interactives/rsa-decryption/>

RSA Decryption

Mode: Key Format Scheme:

Warning: Do not trust this interactive for any real decryption purposes.

Key:

```
-----BEGIN RSA PRIVATE KEY-----
MIGrAgEAAIEAu5JyOivWB+C6pPrkqR5pMwgs01XWZIPRkkn1m9p75MCAwEAAQIh
AIh2wyQPQGf618+vuSxRM7W9yUqxenifriChe8F2z1YBAHEA8uatdHFF6pWh38+P
50Lg5QIRAMWv702VX5MvOU00pVu8HxcCEHBospL4rt39+1ixHL6AY8UCEQCfYCvp
vJefLfgaoJd5tiZahB43zZYThkP+XGM2A1rGtdQ
-----END RSA PRIVATE KEY-----
```

Encrypted Message:

```
cHcTmNqos // fbEZdmp2Xxz dJz0LIGWfqfBH3NzncDskfd5HdfqXfHFocL5GitfGkV/tIt4xdvU
wb1Z+It006g==
```


Decryption successful!

Decrypted Message:

```
BOB MEET ME AT CIVIC SQUARE AT 2PM TODAY.
```

Frustrating Eve

- Bob has publicly announced his key
- Eve intercepts the encrypted message
- She tries decrypting using the public key



Decrypt

Decryption failed! There is a problem with the given key or data.

- She must have the private key that no-one but Bob has possession.

POLL: RSA

- **Bob** wants to send **Alice** a message saying, “*I am Bob, trust me*”. He encrypts the message with **her public key**. **Eve** is listening to the conversation.
 - **CLAIM#1** Only Alice can read the message.
 - **CLAIM#2** Eve can read the message.
 - **CLAIM#3** Only Bob can send Alice a message “*I am Bob, trust me*”.
 - **CLAIM#4** Eve can send Alice a message “*I am Bob, trust me*”.
- a) 1 + 3
b) 1 + 4
c) 2 + 3
d) 2 + 4

PART II:

**Authenticity, non-repudiation and
integrity of messages**

Encrypting Using Private Key

- Bob can also encrypt using his **private** key.
- Anyone possessing his **public** key can decrypt the message.

RSA Encryption

Mode: Key Format Scheme: Padding:

Warning: Do not trust this interactive for any real encryption purposes.

Key:

```
-----BEGIN RSA PRIVATE KEY-----
MIGrAgEAAIEAu57y0ivvWB+C6pPrkqR5pMwgs0LXWZIPRkkn1m9p75MCAwEAAQIh
AIh2wyQPQGf618+vu5xRM7M9yUqxeniFr:IChe8F2z1YBAhEABuatdHFF6plwh38+P
50Lg5QIRAMWv702VX5Hv0U00pVu8HxcCEHBospL4rt39+1ixHL6AY8UCEQCfYcVp
vJJeFLf1gaoJd5tiZAhB43zZYThkP+XGM2A1rGtdQ
-----END RSA PRIVATE KEY-----
```

Plain Message:

HELLO WORLD

Encryption successful! Result displayed in base64.

Encrypted Message:

```
eIIGm864FBhDx8cGjek+grDy5LVKRcWbEjZyy2YTiU=
```

RSA Decryption

Mode: Key Format Scheme:

Warning: Do not trust this interactive for any real decryption purposes.

Key:

```
-----BEGIN RSA PUBLIC KEY-----
MCGCIQC7knIGK9YH4Lqk+uSpHmkxAcZSVdZkg9GSaSeWb2nvkwIDAQAB
-----END RSA PUBLIC KEY-----
```

Encrypted Message:

```
eIIGm864FBhDx8cGjek+grDy5LVKRcWbEjZyy2YTiU=
```

Decryption successful!

Decrypted Message:

HELLO WORLD

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Mode: Key Format Scheme: Padding:

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Key:

```
-----BEGIN RSA PRIVATE KEY-----
MIGrAgEAAIEAu57y0ivvWB+C6pPrkqR5pMwgs0LXWZIPRkmkn1m9p75MCAwEAAQIh
AIh2wyQPQGf618+vu5xRM7W9Uqxenifr:IChe8F2z1YBAhEABuatdHFF6plwh38+P
50Lg5QIRAMw702VX5HvOU00pVu8HccCEHBospL4rt39+1ixHL6AY8UCEQCfYCvp
v3pEh...o3d5...hB43...kTh...XGM2A1...G...
-----END RSA PRIVATE KEY-----
```

Plain Message:

HELLO WORLD

Encryption successful! Result displayed in base64.

Encrypted Message:

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RSA Decryption

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-----BEGIN RSA PUBLIC KEY-----
MCGCIQC7knIGK9YH4Lqk+uSpHmkxAcZSVdZkg9GSaSeWb2nvkwIDAQAB
-----END RSA PUBLIC KEY-----
```

Encrypted Message:

```
eIIGm864FBhDx8cGjek+grDy5LVKRcWbEjZyy2YTiU=
```

Decryption successful!

Decrypted Message:

HELLO WORLD

WHY WOULD YOU DO THIS?

Encrypting Using Private Key

- **Only** person able to encrypt a message using the private key is the owner of the associated public key.
- The public key will only decrypt successfully if it corresponds to the private key used for encryption.
- Decryption fails if a **different** private key is used.
- Only authentic private key holder could have encrypted message.

Authenticating Bob

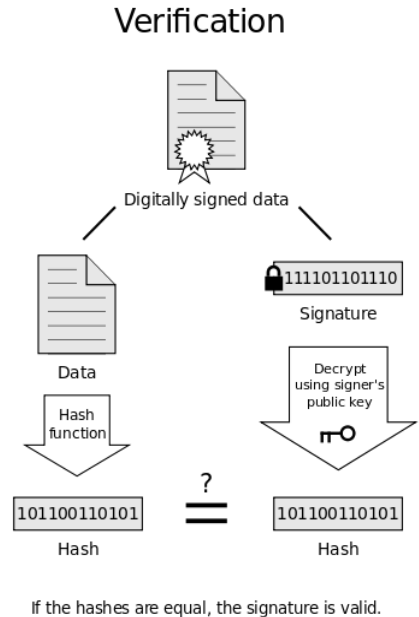
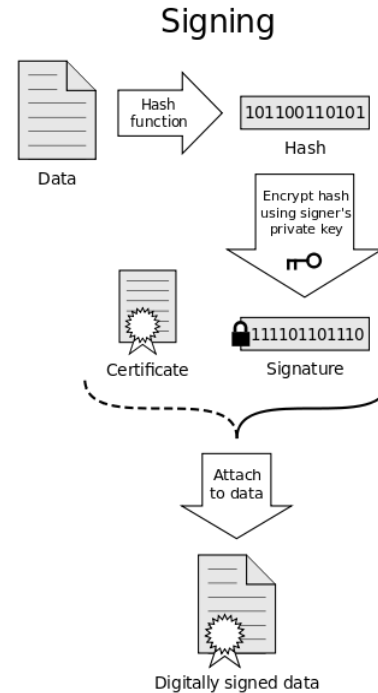
- Alice is suspicious that Eve is carrying out a MiTM attack
- Alice sends Bob a challenge phrase to encrypt with his private key
 - E.g. THE MOON IS GREEN TONIGHT
- Alice decrypts response using Bob's public key to check that Bob was able to do the encryption
- **Would Eve ever be able to create a response that would decrypt successfully?**

Digital Signatures

- Encrypting using private key can be used as a **digital signature**.
- Three key properties:
 - **Authenticate** who encrypted a message.
 - **Non-repudiation** of the sending of the message.
 - Check that **integrity** hasn't been compromised.
- Legally binding in New Zealand
 - Electronic Transactions Act of 2002
 - Part 4 of the Contract and Commercial Law Act 2017

Digital Signatures (cont.)

- Encrypt and decrypt a hash of a message (data) for **efficiency** reasons.
- Note signature is attached to the **plaintext message**.
- Only encrypt the message if it must be **confidential**.



https://commons.wikimedia.org/wiki/File:Digital_Signature_diagram.svg

POLL: DIGITAL SIGNATURE

- Alice **received** a message from Bob with a **digital signature**. The message has been **altered** by Mallory. What would be the result of **checking the digital signature**?
 - a) Decryption of digital signature fails.
 - b) Decryption of message fails.
 - c) Hash of message matches decrypted signature hash.
 - d) Hash of message doesn't match the decrypted signature hash.

PART III:

Establishing Trust

MitM attack on RSA cryptography

- Resistance to MiTM based on *trust in public keys*
- Example:
 - Eve might perform MiTM on Alice and Bob
 - **Eve** gives Alice **her public key** saying "I AM BOB"
 - Alice encrypts the message using "Bob's" public key
 - Eve decrypts and re-encrypts with Bob's **real key**
 - Eve **passes** the message to Bob pretending to be Alice
- **We're back to the same problem was with DHKE**
 - Alice and Bob believe they are communicating securely
 - Eve can read and modify any message

How do we establish trust in public key?

- Authenticate of the binding between a public key and its owner
- Similar to authenticating a message is from the claimed sender
- Method:
 - **Publish** the **public key** in a public place
 - **Adds information** identifying the owner
 - **Sign** using a digital signature (fingerprint it)
 - The recipient can **verify** the digital signature to check integrity and non-repudiation

Establishing trust

- Example:
 - "THIS IS BOB'S KEY" + Bob's Public Key
 - Digitally sign "THIS IS BOB'S KEY" + Bob's Public Key
 - Alice receives and verifies signed public key
 - we can't replace key with her own because will fail verification step

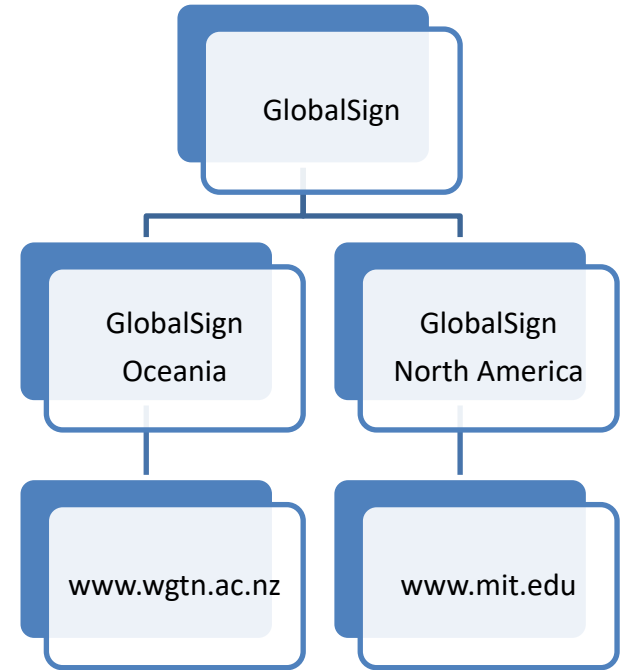
QUESTION: WHOSE KEY DO WE USE TO DO THE SIGNING?

Two Key *Approaches*

- Public Key Infrastructure (PKI)
 - Establishment of trust
 - People who have never met can establish trust
- Certificates and certificate authorities
 - Web browsers, corporate email systems, government systems
- Web of trust
 - PGP, GnuPGP, OpenPGP-compatible systems

Public Key Certificate Authority

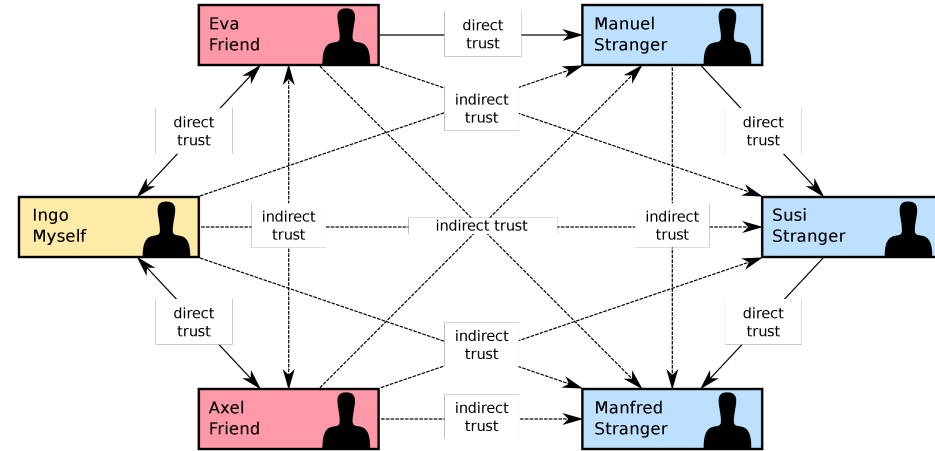
- Hierarchical relationships
 - Trusted authorities called certificate authorities (CAs)
 - CA signs organisation or person's public key
 - That key used to sign other trusted people's keys
 - Repeat ...
- Root CA and all children are trusted 100%



More later under network security

Web of trust

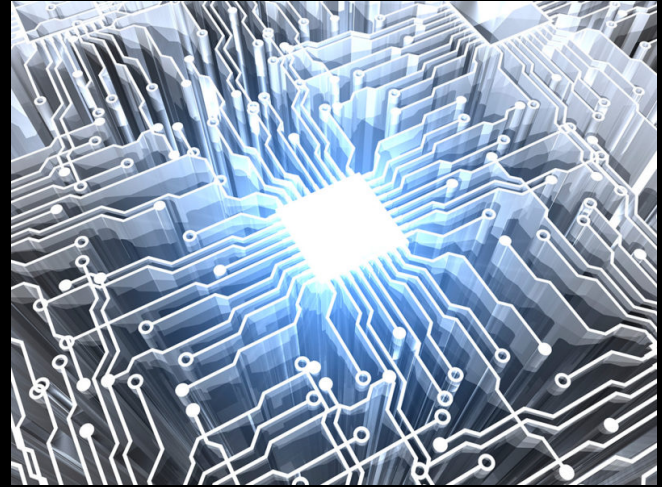
- You decide who to trust based upon set of relationships
 - Direct trust where two people have met and can sign each other's key
 - Indirect trust where someone trust signs someone else's key
 - Assign trust values to each and mathematical formula for calculating overall trust
- Backed by key servers storing signed public keys



https://en.wikipedia.org/wiki/Web_of_trust

PART III:

The Quantum Threat



Quantum computers

- **Ordinary** computers use **bits** for computation – 0 or 1.
- **Quantum** computers use a property allowing atomic particles to **exist in more than one state** at a time.
- Quantum bits (**qubits**) have 3 states available to them - 0, 1 and both “superposition”
- You can carry out a massive number of tasks in **parallel**.
- Require supercold temperatures (-273 Celsius).
- Google, Intel and IBM have built machines.



Quantum computers

- Google provides access to its computer via a service.
 - Sycamore processor
 - 54 superconducting qubits



Quantum Computing Service

The platform enabling researchers to access beyond-classical computational resources

Our quantum computing service provides chaperoned access to NISQ processors and our simulator for researchers who aim to advance the state-of-the-art in quantum computing and publicly share their results in algorithms, applications, tools, and processor characterizations.

[See datasheet](#)

[Read the docs](#)

```
import cirq
import sympy

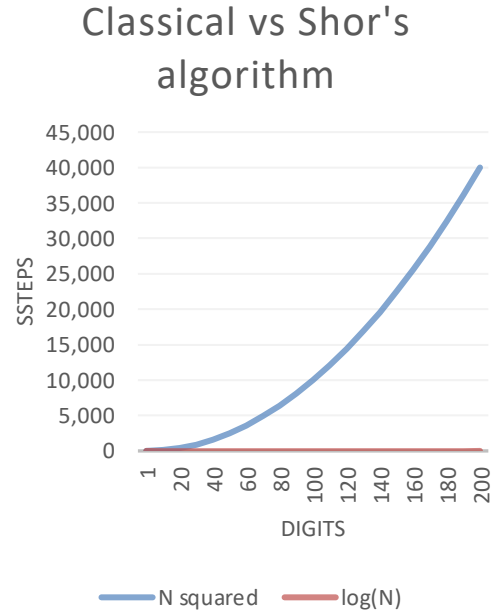
sampler = cirq.google.get_engine_sampler(
    project_id=PROJECT_ID,
    processor_id=PROCESSOR_ID,
    gate_set_name='sqrt_iswap')

circuit = cirq.Circuit(
    cirq.XPowGate(exponent=sympy.Symbol('t'))(cirq.GridQubit(5,4)),
    cirq.measure(cirq.GridQubit(5,4), key='meas'))
rabi_sweep = cirq.Linspace('t', start=0, stop=1, length=20)

results = sampler.run_sweep(circuit,
    repetitions=1000, params=rabi_sweep)
for t in range(20):
    print(results[t].histogram(key='meas'))
```

Threat to cryptography I

- Asymmetric cryptography **depends** upon **solving hard number problems** (primes).
 - Brute forcing a 2048-bit RSA key = 6.4 **quadrillion years**
 - Peter Shor's (1995) **quantum algorithm** for a sufficiently powerful quantum computer
 - **log N** steps versus **N^2** for classical
- 2048-bit key has approximately 616 digits
 - 3 steps versus 379,456 steps



Threat to cryptography II

- What is a **sufficiently powerful** computer?
 - Factoring 2048-bit key reliably requires billion of qubits due to noise from electronics or materials
- Security researchers felt safe until 2019, when a new more efficient algorithm developed
 - Gidney (Google) and Ekerå (KTH Sweden)
 - 20 million qubits versus billions of qubits
- Predicted lifetime of 2048-keys around **20 years**
 - IBM and Google predicted to build a 1 million qubits computer by 2030.
 - Probably doesn't matter for **credit card details** but what about **military or government secrets**?

Post quantum computing

- Some problems are **hard to solve using the quantum** approach – lattices and packing problems.
- Cryptography researchers exploiting these to make quantum-safe algorithms.
- Some old algorithms might be safe (McEliece 1978 system).
- **Bad news** is we have to change our **protocols** to use these and it isn't known yet whether you really cannot solve these problems.

<https://www.wired.com/2015/09/tricky-encryption-stump-quantum-computers/>

PART IV:

What's next

What's up next

- Assignment 1 was released yesterday.
 - Four weeks to complete
 - 20% of the final grade

- Next
 - Guest lecture
 - Application of cryptography (Blockchain and bitcoin)