Common Laplace Transforms

$$\delta(t) \Longleftrightarrow 1$$
$$\delta^{(n)}(t) \Longleftrightarrow s^n$$
$$\mathrm{u}(t) \Longleftrightarrow \frac{1}{s}$$
$$\mathrm{e}^{-at}\,\mathrm{u}(t) \Longleftrightarrow \frac{1}{s+a}$$
$$t\mathrm{e}^{-at}\,\mathrm{u}(t) \Longleftrightarrow \frac{1}{(s+a)^2}$$
$$\frac{t^n}{n!}\mathrm{e}^{-at}\,\mathrm{u}(t) \Longleftrightarrow \frac{1}{(s+a)^{n+1}}$$
$$\sin(\omega t)\,\mathrm{u}(t) \Longleftrightarrow \frac{\omega}{s^2+\omega^2}$$
$$\cos(\omega t)\,\mathrm{u}(t) \Longleftrightarrow \frac{s}{s^2+\omega^2}$$
$$\mathrm{e}^{-at}\sin(\omega t)\,\mathrm{u}(t) \Longleftrightarrow \frac{\omega}{(s+a)^2+\omega^2}$$
$$\mathrm{e}^{-at}\cos(\omega t)\,\mathrm{u}(t) \Longleftrightarrow \frac{s+a}{(s+a)^2+\omega^2}$$

Properties of the Laplace Transform

$$\mathcal{L}\left\{f(t)\right\} := \int_{0^-}^{\infty} f(t)\mathrm{e}^{-st}\,\mathrm{d}t \qquad \mathcal{L}^{-1}\left\{F(s)\right\} := \frac{1}{2\pi\mathrm{j}}\int_{c-\mathrm{j}\infty}^{c+\mathrm{j}\infty} F(s)\mathrm{e}^{st}\,\mathrm{d}s$$

Definition:	$f(t) \Longleftrightarrow F(s)$
Linearity:	$af(t)+bg(t) \Longleftrightarrow aF(s)+bG(s)$
t-scaling	$f(ct) \Longleftrightarrow \frac{1}{\lvert c\rvert}F\left(\frac{s}{c}\right)$
t-shifting: s-shifting:	$f(t-t_0)\mathrm{u}(t-t_0) \Longleftrightarrow \mathrm{e}^{-st_0}F(s)$ $\mathrm{e}^{-s_0 t}f(t) \Longleftrightarrow F(s+s_0)$
Differentiation in t: Integration in t:	$f'(t) \Longleftrightarrow sF(s)-f(0)$ $f''(t) \Longleftrightarrow s^2F(s)-sf(0)-f'(0)$ $f^{(k)} \Longleftrightarrow s^kF(s)-s^{k-1}f(0)-s^{k-2}f'(0)\ldots-f^{(k-1)}(0)$ $\int_0^t f(\tau)\,\mathrm{d}\tau \Longleftrightarrow \frac{1}{s}F(s)$
Differentiation in s: Integration in s:	$tf(t) \Longleftrightarrow -F'(s)$ $\frac{f(t)}{t} \Longleftrightarrow \int_s^\infty F(\tilde{s})\,\mathrm{d}\tilde{s}$
Convolution:	$f(t)*g(t) \Longleftrightarrow F(s)G(s)$ $f(t)g(t) \Longleftrightarrow \frac{1}{2\pi\mathrm{j}}F(s)*G(s)$
Periodicity	$f(t) \Longleftrightarrow F_1(s)\times\frac{1}{1-\mathrm{e}^{-sp}}$ for $f_1(t)$ one cycle of $f(t)$ with period p.
Initial value theorem: Final value theorem:	$f(0^+) = \lim_{s\to\infty} sF(s)$ $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

(for $a, b, t_0, s_0 \in \mathbb{R}, c \in \mathbb{R}_{++}$).

Partial Fractions Expansion

If a partial fraction expansion of $Y(s)$ includes terms $\dfrac{A_m}{(s-a)^m} + \dfrac{A_{m-1}}{(s-a)^{m-1}} + \ldots + \dfrac{A_1}{s-a}$, then the coefficients of factors having multiplicity $m > 1$ are given by the following expressions, where $k \neq m$.

$$A_m = \lim_{s \to a} (s-a)^m Y(s)$$
$$A_k = \frac{1}{(m-k)!} \lim_{s \to a} \frac{\mathrm{d}^{m-k}}{\mathrm{d}s^{m-k}} (s-a)^m Y(s)$$

Trigonometric Identities

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi \implies \begin{cases} \sin(\theta + \frac{\pi}{2}) & = \cos(\theta) \\ \sin(\theta - \frac{\pi}{2}) & = -\cos(\theta) \end{cases}$$

$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi \implies \begin{cases} \cos(\theta + \frac{\pi}{2}) & = -\sin(\theta) \\ \cos(\theta - \frac{\pi}{2}) & = \sin(\theta) \end{cases}$$

$$\begin{aligned} \sin(2\theta) &= 2\sin\theta\cos\theta \\ \cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin(\theta) \end{aligned}$$

Modelling of Electrical Systems

Resistors	$v(t) = i(t)R$	$i(t) = \frac{v(t)}{R}$
Capacitors	$v(t) = \dfrac{1}{C}\int_0^t i(\tau)\,\mathrm{d}\tau$	$i(t) = C\dfrac{\mathrm{d}v(t)}{\mathrm{d}t}$
Inductors	$v(t) = L\dfrac{\mathrm{d}i(t)}{\mathrm{d}t}$	$i(t) = \dfrac{1}{L}\int_0^t v(\tau)\,\mathrm{d}\tau$

Resonant Circuits

	Series	Parallel
Resonant Frequency ω_0	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
Quality factor Q	$\frac{1}{R}\sqrt{\frac{L}{C}}$	$R\sqrt{\frac{C}{L}}$
Bandwidth B	$\frac{\omega_0}{Q}$	$\frac{\omega_0}{Q}$
Cutoff frequencies $\omega_{1,2}$	$\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$	$\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$
For $Q \geq 10$	$\omega_0 \pm \frac{\omega_0}{2Q}$	$\omega_0 \pm \frac{\omega_0}{2Q}$