TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI

VICTORIA UNIVERSITY OF WELLINGTON

EXAMINATIONS – 2018, TRIMESTER 1

Surname ....

First Name ....

Student Number ....

ENGR 121 ENGINEERING MATHEMATICS FOUNDATIONS

Time Allowed: THREE HOURS

CLOSED BOOK

Permitted materials: Silent non-programmable calculators or silent programmable calculators with their memories cleared are permitted.
Non-electronic foreign language to English dictionaries are permitted.
A formula sheet is provided separately.
No other material is permitted.

Instructions: Answer all 8 questions. The exam will be marked out of a total of 100.
Answer in the appropriate boxes if possible — if you write your answer elsewhere, make it clear where your answer can be found. Please use the blank reverse sides of pages and/or the 2 blank end-pages for any extra space you need, for working or for answers.

Questions	Marks
1. Set Theory	[10]
2. Functions & Relations	[20]
3. Logic	[10]
4. Series	[10]
5. Differentiation	[15]
6. Integration	[10]
7. Vectors & Matrices	[15]
8. Probability	[10]

For marking use only

1	
2	
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8	
Total	

1. Set Theory **(10 marks)**

(a) **(5 marks)** State whether each of the following is true or false:

$3.14 \in \mathbb{Q}$	$\sqrt{2} \in \mathbb{R}$	$\sqrt{9} \in \mathbb{N}$
$1/3 \in \mathbb{Z}$	$\mathbb{Q} \subset \mathbb{N}$	

(b) **(5 marks)** Simplify where possible the following operations on sets:

$A \cup \mathbb{E}$	$A \cup \phi$	$A \cap \overline{A}$
$\overline{A} \cap \phi$	$A \cup (A \cap B)$	

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2. Functions and Relations **(20 marks)**

(a) **(4 marks)** Sketch a graph of the two functions $f(x) = x - 1$ and $g(x) = 1 - ax$, on the same axes, assuming $a > 1$ is a given constant. Solve $f(x) = g(x)$ and use your solution to mark on the graph the x-interval where $g(x) > f(x)$.

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(b) **(3 marks)** Is the function $f(x) = \dfrac{1}{x}$ one-to-one, one-to-many, or many-to-one?

What is the domain of $f(x)$? What is the range of $f(x)$?

(c) **(3 marks)** Show that $x = -1$ is a root of the equation $2x^3 - 3x^2 - 3x + 2 = 0$. Find the other roots.

(d) **(1 mark)** Solve $\sqrt{e^{3y}} = 1$.

(e) **(1 mark)** Simplify the following expression as much as possible:

$$e^{-3\ln w + \ln w}\ .$$

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(f) **(1 mark)** The response of a system is given by a signal of amplitude: $A(x) = \dfrac{2}{1 + \left(\frac{x+2}{3}\right)^2}$.

What value of x maximizes the amplitude?

(g) **(1 mark)** Find the value for the angle $x = \sin^{-1}(0.5)$.

(h) **(2 marks)** Find all solutions of the equation $\sin(x) = 0.5$, for $x \in [-\pi, \pi]$.

(i) **(1 mark)** Consider the two functions, $f(x) = x - 1$ and $g(x) = \sin(ax)$ for some real number, a. Write down the function $f(g(x))$.

(j) **(3 marks)** Find the set of real numbers that satisfies the inequality $(1 - x)^2 < 1$.

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3. Logic **(10 marks)**

(a) **(2 marks)** Prove that $\overline{A} + B = \overline{A} + A \cdot B$ using truth tables.

(b) **(2 marks)** Draw a circuit diagram for the expression: $A + \overline{B} \cdot C$

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(c) **(2 marks)** Simplify where possible the expression: $A \cdot B + A \cdot (A + \overline{B})$.

(d) **(2 marks)** Write the disjunctive normal form for a boolean expression with inputs A, B, and C, and with output X, that has the truth table

A	B	C	X
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	0

(e) **(2 marks)** Simplify the disjunctive normal form as much as you can.

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4. Series **(10 marks)**

(a) **(2 marks)** Write down the first three terms, then sum the arithmetic series $\sum_{i=0}^{10}(2+i)$.

(b) **(2 marks)** Evaluate the geometric series $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$.

(c) **(2 marks)** By considering the power series expansion of e^x, find an exact value for the infinite series $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!}$

(d) **(2 marks)** Using the power series expansion for $\cos x$, write down the first three terms of the power series expansion for $\cos(3x)$

(e) **(2 marks)** Use the extended binomial theorem to write down the first three terms of the power series for $\sqrt{1+x/2}$.

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5. Differentiation **(15 marks)**

(a) **(2 marks)** Find the derivative of $f(t) = t^2 + 1$ from first principles, that is, from the definition of the derivative.

(b) **(1 mark)** A function $y(x)$ is such that dy/dx is equal to 1 for all values of x. What can you say about $y(x)$?

(c) **(4 marks)** Find the derivatives y' for the following four functions. You may use the table of derivatives provided in the formula sheet.

(i) $y = 2x^{-3}$

(ii) $y = \frac{x}{\pi}$

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(iii) $y = 3e^{-2x}$

(iv) $y = 2 + \sin(-3x)$

(d) **(5 marks)** Differentiate the five functions

(i) $y = \cos(2x)\sin(5x)$

(ii) $y = (x + x^2)\ln x$

(iii) $$y = \frac{\sin(t)}{t - \frac{1}{t}}$$

(iv) $y = (t^3 - t^2 + 1)^{-5}$

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(v) $y = e^{-x^2}$

(e) **(1 mark)** Calculate d^2y/dt^2, given that $y = t^3 - 2t^2 + 5t$.

(f) **(2 marks)** Find all local maxima and minima of the function

$$y = x^2(4 - x^2)$$

Using the second derivative, show which are minima and which are maxima.

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6. Integration **(10 marks)**

(a) **(5 marks)** Find the following integrals

(i) $\int (x-1)\,\mathrm{d}x$

(ii) $\int \sin(3x)\,\mathrm{d}x$

(iii) $\int -4x^{-6}\,\mathrm{d}x$

(iv) $\int \frac{\pi}{x^2}\ dx$

(v) $\int 5e^{-2x}\,\mathrm{d}x$

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(b) **(2 marks)** Evaluate the integral

$$\int_1^2 x^3 \ln(x)\,\mathrm{d}x$$

using *integration by parts*

$$\int u \left(\frac{\mathrm{d}v}{\mathrm{d}x}\right) \mathrm{d}x \;=\; uv - \int \left(\frac{\mathrm{d}u}{\mathrm{d}x}\right) v\,\mathrm{d}x$$

(c) **(3 marks)** Perform an integration to determine the *average value* of the function

$$f(t) \;=\; t + t^2$$

over the interval $0 \le t \le 1$.

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7. Vectors and Matrices **(15 marks)**

(a) **(4 marks)**

Given

$$\boldsymbol{a} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \qquad \boldsymbol{c} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Find

(i) $2\boldsymbol{a} + \boldsymbol{b}$

(ii) $|\boldsymbol{a} - 2\boldsymbol{c}|$

(iii) $\boldsymbol{a} \cdot \boldsymbol{c}$

(iv) the angle θ between $\boldsymbol{a}$ and $\boldsymbol{c}$

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(b) **(4 marks)** Consider the points $P(2,-2,-1)$, $Q(1,-1,1)$ and $R(2,1,0)$ in $\mathbb{R}^3$.

(i) Determine the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

(ii) Write down a vector equation for the line through P and Q.

(iii) Using the vectors computed in (i), compute a vector which is perpendicular to the plane containing P, Q and R.

(iv) Write down a vector equation for the plane containing P, Q and R.

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(c) **(4 marks)** Given

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} -4 & 3 \\ 1 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} -1 & 2 \\ 2 & 0 \end{bmatrix}.$$

(i) Find $A + B - C$.

(ii) Find BC.

(iii) Find A^{-1}.

(iv) Calculate $det(B^3)$

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(d) **(3 marks)**
(i) Write down a 2x2 transformation matrix A which will scale an object by 2 in the x-direction and keep the y-direction unchanged.

(ii) Write down a 2x2 transformation matrix B which will reflect an object in the y-axis.

(iii) Using matrix multiplication, check that your transformation matrices A and B have the desired effect on the unit square.

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8. Probability **(10 marks)**

(a) **(5 marks)** Given $P(\overline{B}) = 0.5$, $P(A \cap \overline{B}) = 0.4$, $P(A \cap B) = 0.2$, answer the following questions:

(i) Find $P(A)$.

(ii) Find $P(\overline{A} \cap B)$.

(iii) Find $P(A \cup B)$.

(iv) Find $P(B|A)$.

(v) Are A and B independent? *Briefly justify your answer*.

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(b) **(5 marks)** Computer chips are manufactured by either machine A or B where machine A makes 60% of the chips. The probability that a chip is faulty is 0.1 when made by machine A and 0.05 when made by machine B.

(i) Draw a tree diagram for this situation. Include labels for the nodes in your diagram and label the arcs with the relevant probabilities. Also record the overall probabilities on the right hand side of the tree corresponding to each outcome.

(ii) What is the probability of a chip being faulty?

(iii) What is the probability that a chip being made by machine A given that it is found faulty?

(iv) What is the probability that a chip is made by machine B and is not faulty?

(v) For any two events A and B, is it possible that $P(A|B) = 1$? If so, when will it happen?

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SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

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