

ENGR121 Test One

During lecture, 1 April 2016

Name:

ID Number:

Please use the spaces provided in this test booklet next to the questions, to give your answers. Please show all working. You may use page five for rough working if you need more space, plus the reverse sides of all pages.

Attempt all SEVEN questions. All questions are of equal value, ten marks each. The marks for parts of questions are given in square brackets, e.g. [1].

Silent calculators may be used. A table of formulae is attached; you can detach it if you want.

For marking use only

1	
2	
3	
4	
5	
6	
7	
Total	

1. State whether each of the following is true or false [1 mark each]:

(a) $-1.414 \in \mathbb{Z}$

(b) $-1.414 \in \mathbb{R}$

(c) $-1.414 \in \mathbb{N}$

(d) $-1.414 \in \mathbb{Q}$

(e) $\sqrt{2} \in \mathbb{Z}$

(f) $100 \in \mathbb{R}$

(g) $100 \in \mathbb{Q}$

(h) $\mathbb{Q} \subset \mathbb{R}$

(i) $\mathbb{N} \subset \mathbb{Q}$

(j) $\mathbb{N} \cap \mathbb{R} = \mathbb{R}$

2. Simplify where possible the following operations on sets:

(a) $A \cup \mathbb{E}$ [1]

(b) $B \cup \overline{B}$ [1]

(c) $A \cap \phi$ [1]

(d) $B \cap \mathbb{E}$ [2]

(e) $A \cup (A \cap B)$ [2]

(f) $\overline{\overline{C}}$ [2]

(g) $A \cap \overline{\mathbb{E}}$ [1]

3. (a) Sketch a graph of the function $f(x) = x - 1$. [2]

(b) Is the function $f(x) = x + 2$ one-to-one? [1]

(c) Is the function $f(x) = x^2 - 7$ one-to-one? [1]

(d) Can a function be one-to-many? [1]

(e) Write down the inverse of the function $f(x) = 3x - 1$. [2]

(f) Write down and simplify the composition $f(g(x))$ if $f(x) = 2/x$ and $g(x) = 2x - 2$. [2]

4. (a) Write down the graphical symbol for an OR gate with two inputs and one output. [1]

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- (b) Write down the truth table for an OR gate with inputs A and B. [1]

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- (c) Construct the truth table for $A \cdot B + \bar{A}$. [3]

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- (d) Draw a circuit diagram for $A \cdot (B + C)$ using AND and OR gates. [3]

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- (e) Simplify the logical expression $A \cdot A$ [1]

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- (f) Simplify the logical expression $A + \bar{A}$ [1]

5. (a) Write the disjunctive normal form for a boolean expression that has the truth table [5]

A	B	C	X
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

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- (b) Simplify the d.n.f obtained in part (a), if possible. Show all working. [4]

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- (c) Draw a circuit diagram for the boolean expression you obtained in part (b) [1]

6. (a) Simplify $y^2(y^{-1})^3$. [2]

7. (a) Use the Fundamental Laws of Set Algebra (see Formula Sheet) to prove that, for any sets A and B , [5]

$$A \cup (A \cap B) = A$$

(b) Find the roots of $x + 2 = 4x - 1$, showing your working, without using a calculator. [2]

(c) Solve the quadratic equation $x^2 - x - 12 = 0$ using any method except a calculator. Show your working. [2]

(b) Use the Fundamental Laws of Boolean Algebra (see Formula Sheet) to prove that, for any logical inputs A and B , [5]

$$A + \bar{A} \cdot B = A + B$$

Hint: You can start with the distributive law $A + B \cdot C = (A + B) \cdot (A + C)$

(d) Solve the quadratic equation $x = x^2$. [2]

(e) Solve the inequality $x^2 > 1$ [2]

Use this page and the other side for rough working if needed.