ENGR 121 Test One | Model Answers 2016 | Mark McGuinness

1. State whether each of the following is true or false [1 mark each]:

(a)	$-1.414 \in \mathbb{Z}$	F
(b)	$-1.414 \in \mathbb{R}$	T
(c)	$-1.414 \in \mathbb{N}$	F
(d)	$-1.414 \in \mathbb{Q}$	T
(e)	$\sqrt{2} \in \mathbb{Z}$	F
(f)	$100 \in \mathbb{R}$	T
(g)	$100 \in \mathbb{Q}$	T
(h)	$\mathbb{Q} \subset \mathbb{R}$	T
(i)	$\mathbb{N} \subset \mathbb{Q}$	T
(j)	$\mathbb{N} \cap \mathbb{R} = \mathbb{R}$	F

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2. Simplify where possible the following operations on sets:

(a)	$A \cup \mathbb{E}$	$\mathbb{E}$	[1]
(b)	$B \cup \overline{B}$	$\mathbb{E}$	[1]
(c)	$A \cap \phi$	ϕ	[1]
(d)	$B \cap \mathbb{E}$	B	[2]
(e)	$A \cup (A \cap B)$	A	[2]
(f)	$\overline{\overline{\overline{C}}}$	$\overline{C}$	[2]
(g)	$A \cap \overline{\mathbb{E}}$	ϕ	[1]

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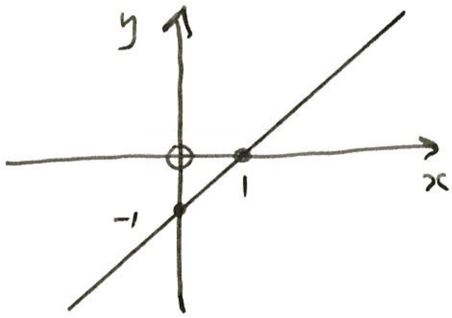
3. (a) Sketch a graph of the function $f(x) = x - 1$. [2]



scale
labels
straight
slope
intercept

(b) Is the function $f(x) = x + 2$ one-to-one? [1]

yes

(c) Is the function $f(x) = x^2 - 7$ one-to-one? [1]

no

(d) Can a function be one-to-many? [1]

no

(e) Write down the inverse of the function $f(x) = 3x - 1$. [2]

$y = 3x - 1$

$\Rightarrow x = \frac{y+1}{3}$

$\Rightarrow f^{-1}(x) = \frac{x+1}{3}$

(f) Write down and simplify the composition $f(g(x))$ if $f(x) = 2/x$ and $g(x) = 2x - 2$. [2]

$f(g) = \frac{2}{g} = \frac{2}{2x-2}$

$= \frac{1}{x-1}$ (simplified)

(Total is only 9, so 1 point given for free) 10

4. (a) Write down the graphical symbol for an OR gate with two inputs and one output. [1]

(b) Write down the truth table for an OR gate with inputs A and B. [1]

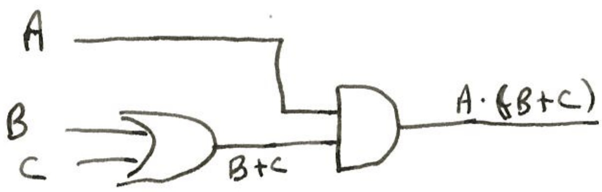
A	B	A+B
1	1	1
1	0	1
0	1	1
0	0	0

(c) Construct the truth table for $A \cdot B + \overline{A}$. [3]

A	B	A·B	$\overline{A}$	AB+$\overline{A}$
1	1	1	0	1
1	0	0	0	0
0	1	0	1	1
0	0	0	1	1

1 point for (one) working columns

(d) Draw a circuit diagram for $A \cdot (B + C)$ using AND and OR gates. [3]



(e) Simplify the logical expression $A \cdot A$ [1]

A

(f) Simplify the logical expression $A + \overline{A}$ [1]

1

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5. (a) Write the disjunctive normal form for a boolean expression that has the truth table [5]

A	B	C	X	
1	1	1	1	← ABC
1	1	0	1	← AB$\overline{C}$
1	0	1	0	
1	0	0	0	
0	1	1	1	← $\overline{A}$BC
0	1	0	0	
0	0	1	0	
0	0	0	0	

so the dnf is

$$ABC + AB\overline{C} + \overline{A}BC$$

(b) Simplify the d.n.f obtained in part (a), if possible. Show all working. [4]

$$ABC + AB\overline{C} + \overline{A}BC =$$

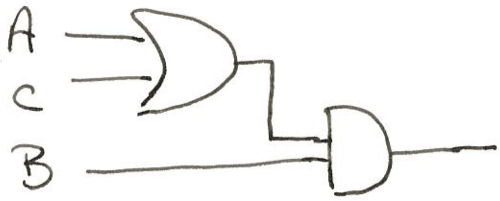
$$ABC + ABC + AB\overline{C} + \overline{A}BC$$

$$= AB(C + \overline{C}) + (A + \overline{A})BC$$

$$= AB \cdot 1 + 1 \cdot BC$$

$$= AB + BC = B(A + C)$$

(c) Draw a circuit diagram for the boolean expression you obtained in part (b) [1]



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6. (a) Simplify $y^2(y^{-1})^3$. [2]

$= y^2 y^{-3}$

$= y^{-1}$ or $\frac{1}{y}$

(b) Find the roots of $x + 2 = 4x - 1$, showing your working, without using a calculator. [2]

$\Rightarrow x - 4x = -1 - 2$

$\Rightarrow -3x = -3$

$\Rightarrow x = 1$

(c) Solve the quadratic equation $x^2 - x - 12 = 0$ using any method except a calculator. Show your working. [2]

$x^2 - x - 12 = (x-4)(x-3)$

is zero when $x = 4$ or -3

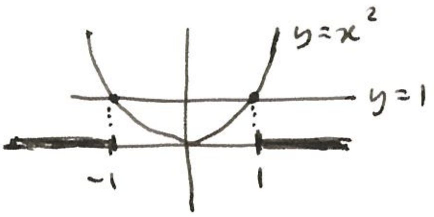
(d) Solve the quadratic equation $x = x^2$. [2]

$\Rightarrow x^2 - x = 0$

$\Rightarrow x(x-1) = 0$

$\Rightarrow x = 0$ or 1

(e) Solve the inequality $x^2 > 1$ [2]



$x < -1$

or

$x > 1$

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or

$x^2 - 1 > 0 \Rightarrow (x+1)(x-1) > 0$

$\Rightarrow$ both +ve or both −ve

$\Rightarrow x < -1$ or $x > 1$

7. (a) Use the Fundamental Laws of Set Algebra (see Formula Sheet) to prove that, for any sets A and B, [5]

$$A \cup (A \cap B) = A$$

name the laws

LHS $= A \cup (A \cap B)$

$= (A \cap E) \cup (A \cap B)$ identity law

$= A \cap (E \cup B)$ distributive law

$= A \cap E$ definition of E

$= A$ identity law

$=$ RHS

(logic sets 1 or 2 points)

(b) Use the Fundamental Laws of Boolean Algebra (see Formula Sheet) to prove that, for any logical inputs A and B, [5]

$$A + \overline{A} \cdot B = A + B$$

Hint: You can start with the distributive law $A + B \cdot C = (A + B) \cdot (A + C)$

LHS $= A + \overline{A} \cdot B$

$= (A + \overline{A}) \cdot (A + B)$ by distributive law

$= 1 \cdot (A + B)$ complement law

$= A + B$ identity law

$=$ RHS

name laws: 1 point

logic: 1 or 2 points

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Use this page and the other side for rough working if needed.

Formula Sheet for ENGR121 Engineering Mathematics

Fundamental Laws of Set Algebra (A, B, C are sets; $\mathbb{E}$ is a universal set)

$$\left.\begin{array}{l} A \cup B = B \cup A \\ A \cap B = B \cap A \end{array}\right\} \text{Commutative laws}$$

$$\left.\begin{array}{l} A \cup (B \cup C) = (A \cup B) \cup C \\ A \cap (B \cap C) = (A \cap B) \cap C \end{array}\right\} \text{Associative laws}$$

$$\left.\begin{array}{l} A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \end{array}\right\} \text{Distributive laws}$$

$$\left.\begin{array}{l} A \cup \emptyset = A \\ A \cap \mathbb{E} = A \end{array}\right\} \text{Identity laws}$$

$$\left.\begin{array}{l} A \cup \overline{A} = \mathbb{E} \\ A \cap \overline{A} = \emptyset \\ \overline{\overline{A}} = A \end{array}\right\} \text{Complement laws}$$

Set Laws derivable from the above:

$$\left.\begin{array}{l} A \cup (A \cap B) = A \\ A \cap (A \cup B) = A \end{array}\right\} \text{Absorption laws}$$

$$\left.\begin{array}{l} (A \cap B) \cup (A \cap \overline{B}) = A \\ (A \cup B) \cap (A \cup \overline{B}) = A \end{array}\right\} \text{Minimization laws}$$

$$\left.\begin{array}{l} \overline{A \cup B} = \overline{A} \cap \overline{B} \\ \overline{A \cap B} = \overline{A} \cup \overline{B} \end{array}\right\} \text{De Morgan's laws}$$

Fundamental Laws of Boolean Algebra.

$A + B = B + A$ $A \cdot B = B \cdot A$	Commutative laws
$A + (B + C) = (A + B) + C$ $A \cdot (B \cdot C) = (A \cdot B) \cdot C$	Associative laws
$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$ $A + (B \cdot C) = (A + B) \cdot (A + C)$	Distributive laws
$A + 0 = A$ $A \cdot 1 = A$	Identity laws
$A + \overline{A} = 1$ $A \cdot \overline{A} = 0$ $\overline{\overline{A}} = A$	Complement laws

The laws of Boolean Algebra that may be derived from the laws above.

$A + (A \cdot B) = A$ $A \cdot (A + B) = A$	Absorption laws
$(A \cdot B) + (A \cdot \overline{B}) = A$ $(A + B) \cdot (A + \overline{B}) = A$	Minimisation laws
$\overline{A + B} = \overline{A} \cdot \overline{B}$ $\overline{A \cdot B} = \overline{A} + \overline{B}$	De Morgan's laws
$A + 1 = 1$ $A \cdot 0 = 0$	

Logarithm Laws

$$\begin{aligned} \log_a(AB) &= \log_a A + \log_a B \\ \log_a\left(\frac{A}{B}\right) &= \log_a A - \log_a B \\ \log_a(A^n) &= n \log_a A \\ \log_a a &= 1 \\ \log_a X &= \frac{\log_b X}{\log_b a}. \end{aligned}$$

Hyperbolic Functions

$$\begin{aligned} \cosh^2 x - \sinh^2 x &= 1 \\ 1 - \tanh^2 x &= \operatorname{sech}^2 x \\ \coth^2 x - 1 &= \operatorname{cosech}^2 x \\ \sinh(x \pm y) &= \sinh x \cosh y \pm \cosh x \sinh y \\ \cosh(x \pm y) &= \cosh x \cosh y \pm \sinh x \sinh y \\ \sinh 2x &= 2 \sinh x \cosh x \\ \cosh 2x &= \cosh^2 x + \sinh^2 x \\ \cosh^2 x &= \frac{\cosh(2x) + 1}{2} \\ \sinh^2 x &= \frac{\cosh(2x) - 1}{2} \end{aligned}$$

The quadratic formula

The solutions to the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$