

ENGR121 Test One

During lecture, 7 April 2017

Name:

ID Number:

Please use the spaces provided in this test booklet next to the questions, to give your answers. Please show all working. You may use the last page for rough working if you need more space. Attempt all questions. The marks for parts of questions are given in square brackets, e.g. [1]. Silent calculators may be used. A table of formulae is provided, you can detach it if you wish.

1. State whether each of the following is true or false [1 mark each]:

(a) $-2.5 \in \mathbb{Z}$	F
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(b) $300 \in \mathbb{R}$	T
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(c) $-2.3 \in \mathbb{N}$	F
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(d) $-2.3153 \in \mathbb{Q}$	T
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(e) $\sqrt{2} \in \mathbb{Q}$	F
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(f) $\mathbb{Z} \subset \mathbb{Q}$	T
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(g) $\mathbb{Q} \cap \mathbb{N} = \mathbb{N}$	T
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2. Simplify where possible the following operations on sets [1 mark each]:

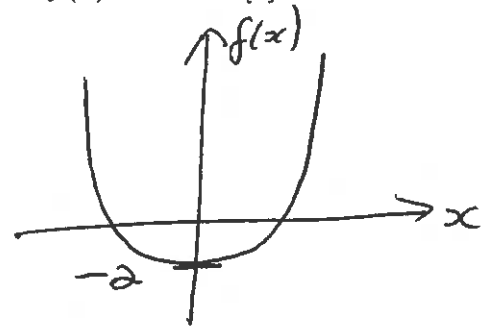
(a) $A \cap \emptyset$	\emptyset
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(b) $B \cup \bar{B}$	E
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(c) $\bar{\phi}$	E
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(d) $(A \cap B) \cup (\bar{A} \cap B)$	B
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3. (a) Sketch a graph of the function $f(x) = x^2 - 2$ [2].



- (b) Is the function $f(x) = x^2 - 7$ one-to-one, one-to-many or many-to-one? [1]

m-to-1

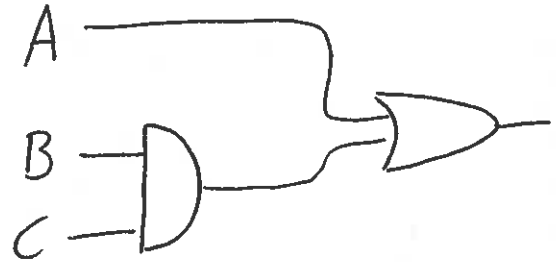
- (c) Write down the inverse of the function $f(x) = 2 + \frac{1}{x}$ [2].

$$\begin{aligned}
 y &= 2 + \frac{1}{x} \\
 y - 2 &= \frac{1}{x} \\
 \frac{1}{y-2} &= x \\
 \Rightarrow f^{-1}(x) &= \frac{1}{x-2}
 \end{aligned}$$

- (d) Write down and simplify the composition $f(g(x))$ if $f(x) = (x + 1)^2$ and $g(x) = 6x - 1$ [2].

$$f(g(x)) = f(6x-1) = (6x-1+1)^2 = 36x^2$$

- (d) Draw a circuit diagram for $A + B \cdot C$ using AND and OR gates [2].



4. (a) Write down the graphical symbol for an OR gate [1].



5. Write the disjunctive normal form for a boolean expression that has the truth table [4]:

A	B	C	X
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	1

$$- ABC$$

$$- \bar{A}BC$$

$$- \bar{A}\bar{B}\bar{C}$$

$$ABC + \bar{A}BC + \bar{A}\bar{B}\bar{C}$$

- (b) Write down the truth table for an OR gate with inputs A and B [2].

A	B	answer
1	1	1
1	0	1
0	1	1
0	0	0

- (c) Construct the truth table for $A \cdot (\bar{B} + \bar{A})$ [2].

A	B	\bar{A}	\bar{B}	$\bar{A} + \bar{B}$	answer
1	1	0	0	0	0
1	0	0	1	1	1
0	1	1	0	1	0
0	0	1	1	1	0

6. (a) Simplify $(x^2x^{-4})^5$ [1].

$$x^{-10}$$

(b) Find the roots of $6x - 1 = 3x - 7$, showing your working, without using a calculator [1].

$$\begin{aligned}6x - 1 &= 3x - 7 \\6x - 3x &= -7 + 1 \\3x &= -6 \\x &= -2\end{aligned}$$

(c) Solve the quadratic equation $x^2 - 2x - 3 = 0$ using any method except a calculator. Show your working [2].

$$\begin{aligned}(x-3)(x+1) &= 0 \\ \Rightarrow x &= 3 \text{ or } -1\end{aligned}$$

(d) Solve the polynomial equation $x^3 = x$ [3].

$$\begin{aligned}x^3 - x &= 0 \\ x(x^2 - 1) &= 0 \\ x(x+1)(x-1) &= 0 \\ x &= -1 \text{ or } 1 \text{ or } 0\end{aligned}$$

(e) Solve the inequality $x^2 > 1$ [2].

$$\begin{aligned}x &> 1 \\ \text{or} \\ x &< -1\end{aligned}$$

7. Consider the events A and B which satisfy the following probabilities: $P(\bar{A}) = 0.3$, $P(B \cap \bar{A}) = 0.1$ and $P(B) = 0.5$. Answer the following questions [1 mark each]:

(a) What is $P(A \cap B)$?

(b) What is $P(A \cup B)$?

(c) What is $P(A)$?

(d) What is $P(A|B)$?

(e) Are A and B independent?
