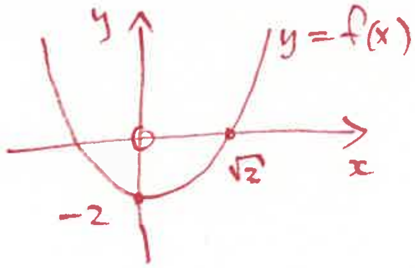
1. State whether each of the following is true or false [1 mark each]:

(a) $-7 \in \mathbb{Z}$	T
(b) $\pi \in \mathbb{R}$	T
(c) $-7.7 \in \mathbb{N}$	F
(d) $-7.777 \in \mathbb{Q}$	T
(e) $\sqrt{4} \in \mathbb{Z}$	T
(f) $\mathbb{Z} \subset \mathbb{Q}$	T
(g) $\mathbb{N} \cap \mathbb{R} = \mathbb{Q}$	F

2. Simplify where possible the following operations on sets:

(a) $A \cap \mathbb{E}$	A	[1]
(b) $B \cup \overline{B}$	$\mathbb{E}$ (or U)	[1]
(c) $A \cup \phi$	A	[1]
(d) $\overline{\phi}$	E	[2]
(e) $A \cup (A \cap \overline{B})$	A	[2]

3. (a) Sketch a graph of the function $f(x) = x^2 - 2$. [2]



(b) Is the function $f(x) = 3x + 1$ one-to-one? [1]

Yes

(c) Is the function $f(x) = x^2 - \pi$ one-to-one? [1]

No

(d) Can a function be many-to-one? [1]

Yes

(e) If $f(x) = 3 - 1/x$, find the inverse function $f^{-1}(x)$. [1]

$y = 3 - \frac{1}{x}$

$\Rightarrow \frac{1}{x} = 3 - y$

$\Rightarrow x = \frac{1}{3-y} = f^{-1}(y)$

$\Rightarrow f^{-1}(x) = \frac{1}{3-x}$

(f) Write down the composition $f(g(x))$ if $f(x) = 3x^2$ and $g(x) = \sin(x-1)$. [1]

$3 \sin^2(x-1)$

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4. (a) Write down the graphical symbol for an AND gate. [1]

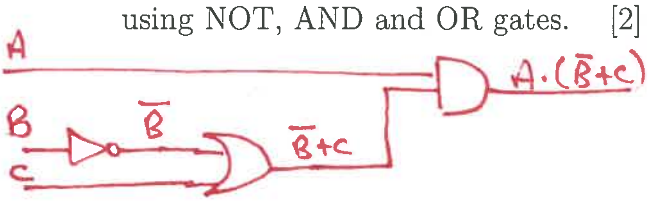
(b) Write down the truth table for an AND gate with inputs A and B. [1]

A	B	A·B
1	1	1
1	0	0
0	1	0
0	0	0

(c) Construct the truth table for $\overline{A}\cdot B + A$. [2]

A	B	$\overline{A}$	$\overline{A}\cdot B$	$\overline{A}\cdot B + A$
1	1	0	0	1
1	0	0	0	1
0	1	1	1	1
0	0	1	0	0

(d) Draw a circuit diagram for $A\cdot(\overline{B}+C)$ using NOT, AND and OR gates. [2]



(e) Simplify the logical expression $A\cdot\overline{A}$ [1]

0

(no mark for ϕ)

5. (a) Write the disjunctive normal form for a boolean expression that has the truth table [3]

A	B	C	X
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

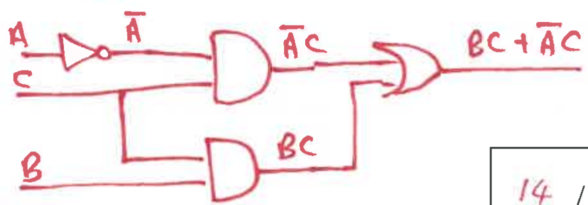
ABC
$+$
$\overline{A}BC$
$+$
$\overline{A}\,\overline{B}C$

(b) Simplify the d.n.f obtained in part (a), if possible. Show all working. [3]

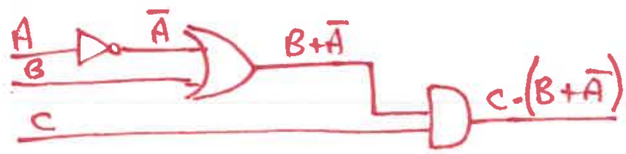
$$ABC + \overline{A}BC + \overline{A}\,\overline{B}C =$$
$$ABC + \overline{A}BC + \overline{A}BC + \overline{A}\,\overline{B}C$$
$$= (A+\overline{A})BC + \overline{A}CB + \overline{A}C\overline{B}$$
$$= 1.BC + \overline{A}C(B+\overline{B})$$
$$= BC + \overline{A}C.1$$
$$= BC + \overline{A}C \quad \text{or } C.(B+\overline{A})$$

(c) Draw a circuit diagram for the boolean expression you obtained in part (b) [1]

for $BC+\overline{A}C$:



or for $C.(B+\overline{A})$:



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6. (a) Simplify $w^3(w^{-2})^2$. [1]

$= w^3 w^{-4} = w^{-1} \left(\text{or } \frac{1}{w}\right)$

(b) Find the roots of $2x - 1 = 3x - 3$, showing your working, without using a calculator. [2]

$2x - 1 = 3x - 3$

$\Rightarrow -1 + 3 = 3x - 2x = x$

$\Rightarrow x = 2$

(c) Solve the quadratic equation $x^2 + 2x - 1 = 0$ using any method except a calculator. Show your working. [2]

$x = \dfrac{-2 \pm \sqrt{4+4}}{2}$

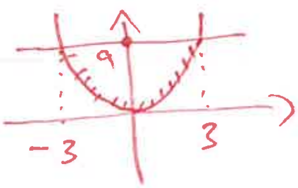
$= -1 \pm \frac{1}{2}\sqrt{8}$

$= -1 \pm \sqrt{2}$

(d) Solve the polynomial equation $x^3 = x^2$. [1]

$x = 0$ or $x = 1$

(e) Solve the inequality $x^2 < 9$ [1]



so $-3 < x < 3$

7. (a) Use the Fundamental Laws of Set Algebra (see Formula Sheet) to prove that, for any sets A and B, [5]

$$A \cap (A \cup B) = A$$

LHS $= (A \cup \phi) \cap (A \cup B)$ identity law

$= A \cup (\phi \cap B)$ distributive law

$= A \cup \phi$

$= A$ identity law

$=$ RHS

(b) Describe the vertical asymptotes of the rational function [2]

$$\frac{x+1}{x^2+2x}.$$

the zeros of x^2+2x, which are $x=0$ and $x=-2$, are the vertical asymptotes.

(c) Find a formula for the oblique asymptote of the rational function [1]

$$\frac{2x^2+x+1}{x+2}.$$

divide

$$\begin{array}{r|l} & 2x \quad -3 \\ \hline x+2 & 2x^2 + x + 1 \\ & 2x^2 + 4x \\ \hline & \quad -3x + 1 \\ & \quad -3x - 6 \\ \hline & \qquad 7 \end{array}$$

so rational function is

$2x - 3 + \dfrac{7}{x+2}$

which $\to 2x-3$ as $|x| \to \infty$. Hence oblique asymptote is $\boxed{y = 2x - 3}$

************* END OF TEST *************

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