ENGR121 - Test 2 - 2019 (50 minutes in class)

Surname: ....

First Name: ....

Student Number: ....

Please use the spaces provided in this test booklet next to the questions, to give your answers. Please show all working. You may use the last page for rough working if you need more space. Attempt all questions. Silent calculators may be used although will be no help.

A table of formulae is provided, you can detach it if you wish.

Question totals, for marking use only

Question	Mark	max
Q. 1		10
Q. 2		10
Total		20

1. Differentiation **(10 marks)**

Find the derivatives y' for the following functions of x. You may use the table of derivatives provided in the formula sheet.

(a) $y = \frac{1}{x^2}$

$y = x^{-2}$

$y' = -2x^{-3}$

(b) $y = -4e^{2x}$

$y' = -4 \times 2e^{2x}$

$y' = -8e^{2x}$

(c) $y = \pi^{2e}$

π^{2e} is constant so

$y' = 0$

(d) $y = \cos(2x)\ln(x)$ (use product rule)

~~[illegible]~~

$y' = -2\sin(2x)\ln(x) + \cos(2x) \times \frac{1}{x}$

$= -2\sin(2x)\ln(x) + \frac{\cos(2x)}{x}$

2. Integration **(10 marks)**

Find the following integrals

(a) $\int x^{-5}dx$ $= \frac{1}{-5+1} x^{-5+1} + c$

$= \frac{-1}{4} x^{-4} + c$

(b) $\int \sin(2x+3)dx$ $= \frac{-1}{2}\cos(2x+3) + c$

(c) $\int \frac{1}{e^{7x}}dx$ $= \int e^{-7x}\, dx$

$= \frac{e^{-7x}}{-7} + c$

(d) $\int (x^3 - e^{2x})\, dx$ $= \frac{x^4}{4} - \frac{1}{2}e^{2x} + c$

(e) $\int_0^1 x^5 dx$ $= \cancel{4}\left[\frac{1}{6}x^6\right]_0^1$

$= \frac{1}{6}1^6 - \left(\frac{1}{6}0^6\right)$

$= \frac{1}{6}$

Note: we do not have a + c in this question since we have limits

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(e) $y = \frac{\cos x}{x^2}$ (use quotient rule)

$$y' = \frac{-\sin(x) \times x^2 - \cos(x) \times 2x}{x^4}$$

$$= \frac{-x\sin(x) - 2\cos(x)}{x^3}$$

(f) $y = \cos(3x^3 + 2)$ (use chain rule)

$$y' = -\sin(3x^3+2) \times 9x^2$$

$$= -9x^2 \sin(3x^3+2)$$

(g) Find all local maxima and minima of $y = 2x^3 - 3x^2$. Which are maxima and which are minima?

$y' = 6x^2 - 6x$

to find max/min set $y' = 0$

$0 = 6x^2 - 6x$

$0 = x^2 - x$

so $x^2 = \cancel{x}$ & $\therefore x = 0$ or 1

when $x = 0$ $y = 0$

$x = 1$ $y = 2 - 3 = -1$

so pt are $(0,0)$ & $(1,-1)$

check y' ~~(x)~~ when $x = -1$, get $y' = 6 + 6 > 0$

check y' when $x = 0.5$ (between 0 & 1) $= \frac{6}{4} - \frac{6}{2} < 0$

check y' when $x = 2$ get $y' = 24 - 18 > 0$

so $x = 0$ is a maximum & $x = 1$ is a minimum

(h) Use implicit differentiation to find $\frac{dy}{dx}$ of $2x = 3x^2 - 4y^3$.

~~$2x =$~~ $2 = 6x - 12y^2 \frac{dy}{dx}$

$12y^2 \frac{dy}{dx} = 6x - 2$

$\frac{dy}{dx} = \frac{6x - 2}{12y^2}$

(f) Using integration by substitution, find the following integral

$$\int x^2 \sin(x^3) dx.$$

set $u = x^3$, then $\frac{du}{dx} = 3x^2$ so $dx = \frac{du}{3x^2}$

$\therefore \int x^2 \sin u \frac{du}{3x^2} = \frac{1}{3} \sin(u) \, du$

$= -\frac{1}{3}\cos(u) + c = \frac{-\cos(x^3)}{3} + c$

(g) Using integration by parts, find the following integral

$$\int x \cos(3x) dx.$$

let $u = x$ $\quad \frac{dv}{dx} = \cos(3x)$

$u' = 1$ $\quad v = \frac{\sin(3x)}{3}$

$\int x \cos(3x) = \frac{x\sin(3x)}{3} - \int 1 \times \frac{\sin(3x)}{3}$

$= \frac{x\sin(3x)}{3} - -\frac{\cos(3x)}{9} + c$

$= \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + c$

(h) Find the average of the function $f(t) = e^{2t}$ across the interval $[-1, 1]$.

$\frac{1}{1--1}\int_{-1}^{1} e^{2t} = \frac{1}{2}\int_{-1}^{1} e^{2t}$

$= \frac{1}{2}\left[e^{2t}\right]_{-1}^{1}$

$= \frac{1}{2}\left(e^{2} - e^{-2}\right)$

again, since the integral has limits, there is no $+c$

SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Formula sheet

Function $f(x)$	*Derivative* $f'(x)$	*Indefinite integral* $\int f(x)\,dx$
k (constant)	0	$kx + c$
x	1	$\frac{1}{2}x^2 + c$
x^2	$2x$	$\frac{1}{3}x^3 + c$
x^n where $n \neq -1$	nx^{n-1}	$\frac{1}{n+1}x^{n+1} + c$
$\frac{1}{x}$ $(= x^{-1})$	$-x^{-2}$ $(= -\frac{1}{x^2})$	$\ln\|x\| + c$
$\sin x$	$\cos x$	$-\cos x + c$
$\sin(ax+b)$	$a\cos(ax+b)$	$-\frac{1}{a}\cos(ax+b) + c$
$\cos x$	$-\sin x$	$\sin x + c$
$\cos(ax+b)$	$-a\sin(ax+b)$	$\frac{1}{a}\sin(ax+b) + c$
$\operatorname{cosec}(ax+b)$	$-a\operatorname{cosec}(ax+b)\cot(ax+b)$	$\frac{1}{a}\ln(\operatorname{cosec}(ax+b) - \cot(ax+b)) + c$
$\sec(ax+b)$	$a\sec(ax+b)\tan(ax+b)$	$\frac{1}{a}\ln(\sec(ax+b) + \tan(ax+b)) + c$
$\tan^{-1}(x)$	$\dfrac{1}{1+x^2}$	$x\tan^{-1}(x) - \frac{1}{2}\ln(x^2+1)$
e^x	e^x	$e^x + c$
e^{kx}	ke^{kx}	$\frac{1}{k}e^{kx} + c$
$\ln x$	$\frac{1}{x}$	$x\ln x - x + c$
$\ln(kx)$	$\frac{1}{x}$	$x\ln(kx) - x + c$

Integration by parts

$$\int u\left(\frac{dv}{dx}\right)dx = uv - \int\left(\frac{du}{dx}\right)v\,dx$$

Derivative Rules:

Product Rule: if $y = uv$, then $y' = u'v + uv'$

Quotient Rule: if $y = u/v$, then

$$y' = \frac{(vu' - uv')}{v^2}$$

Chain Rule: if $y = y(z)$ and $z = z(x)$, then

$$\frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx}$$

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