School of Mathematics and Statistics
Te Kura Mātai Tatauranga

ENGR 123 **Assignment 1**

This assignment is out of 30.

1 *Proofs using truth-tables*

(a) Use a truth-table to show that
$P \vee Q \equiv \neg P \to Q.$ *(2 marks)*

Solution: Construct the truth table below and note that the columns in bold are identical

P	Q	$\mathbf{P \vee Q}$	$\neg P$	$\mathbf{\neg P \to Q}$
0	0	**0**	1	**0**
0	1	**1**	1	**1**
1	0	**1**	0	**1**
1	1	**1**	0	**1**

(b) Construct truth tables for the following propositions, and determine whether each is a tautology, contradiction, or contingent:

(i) $\neg P \to (P \vee Q)$ *(2 marks)*

Solution: Construct the truth table below

P	Q	$\neg P$	$\to$	$(P \vee Q)$
0	0	1	**0**	0
0	1	1	**1**	1
1	0	0	**1**	1
1	1	0	**1**	1
		1	**3**	2

The proposition is contingent since the final (3rd) column contains both ones and zeros.

(ii) $[(Q \wedge R) \to P] \to (\neg P \to \neg R)$ *(3 marks)*

Solution: from the truth table below, we see that the compound proposition is also contingent.

P	Q	R	$((Q \wedge R)$	$\to P)$	$\to$	$(\neg P$	$\to$	$\neg R)$
0	0	0	0	1	**1**	1	1	1
0	0	1	0	1	**0**	1	0	0
0	1	0	0	1	**1**	1	1	1
0	1	1	1	0	**1**	1	0	0
1	0	0	0	1	**1**	0	1	1
1	0	1	0	1	**1**	0	1	0
1	1	0	0	1	**1**	0	1	1
1	1	1	1	1	**1**	0	1	0
			1	*2*	**6**	3	*5*	4

2 *Translations*

(a) Using the propositional variable t to mean *she drinks tea* and c to mean *she drinks coffee* and h to mean *she drinks hot chocolate*, turn the following sentences into compound propositions:

(i) She drinks both coffee and hot chocolate, but not tea. *(1 mark)* **Solution:** $(c \wedge h) \wedge \neg t$

(ii) Drinking hot chocolate implies she drinks tea or coffee, but not both! *(2 marks)* **Solution:** $h \rightarrow (tXORc)$

(b) Using the same variables, turn the following propositions directly into English:

(i) $\neg(t \wedge h)$ *(2 marks)* **Solution:** It isn't the case that she drinks tea and hot chocolate.

(ii) $(t \vee c) \rightarrow h$ *(2 marks)* **Solution:** If she drinks tea or coffee, then she drinks hot chocolate.

3 *Building circuits with NAND-gates*

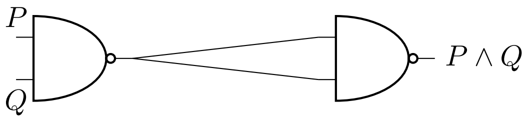
The truth-table for a $NAND$-gate is

P	Q	$\neg(P \wedge Q)$
0	0	1
0	1	1
1	0	1
1	1	0

Construct the following logic gates, using only $NAND$-gates:

(a) AND *(2 marks)*

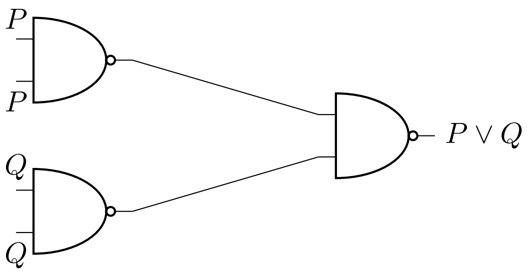
Solution:



Check: $\neg\Big(\neg(P \wedge Q)\Big) \equiv P \wedge Q$

(b) OR *(2 marks)*

Solution:



Check: $\neg(\neg P \wedge \neg Q) \equiv \neg\neg P \vee \neg\neg Q \equiv P \vee Q$

4 *Minesweeper*

In the following 3×3 grid, there is at most 1 mine in each uncovered square, and the number in a square is the number of mines in neighbouring squares (maximum: 8).

Now, four squares have been revealed, and there is no mine in them. Let P_i be the proposition "square S_i has a mine".

S_1	1	0
S_2	4	2
S_3	S_4	S_5

(a) What does the "2" tell you (without considering the "1" and "4")? Write the proposition using notation from class. *(1 mark)*

Solution: $P_4 \wedge P_5$

(b) What does the "1" tell you (without considering the "2" and "4")? Write the proposition using notation from class. *(1 mark)*

Solution: $P_1 \text{xor} P_2$, i.e., $(P_1 \wedge \neg P_2) \vee \neg(P_2 \wedge P_1)$

(c) What does the "4" tell you (without considering the "1" and "2")? Write the proposition using notation from class. *(2 marks)*

Solution: $C = (P_1 \wedge P_2 \wedge P_3 \wedge P_4 \wedge \neg P_5) \vee (\neg P_1 \wedge P_2 \wedge P_3 \wedge P_4 \wedge P_5) \vee (P_1 \wedge \neg P_2 \wedge P_3 \wedge P_4 \wedge P_5) \vee (P_1 \wedge P_2 \wedge P_3 \wedge \neg P_4 \wedge P_5) \vee (P_1 \wedge P_2 \wedge \neg P_3 \wedge P_4 \wedge P_5)$

(d) Write down a compound proposition that expresses the combined information in the "1", "2" and "4". *(1 mark)*

Solution: $(P_4 \wedge P_5) \wedge (P_1 \text{xor} P_2) \wedge C$, where C is the expression above.

(e) Can you decide the truth-values of propositions $P_1, \ldots, P_5$ using the information provided? List all possible solutions. *(2 marks)*

Solution: No. The possible options are

P_1	P_2	P_3	P_4	P_5
1	0	1	1	1
0	1	1	1	1

In particular, we don't know the value for squares "1" and "2".

5 *Add to multiply*

Construct a circuit that multiplies a double-digit binary number by three, using only half-adders, which were described in class. Please use a dotted line for the result digit, and a solid line for the carry digit. *(5 marks)*

		P	Q
×		1	1
T	U	V	W

Solution: Recall that the half-adder adds two one-digit binary numbers, whereas the full-adder adds three one-digit binary numbers.

The computation requires some cross multiplication and carrying of digits