

1 Propositions and quantifiers Let $x, y \in \{2, 3, 4\}$. For the following propositions provide a proof or counterexample to the statements below:

- $P_0(x, y)$: $x < y$
- $P_1(x, y)$: both x and y are even
- $P_2(x, y)$: $x \times y > 7$
- $P_3(x, y)$: x is even and $x + y$ is a multiple of x .
- $P_4(x, y)$: $x \neq y$

(a) for all propositions P in P_1, \dots, P_5 ,
 $[\exists x \forall y P] \rightarrow [\forall y \exists x P]$ (3 marks)

(b) for all propositions P in P_1, \dots, P_5 ,
 $[\forall x \exists y P] \rightarrow [\exists y \forall x P]$ (3 marks)

2 Arguments

(a) Let $A(x, y) =$ “ x agrees with y ”.
Rewrite the following statement and its negation formally. Rewrite the negation in English.

Everybody agrees with someone. (3 marks)

(b) Using A from the previous question, and letting $D(x, y) =$ “ x demurs to y ”, rewrite the following statement.
Somebody demurs to someone who everyone agrees with. (3 marks)

(c) Rewrite the following statement.
Everyone demurs to exactly one person. (3 marks)

3 Proofs

Rewrite the following arguments formally and state which are valid/invalid.

Dictionary Use names for individuals (Mac and Ashley) and the following predicates

LovesFC(a) = “a loves fish and chips”; LikesTS(a) = “a likes tomato sauce”;

Loud(a) = “a is a loud flier”; LikesCold(a) = “a likes the cold”;

CITui(a) = “a is a Chatham island Tui”

Bird(a) = “a is a bird”; Kakapo(a) = “a is a Kakapo”

All lovers of fish and chips like tomato sauce.

- (a) $\frac{\text{Ashley loves fish and chips.}}{\text{Ashley likes tomato sauce.}}$

(3 marks)

No loud fliers like the cold.

- (b) $\frac{\text{All Chatham island Tui are loud fliers.}}{\text{No Chatham island Tui like the cold.}}$

(3 marks)

All kakapo are birds.

- (c) $\frac{\text{Mac is a bird.}}{\text{Mac is a kakapo.}}$

(3 marks)

4 Rearrangement

Using this dictionary

Python(s) = Codes in Python; Indent(s) = Likes indenting code;

Baking(s) = Finds baking agreeable; Dishes(s) = Likes clean dishes;

Tramping(s) = Enjoys tramping; Outdoors(s) = Likes the outdoors.

rewrite the following argument to show that the conclusion follows logically.

That is, reorder the premises, and rewrite statements as “if-then’s” or contrapositives where necessary. (6 marks)

1. If you don’t enjoy tramping, then you will like dirty dishes.
2. Everyone that enjoys tramping likes the outdoors.
3. No one that likes indenting code finds baking disagreeable.
4. Everyone that codes in python likes indenting code.
5. Only those that find baking disagreeable like dirty dishes.

$\frac{\text{Everyone that codes in python likes the outdoors.}}$

1 *Propositions and quantifiers* Given $x, y \in \{2, 3, 4\}$ and the following propositions:

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- $P_1(x, y)$: both x and y are even
- $P_2(x, y)$: $x \times y > 7$
- $P_3(x, y)$: x is even and $x + y$ is a multiple of x .
- $P_4(x, y)$: $x \neq y$

(a) Based on the following example,

P_0	$x = 2$	$x = 3$	$x = 4$
$y = 2$	0	0	0
$y = 3$	1	0	0
$y = 4$	1	1	0

complete the tables below:

(i)

P_1	$x = 2$	$x = 3$	$x = 4$
$y = 2$			
$y = 3$			
$y = 4$			

(ii)

P_2	$x = 2$	$x = 3$	$x = 4$
$y = 2$			
$y = 3$			
$y = 4$			

(b) Complete the following table

	P_0	P_1	P_2	P_3	P_4
$\exists x \exists y P_i$	1	1	1		
$\exists x \forall y P_i$	0	0	1		
$\forall x \exists y P_i$	0	0	1		
$\forall x \forall y P_i$	0	0	0		

(c) Provide a proof or counterexample to the following statements:

- (i) for all $P \in \{P_0, \dots, P_4\}$,
 $[\exists x \forall y P] \rightarrow [\forall x \exists y P]$
- (ii) for all propositions P ,
 $[\exists x \exists y P] \rightarrow [\exists x \forall y P]$

2 Arguments

- (a) Rewrite the following statement and its negation formally. Rewrite the negation in English.

Let $T(x, y) = \text{"}x \text{ trusts } y\text{"}$.

- (i) Everybody trusts somebody.
- (b) Rewrite the following arguments formally. State which are valid/invalid:

- If code has compile errors, it has bugs.
- (i) $\frac{\text{My code has no compile errors.}}{\text{My code has no bugs.}}$
- All logicians spot invalid arguments.
- (ii) $\frac{\text{Some politicians spot invalid arguments.}}{\text{Some politicians are logicians.}}$
- Only logicians spot invalid arguments.
- (iii) $\frac{\text{Some politicians spot invalid arguments.}}{\text{Some politicians are logicians.}}$

3 Rewrite the following arguments to show that the conclusion follows logically. That is, reorder the premises, and rewrite statements as “if-then’s” or contrapositives where necessary.

1. I trust every animal that belongs to me.
 2. Dogs gnaw bones.
 3. I admit no animals into my study unless they beg when told to do so.
 4. All the animals in the yard are mine.
 5. I admit every animal that I trust into my study.
 6. The only animals that beg when told to do so are dogs.
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- All the animals in the yard gnaw bones.