SCHOOL OF MATHEMATICS AND STATISTICS
Te Kura Mātai Tatauranga

ENGR 123 **Tutorial 1**

1 *Proofs using truth tables*

(a) Show that $(P \to Q) \wedge (Q \to P)$ and $P \leftrightarrow Q$ are logically equivalent.

Solution: Construct the truth table below and note that the columns in bold are identical

P	Q	$P \to Q$	$Q \to P$	$(\mathbf{P} \to \mathbf{Q}) \wedge (\mathbf{Q} \to \mathbf{P})$	$\mathbf{P} \leftrightarrow \mathbf{Q}$
0	0	1	1	**1**	**1**
0	1	1	0	**0**	**0**
1	0	0	1	**0**	**0**
1	1	1	1	**1**	**1**

(b) Construct truth tables for the following propositions, and determine whether each is a tautology, contradiction, or contingent:

(i) $Q \leftrightarrow (\neg Q \wedge P)$

Solution: construct the truth table for the expression, and observe that the corresponding column (in bold) contains both zeroes and ones, and so is contingent. That is, whether or not the compound proposition $Q \leftrightarrow (\neg Q \wedge P)$ is true depends on the truth-values of the atomic propositions P and Q.

P	Q	$\neg Q \wedge P$	$\mathbf{Q} \leftrightarrow (\neg \mathbf{Q} \wedge \mathbf{P})$
0	0	0	**1**
0	1	0	**0**
1	0	1	**0**
1	1	0	**0**

(ii) $\neg Q \to (Q \to P)$

Solution: Construct the truth table, and observe that the final column (corresponding to the compound proposition in question) is all ones, and so the proposition is a tautology.

P	Q	$Q \to P$	$\neg Q$	$\neg \mathbf{Q} \to (\mathbf{Q} \to \mathbf{P})$
0	0	1	1	**1**
0	1	0	0	**1**
1	0	1	1	**1**
1	1	1	0	**1**

(iii) $(P \to Q) \vee R$

Solution: Construct the truth-table, and observe from the last column that the proposition is contingent.

P	Q	R	$P \to Q$	$(\mathbf{P} \to \mathbf{Q}) \vee \mathbf{R}$
0	0	0	1	**1**
0	0	1	1	**1**
0	1	0	1	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	1	0	**1**
1	1	0	1	**1**
1	1	1	1	**1**

(iv) $P \to (Q \to R)$ **Solution:** Construct the truth-table, and observe from the last column that the proposition is contingent.

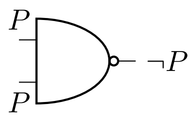
P	Q	R	$Q \to R$	$\mathbf{P} \to (\mathbf{Q} \to \mathbf{R})$
0	0	0	1	**1**
0	0	1	1	**1**
0	1	0	0	**1**
0	1	1	1	**1**
1	0	0	1	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	1	1	**1**

2 *Building circuits with $NAND$-gates*

Construct the following logic gates, using only $NAND$-gates, and check your answers using the "laws of logic". :

(a) NOT

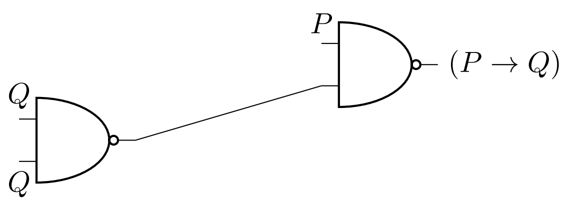
Solution:



Check: $\neg(P \wedge P) \equiv \neg P \vee \neg P \equiv \neg P$

(b) $IMPLIES$

Solution:



Check: $\neg(P \wedge \neg Q) \equiv \neg P \vee \neg\neg Q \equiv \neg P \vee Q \equiv P \to Q$

3 *Knights and Knaves.*

Jeff is either a knight or a knave. Knights always tell the truth; knaves always lie.

Someone asks Jeff: "Are you a knight or a knave?"

He replies: "If I'm a knight, then I code in binary".

(a) Introduce the propositions

$$P = \text{"Jeff is a knight"}$$
$$Q = \text{"Jeff codes in binary"}$$

Translate Jeffs reply into propositional logic.

Solution: $P \to Q$

(b) Use a truth-table to show that Jeff must be a knight (and that he therefore codes in binary).

Solution: Consider the truth table for $P \to Q$

P	Q	$P \to Q$
0	0	1
0	1	1
1	0	0
1	1	1

There are two possible scenarios.

Scenario #1: *If Jeff is a knight* then (obviously) he is a knight and (therefore) he also codes binary because (i) he says so and (ii) he always tells the truth.

Scenario #2: *If Jeff is a knave* then he *lied* and so we know that $P \to Q$ must be false. It follows that we are in row #3 of the truth table (since that is the only row for which $P \to Q$) is false.

However, according to row #3, it must be that $P = 1$ or in other words Jeff is a knight! But this is a *contradiction.* Therefore, only scenario #1 is allowed.

4 *Adders*

Construct a circuit that adds two double-digit binary numbers, using the half- and full-adder described in class.

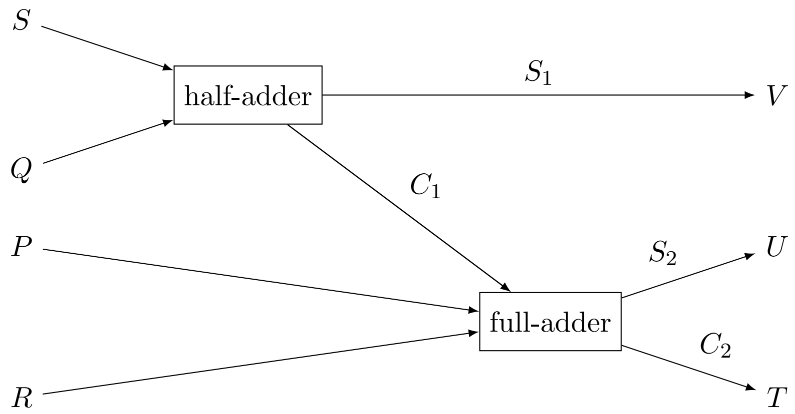
	P	Q
+	R	S
T	U	V

Solution: Recall that the half-adder adds two one-digit binary numbers, whereas the full-adder adds three one-digit binary numbers.

We compute

	P	Q
+	R	S
T	U	V

as follows


S
Q
P
R
half-adder
full-adder
S1
C1
S2
C2
V
U
T