School of Mathematics and Statistics
Te Kura Mātai Tatauranga

ENGR 123 **Tutorial 2**

1 *Propositions and quantifiers* Given $x, y \in \{2, 3, 4\}$ and the following propositions:

- $P_0(x, y)$: $x < y$
- $P_1(x, y)$: both x and y are even
- $P_2(x, y)$: $x \times y > 7$
- $P_3(x, y)$: x is even and $x + y$ is a multiple of x.
- $P_4(x, y)$: $x \neq y$

(a) Based on the following example,

P_0	$x = 2$	$x = 3$	$x = 4$
$y = 2$	0	0	0
$y = 3$	1	0	0
$y = 4$	1	1	0

complete the tables below:

(i)

P_1	$x = 2$	$x = 3$	$x = 4$
$y = 2$			
$y = 3$			
$y = 4$			

SOLN

P_1	$x = 2$	$x = 3$	$x = 4$
$y = 2$	1	0	1
$y = 3$	0	0	0
$y = 4$	1	0	1

(ii)

P_2	$x = 2$	$x = 3$	$x = 4$
$y = 2$			
$y = 3$			
$y = 4$			

SOLN

P_2	$x = 2$	$x = 3$	$x = 4$
$y = 2$	0	0	1
$y = 3$	0	1	1
$y = 4$	1	1	1

(b) Complete the following table

	P_0	P_1	P_2	P_3	P_4
$\exists x\, \exists y\, P_i$	1	1	1		
$\exists x\, \forall y\, P_i$	0	0	1		
$\forall x\, \exists y\, P_i$	0	0	1		
$\forall x\, \forall y\, P_i$	0	0	0		

Solution:

	P_3	P_4
$\exists x\, \exists y\, P_i$	1	1
$\exists x\, \forall y\, P_i$	0	0
$\forall x\, \exists y\, P_i$	0	1
$\forall x\, \forall y\, P_i$	0	0

(c) Provide a proof or counterexample to the following statements:

(i) for all $P \in \{P_0, \ldots, P_4\}$,
$[\exists x\, \forall y\, P] \to [\forall x\, \exists y\, P]$

Solution: The statement is TRUE. Only column P_2 has the second rom ($\exists x\, \forall y$) being true, but in that column the third row is also true.

(ii) for all propositions P,
$[\exists x\, \exists y\, P] \to [\exists x\, \forall y\, P]$

Solution: The statement is FALSE. Columns P_1, P_2, P_4 and P_5 are all counter-examples.

Note that $\exists x\ \exists y\ P_1$ is TRUE but $\exists x\ \forall y\ P_1$ is FALSE, as can be seen from the first and second rows of the truth table respectively.

2 *Arguments*

(a) Rewrite the following statement and its negation formally. Rewrite the negation in English.

Let $T(x, y) =$"x trusts y".

(i) Everybody trusts somebody.

Solution:

"Everybody trusts somebody" is the same as $\forall x\ \exists y\ T(x, y)$.
Negation: "Someone trusts no-one" is the same as $\exists x\ \forall y\ \neg T(x, y)$

(b) Rewrite the following arguments formally. State which are valid/invalid:

(i)

If code has compile errors, it has bugs.
My code has no compile errors.
―――――――――――――――――
My code has no bugs.

Solution:

$\forall x[\text{Error}(x) \to \text{Bug}(x)]$
$\neg\text{Error}(c)$
―――――――――――――――――
$\neg\text{Bug}(c)$

Invalid

(ii)

All logicians spot invalid arguments.
Some politicians spot invalid arguments.
―――――――――――――――――
Some politicians are logicians.

Solution:

$\forall x[\text{Logician}(x) \to \text{Spot}(x)]$
$\exists x[\text{Politician}(x) \wedge \text{Spot}(x)]$
―――――――――――――――――
$\exists x[\text{Politician}(x) \wedge \text{Logician}(x)]$

Invalid

(iii)

Only logicians spot invalid arguments.
Some politicians spot invalid arguments.
―――――――――――――――――
Some politicians are logicians.

Solution:

$\forall x[\text{Spot}(x) \to \text{Logician}(x)]$
$\exists x[\text{Politician}(x) \wedge \text{Spot}(x)]$
―――――――――――――――――
$\exists x\ \text{Politician}(x) \wedge \text{Logician}(x)$

Valid

3 Rewrite the following arguments to show that the conclusion follows logically. That is, reorder the premises, and rewrite statements as "if-then's" or contrapositives where necessary.

1. I trust every animal that belongs to me.
2. Dogs gnaw bones.
3. I admit no animals into my study unless they beg when told to do so.
4. All the animals in the yard are mine.
5. I admit every animal that I trust into my study.
6. The only animals that beg when told to do so are dogs.

All the animals in the yard gnaw bones.

Solution:

4.	$\forall x[\text{Yard}(x) \rightarrow \text{Belong}(x)]$
1.	$\forall x[\text{Belong}(x) \rightarrow \text{Trust}(x)]$
5.	$\forall x[\text{Trust}(x) \rightarrow \text{Admit}(x)]$
3.	$\forall x[\text{Admit}(x) \rightarrow \text{Beg}(x)]$
6.	$\forall x[\text{Beg}(x) \rightarrow \text{Dog}(x)]$
2.	$\forall x[\text{Dog}(x) \rightarrow \text{Gnaw}(x)]$
	$\forall x[\text{Yard}(x) \rightarrow \text{Gnaw}(x)]$