School of Mathematics and Statistics
Te Kura Mātai Tatauranga

ENGR 123 **Tutorial 4**

1 *Truth tables*
Construct truth tables for the following propositions, and determine whether each is a tautology, contradiction, or contingent:

(a) $\neg Q \wedge (P \to Q)$

(b) $(P \to (Q \vee R)) \to (\neg Q \to (Q \to P))$

2 *Translations*

- Using the propositional variable e to mean *she majors in engineering* and m to mean *she majors in maths*, and d to mean *she is doing a double major* at university, turn the following sentences into compound propositions:
  (a) She majors in either Engineering or Maths at university, but not both
  (b) She majors in neither Engineering nor Maths, but is working on a double major at university *(1 mark)*
- Using the same variables, turn the following propositions into English:
  (a) $e \to \neg d$ *(1 mark)*
  (b) $\neg d \wedge m$ *(1 mark)*

3 *Arguments*

(a) Rewrite the following statement and it negation formally. Rewrite the negation in English.

Let $T(x, y) =$"x trusts y".

(i) Somebody trusts everybody.

(b) Rewrite the following arguments formally. State which are valid/invalid:

(i)
All kiwis ride bicycles.
All bicycle riders eat curry.

All kiwis eat curry.

(ii)
All kokakos sing.
Sam sings.

Sam is a kokako.

(iii)
All kakapo are birds.
Max is a bird.

Max is a kakapo.

(c) Rewrite the following argument to show that the conclusion follows logically. That is, reorder the premises, and rewrite statements as "if-then's" or contrapositives where necessary.

1. No students who can't do maths are in ENGR123.
2. All hard-working students can think logically.
3. Only ENGR123 students code.
4. Only students who can't do maths are lazy.

Only students who think logically can code.

4 *Relations*
Let C be the set of all cities in New Zealand. Determine whether the relation R on C is reflexive, symmetric, antisymmmetric and/or transitive where (a, b) in R iff

(a) there is no road between city a and city b

(b) everyone who has visited city a has also visited city b

(c) there is a highway with interchanges to both city a and city b

(d) there is at least one road between city a and city b

In each case, for the properties that don't hold, explain why e.g. it isn't symmetric because ...

5 *Equivalence relations*
Let R be the relation on the set of people who have visited a particular city such that xRy iff person x and person y left the city using the same road. Show that R is an equivalence relation.

6 *Partitions*
Which of the following are partitions of the set of all students at Victoria?

(a) the set of students whose family lives in Wellington, the set of students whose family lives elsewhere in New Zealand, and the set of students whose family lives overseas.

(b) the set of students who can speak a foreign language, the set of students who can play a musical instrument, the set of students who can't do either.

(c) the set of students who are New Zealanders, and the set of students who are not.

(d) the set of students who study engineering, the set of students who study science, the set of students who study arts.

7 *Proofs*

Claim: for all $n \in \mathbb{N}$, if $2n+1$ is a multiple of 3 then n^2+1 is a multiple of 3.

Incorrect Proof: by contrapositive. Assume that $2n+1$ is not a multiple of 3. It follows that

- If $n = 3k+1$ for $k \in \mathbb{N}$ then $n^2+1 = 9k^2+6k+2$ is not a multiple of 3.
- If $n = 3k+2$ for $k \in \mathbb{N}$ then $n^2+1 = 9k^2+12k+5$ is not a multiple of 3.
- If $n = 3k+3$ for $k \in \mathbb{N}$ then $n^2+1 = 9k^2+18k+10$ is not a multiple of 3.

In all cases, we have shown that n^2+1 is not a multiple of 3, and so we have proved the claim.

(a) Provide a counterexample to the claim.

(b) Explain clearly where the "proof" made logical errors.

8 *More proofs*

Use a *proof by contradiction* to show that if x is rational and y is irrational, then $x+y$ is irrational.

9 *Induction*

Use induction to show that $4n < 2^n$ for all $n \geq 5$.