ENGR 123 - Tutorial 5 solutions for 21–25 September 2020

Topic: DISCRETE RANDOM VARIABLES

1. A box contains 3 red marbles and 5 blue marbles. Two marbles are taken at random *without* replacement, and the random variable X is the number of blue marbles obtained.

   (a) Write down the sample space, i.e., the set of all possible outcomes.

   (b) Write down the set of possible *values* of the random variable X and draw up a table showing the *probability distribution* of X, i.e., $P(X = x_i)$ for each value x_i.

   (c) Find $\mathsf{E}(X), \mathsf{E}(X^2), \mathrm{Var}(X)$.

Solution:

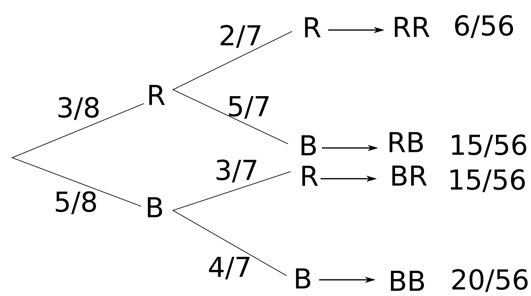

Fig. 1: Tutorial Q1.- Tree diagram

(a) $S = \{RR, RB, BR, BB\}$ or you could say $S = \{0B, 1B, 2B\}$

(b) From the tree diagram, we have

x	$P(X = x)$
0	6/56
1	30/56
2	20/56

(c)

$$\mathsf{E}(X) = \sum_{\text{all } x} xP(X = x) = 0 \times P(X = 0) + 1 \times P(X = 1) + 2 \times P(X = 2)$$
$$= \frac{30}{56} + \frac{40}{56} = \frac{70}{56}$$

(d)

$$\mathsf{E}(X^2) = \sum_{\text{all } x} x^2 P(X = x) = 0 \times P(X = 0) + 1 \times P(X = 1) + 2^2 \times P(X = 2)$$
$$= \frac{30}{56} + \frac{80}{56} = \frac{110}{56}$$

Hence,

$$\mathrm{Var}(X) = \mathsf{E}(X^2) - \mathsf{E}(X)^2 = \frac{110}{56} - \left(\frac{70}{56}\right)^2 = 0.402.$$

2. For each of the scenarios below, select:

$$\begin{aligned} A &: \text{ variable is binomial (approximately)} \\ B &: \text{ variable is Poisson (approximately)} \\ C &: \text{ variable is neither} \end{aligned}$$

In any case, explain your choice!

(a) Interview 100 people, 50 couples and ask "Should we introduce paid paternity leave?" X counts the number of people saying "YES".

(b) The number of errors in a 20ms transmission.

Solution:

(a) This is a binomial type situation. Check 4 assumptions of the binomial model:

- binary outcomes: YES or NO ✓
- fixed number of trials($n = 100$) ✓
- constant probability ?
- independent trial ?

However, since the interview involves couples, it is very unlikely to have independent responses, so C would be the BEST choice.

(b) It is hard to tell. If there are a fixed number of bits in 20ms and bits are independent, we could go with binomial model A. Otherwise, if errors arise randomly and singly in a period of 20ms transmission, we could go with Poisson model B. If the errors fail the usual assumptions badly - by being strongly dependent for example, then we might need to go for C.

3. A set of 10 microprocessing chips from a large population are randomly selected and are independently tested to determine if they are acceptable for a certain application. Eighty percent of chips in the population are acceptable.

(a) What is the probability that there are exactly 8 chips acceptable?

(b) What is the probability of having more than 8 chips acceptable?

(c) What is the probability of having less than 8 chips acceptable?

Solution: Let X = number of acceptable chips in the batch of 10 chips, then $X \sim Bin(10, 0.8)$.

(a) $P(X = 8) = \binom{10}{8} \times 0.8^8 \times (1 - 0.8)^2 = \dfrac{10!}{8!2!} \times 0.8^8 \times 0.2^2 = 0.302$

(b)

$$\begin{aligned}P(X>8) &= P(X=9)+P(X=10)\\ &= \binom{10}{9}\times 0.8^9\times(1-0.8)^1+\binom{10}{10}\times 0.8^{10}\times(1-0.8)^0\\ &= \frac{10!}{9!1!}\times 0.8^9\times 0.2^1+\times 0.8^{10}=0.268+0.107=0.375\end{aligned}$$

(c) $P(X<8)=1-P(X\geq 8)=1-[P(X=8)+P(X>8)]=1-[0.302+0.375]=0.323.$

4. A quality engineer is in charge of testing whether or not 90% of the DVD players produced by his company conform to specifications. To do this, the engineer randomly selected a batch of 12 DVD players from each day's production. The day's production is acceptable provided no more than 1 DVD player failed to meet specifications. Otherwise, the entire day's production has to be tested.

   (a) What is the probability that the engineer incorrectly passes a day's production as acceptable if only 80% of the day's DVD player actually conform to specification?

   (b) What is the probability that the engineer unnecessarily requires the entire's day production to be tested if in fact 90% of the DVD players conform to specifications?

   Solution:

   (a) Let X = number of DVD players in the sample of 12 that fails to meet specification. We want to calculate $P(X\leq 1)$ where $X\sim Bin(12,0.2)$, which is

$$\begin{aligned}P(X\leq 1) &= P(X=0)+P(X=1)\\ &= \binom{12}{0}0.2^0(1-0.2)^{12}+\binom{12}{1}0.2^1(1-0.2)^{11}\\ &= 0.069+0.206=0.275.\end{aligned}$$

   (b) We now want $P(X>1)$ where $X\sim Bin(12,0.1)$

$$\begin{aligned}P(X>1) &= 1-P(X\leq 1)=1-P(X=0)-P(X=1)\\ &= 1-\binom{12}{0}0.1^0(1-0.1)^{12}-\binom{12}{1}0.1^1(1-0.1)^{11}\\ &= 1-0.659=0.341.\end{aligned}$$

5. Records show that on average, three emergency calls per day are received by a service engineer. What is the probability that on a particular day: (a) three; (b) two; (c) four calls will be received?

   Solution: Denote by X the number of calls received on a particular day, $X\sim Poi(3)$.

   (a) $P(X=3)=e^{-3}\dfrac{3^3}{3!}=0.224.$

(b) $P(X = 2) = e^{-3}\dfrac{3^2}{2!} = 0.224.$

(c) $P(X = 4) = e^{-3}\dfrac{3^4}{4!} = 0.168.$