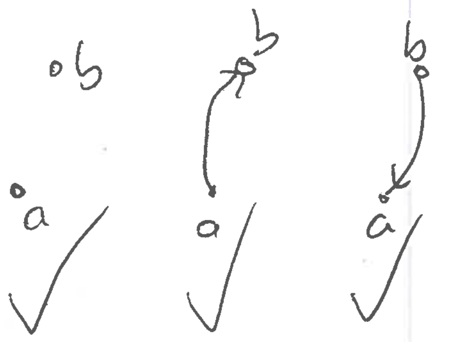
①

Engr ~~#~~123

Antisymmetric

$aRb \wedge bRa \rightarrow a = b$

$a \neq b \rightarrow \neg(aRb \wedge bRa)$



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Proof by Contrapositive

$P \rightarrow C \equiv \neg C \rightarrow \neg P$

Use when good info is in the conclusion

②

Prove

If $x^2$ is even, $P$

Then $x$ is even. $C$

$\neg C$. Suppose $x$ is not even

$x$ is odd

for some $k \in \mathbb{Z}$, $x = 2k+1$

⋮

for some $m \in \mathbb{Z}$, $x^2 = 2m+1$

so $x^2$ is odd

hence $x^2$ is not even. $\neg P$

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Examples of relations from A to B

Empty relation $\{\} = \emptyset$

③

$A \times B$ the full relation

$R$ from $A$ to $B$, $S$ from $B$ to $C$

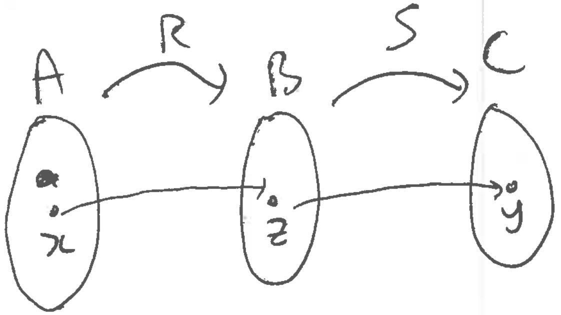
**Inverse relation**

$$R^{-1} = \{(a,b) : \overbrace{(b,a) \in R}^{bRa}\}$$

**Composition of two relations**

$$RS = \{(\underset{x,y}{\cancel{a,b}}) : \exists \underset{z}{\cancel{c}} \in B(\underset{x}{\cancel{a}}R\underset{z}{\cancel{c}} \wedge \underset{z}{\cancel{c}}R\underset{y}{\cancel{b}})\}$$

from $A$ to $C$




$f : A \to B$

④

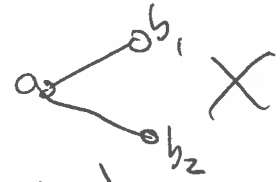
$$f : A \to B \;\; \begin{cases} f(a) = b \\ (a,b) \in f \\ afb \end{cases}$$

1) all inputs are accepted

$$\forall a \in A\ \exists b \in B\ (afb)$$

2) each input has only one ~~input~~ output

$$\forall a \in A\ \forall b_1, b_2 \in B$$




$$(afb_1 \wedge afb_2 \to b_1 = b_2)$$

$$\equiv (\underset{b_1 \neq b_2}{\cancel{a \neq b}} \to \neg(afb_1 \wedge afb_2))$$