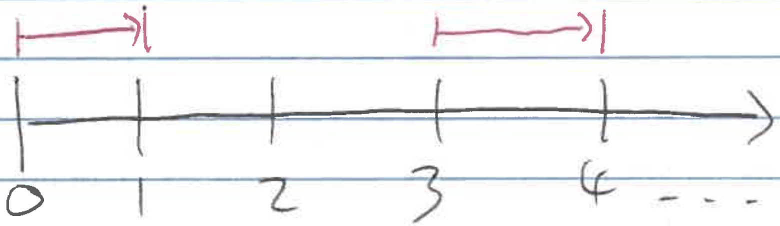
Engr 123 L12

Induction and recursion.

Mathematical Induction works on numbers

, in fact, the natural numbers.



Goal: try to show something $P(n)$ is true, for all natural numbers

1) Check $P(0)$. Base case.
2) Assume $P(k)$ is true. Induction hypothesis
3) Show/Prove that $P(k+1)$ is true Induction step.

Example

$$0+1+2+\cdots+n=\frac{n(n+1)}{2}, n\geq 0$$

Base Case: $n=0$

LHS $=0$ $\qquad$ RHS $=\frac{0(0+1)}{2}=\frac{0(1)}{2}=\frac{0}{2}=0$

LHS = RHS ✓

Induction hypothesis $n=k$

$$0+1+2+\cdots+k=\frac{k(k+1)}{2}$$

Induction step: $n=k+1$

Goal: $0+1+2+\cdots+(k+1)=\frac{(k+1)(k+1+1)}{2}$

$$\text{LHS}=0+1+2+\cdots+(k+1)$$

$$=0+1+2+\cdots+k+(k+1)$$

$$=\frac{k(k+1)}{2}+(k+1) \quad \text{by IH}$$

$$\vdots$$

$$=\frac{(k+1)(k+2)}{2}=\text{RHS}$$

Example 2

$$0^2+1^2+2^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$n \geq 0$

Base Case: $n=0$

$LHS = 0^2 = 0$ $\qquad$ $RHS = \frac{0(0+1)(2(0)+1)}{6} = 0$

$LHS = RHS$

Induction Hypothesis: $n=k$

$$0^2+1^2+2^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6}$$

Induction Step: $n=k+1$

Want to prove that

$$0^2+1^2+2^2+\cdots+(k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$LHS = 0^2+1^2+\cdots+(k+1)^2$$

$$= 0^2+1^2+\cdots+k^2+(k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \quad \text{by IH}$$