ENGR 123 L13

Plan: Factorial def^n
Recursion example
Proof that $\sqrt{2}$ is irrational
" " $n < 2^n$ where $n \geq 0$

Factorial $m!$ – say m factorial
m bang

$$(n+1)! = (n+1) \cdot n!$$
$$1! = 1$$

e.g. $2! = 2 \times 1!$
$= 2 \times 1$
$3! = 3 \times 2!$
$= 3 \times 2 \times 1$
$4! = 4 \times 3!$
$= 4 \times 3 \times 2 \times 1$
$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 10 \times 9!$

Algorithm 1: MinMax(S)

```
input: a set S = (s_1, ..., s_n) of n numbers, where n is a power of two
if n = 2 then
    if s_1 < s_2 then
        return [s_1, s_2]
    else
        return [s_2, s_1]
else
    [sml_1, big_1] = MinMax(s_1, ..., s_{n/2})
    [sml_2, big_2] = MinMax(s_{n/2+1}, ..., s_n)
    big = Max(big_1, big_2)
    sml = Min(sml_1, sml_2)
    return [sml, big]
output: The min and max elements of S
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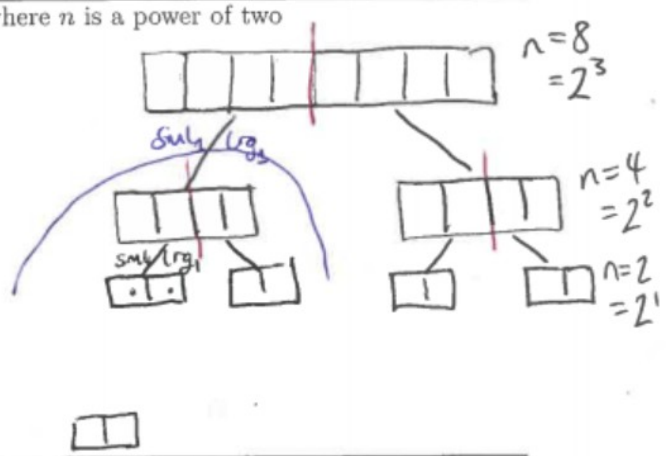

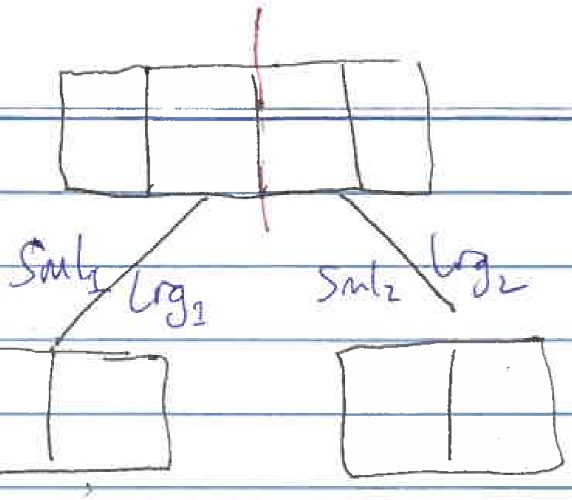

Looks legit

Third type of proof - Direct proof

$P \to C$ - Contrapositive

$\equiv \neg C \to \neg P$

Direct	Contrapositive	Contradiction
P	$\neg C$	$P \wedge \neg C$
$\to$ ⋮	⋮	⋮
$\to$ ⋮	⋮	⋮
$\to C$	$\neg P$	$R \wedge \neg R$

~~New~~ New type - proof by ~~contradict~~ contradiction

Proof that $\sqrt{2}$ is irrational

Contrary statement - $\sqrt{2}$ is rational

Suppose $\sqrt{2} = \frac{p}{q}$ p & q have no common factors (*)

$\rightarrow \sqrt{2}\,q = p$
$\rightarrow 2q^2 = p^2$
$\rightarrow p^2$ is even
$\rightarrow p$ is even (earlier lecture)
$\rightarrow p = 2k$
$\rightarrow 2q^2 = (2k)^2$
$= 4k^2$
$\rightarrow q^2 = 2k^2$
$\rightarrow q^2$ is even
$\rightarrow q$ is even

This contradicts the starred line

So $\sqrt{2}$ <u>must</u> be irrational.