ENGR123 Test One
50 minutes. 5 questions.
40 marks total
19 August 2015

Name:

ID Number:

Please use the spaces provided in this test booklet to give your answers. Attempt all questions. Blank pages for rough work are provided toward the end. A formula sheet is on the last page.

1. Complete the following truth table **[10 marks]**

P	Q	R	$\neg(P \land Q) \lor (R \to P)$	$P \to (R \to (P \lor Q))$
0	0	0	1 0 1 1	1 1 0
0	0	1	1 0 1 0	1 0 0
0	1	0	1 0 1 1	1 1 1
0	1	1	1 0 1 0	1 1 1
1	0	0	1 0 1 1	1 1 1
1	0	1	1 0 1 1	1 1 1
1	1	0	0 1 1 1	1 1 1
1	1	1	0 1 1 1	1 1 1
			2 1 4 3	3 2 1

2. Let $F(p) =$ "printer p is broken"; $B(p) =$ "printer p is busy"; $Q(j) =$ "job j is queued" and $L(j) =$ "job j is lost". **[8 marks]**

Consider the statements:

- $\exists p \; (F(p) \wedge B(p)) \rightarrow \exists j \; L(j)$
- $\forall p \; B(p) \rightarrow \exists j \; Q(j)$

(a) Translate the statements into English.

(b) Negate the statements (symbolically, i.e. not in English).

a) If some busy printer is broken, then some job is lost.

If every printer is busy, then some job is queued

b) $\exists p (F(p) \wedge B(p)) \wedge \forall j \neg L(j)$

$\forall p \; B(p) \wedge \forall j \neg Q(j)$

3. (a) What is a valid argument? **[2 marks]**

(b) Rewrite the following arguments using predicates and *explain* which are valid. **[6 marks]**

i. All convertible cars are fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.

ii. Everyone who eats granola for breakfast is healthy. Lucy is not healthy. Therefore, Lucy does not eat granola for breakfast.

(a) An argument that whenever the premisses are true, the conclusion necessarily follows.

b) (i) $C(x) \to F(x)$

$\neg C(\text{Isaac})$

$\neg F(\text{Isaac})$

Not valid

(ii) $G(x) \to H(x)$

$\neg H(\text{Lucy})$

$\neg G(\text{Lucy})$

Valid, contrapositive of first statement.

4. Let F be the set of Facebook users. For each of the following relations, check:
(i) reflexivity (ii) symmetry (iii) transitivity and (iv) anti-symmetry. **[8 marks]**

(a) $R \subset F \times F$ is the relation aRb iff there is a photo that both a and b have liked.

(b) $S \subset F \times F$ is the relation aSb iff a has as many or more friends than b.

(a)(i) No, someone who has liked <u>no</u> photos.

(ii) Yes, if a & b have liked a photo, so have b & a!

(iii) No, liked <u>different</u> photos.

(iv) No, two people liking the same photo.

(b)(i) Yes, x has as many friends as x.

(ii) No, one has more friends than the other.

(iii) Yes, $\#F(x) \geq \#F(y) \wedge \#F(y) \geq \#F(z)$

$\longrightarrow \#F(x) \geq \#F(z)$

(iv) No, same number of friends.

5. A chain letter works as follows. Stage one: an initial person sends the letter to 2 friends. Stage 2: each of whom sends it to 2 new friends. Stage 3++: and so on.

(a) Give a formula for the *total* number of letters that have been mailed *by the n^{th} stage* of the chain. **[2 marks]**

(b) Prove your formula is correct using induction. **[4 marks]**

(a) $N(1) = 2 = 4-2 = 2^2-2 = 2(2^1-1)$

$N(2) = 2N(1)+2 = 6 = 8-2 = 2^3-2 = 2(2^2-1)$

$N(3) = 2N(2)+2 = 14 = 16-2 = 2^4-2 = 2(2^3-1)$

$\vdots$

$N(n) = \; ? = 2^{n+1}-2 = 2(2^n-1)\,?$

(b) B.C. $\boxed{n=1}$ Def^n says 2 letter

Formula says $2(2^1-1) = 2$ ✓

I.H. $\boxed{n=k}$ Assume

$2(2^k-1)$ letters have been sent.

I.S. $\boxed{n=k+1}$ Send a letter to each friend

$N(k+1) = 2 + N(k) + N(k)$ ← friends onwards

$= 2 + 2^{k+1} - 2 + 2^{k+1} - 2 = 2\cdot 2^{k+1} - 2$

$= 2(2^{k+1}-1)$ ✓

Use this page for rough working if needed.

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List of laws:

1. Double negation: $P \equiv \neg\neg P$
2. De Morgan's laws:
   $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$
   $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
3. $P \to Q \equiv \neg P \vee Q$
4. Commutative laws:
   $P \wedge Q \equiv Q \wedge P$
   $P \vee Q \equiv Q \vee P$
5. Idempotent laws:
   $P \wedge P \equiv P$
   $P \vee P \equiv P$
6. Distributive laws:
   $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
   $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
7. Associative laws:
   $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
   $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
8. Contrapositive: $(P \to Q) \equiv (\neg Q \to \neg P)$
9. Tautology: if $\mathbb{T}$ is a tautology, then
   $P \vee \mathbb{T} \equiv \mathbb{T}$
   $P \wedge \mathbb{T} \equiv P$
10. Contradiction: if $\mathbb{F}$ is a contradiction, then
    $P \vee \mathbb{F} \equiv P$
    $P \wedge \mathbb{F} \equiv \mathbb{F}$