

ENGR123 Test Two
50 minutes. 7 questions.
40 marks total
5th October 2018

Surname:
First names:
ID Number:

Please use the spaces provided in this test booklet to give your answers. Attempt all questions. Blank pages for rough work are provided toward the end. A formula sheet is on the last two pages.

1. Complete the following truth table

[8 marks]

P	Q	R	$\neg P \leftrightarrow (Q \wedge R)$	$P \rightarrow (Q \rightarrow (\neg R \wedge P))$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

2. Consider the following jumbled argument:

- (1) All my sons are slim;
- (2) No child of mine is healthy who takes no exercise;
- (3) All gluttons, who are children of mine, are fat;
- (4) No daughter of mine takes any exercise.

- (a) Rewrite each statement using predicates. [4 marks]
- (b) Derive a conclusion, and order the statements so that the conclusion follows logically from the premises. [4 marks]

3. Determine the truth values of the following statements, where the variables are people, and $\text{happy}(a, b)$ means that they can live happily together. Provide a brief explanation in each case.
[4 marks]

(a) $\forall n \exists m \text{ happy}(m, n)$

(b) $\exists m \forall n \text{ happy}(m, n)$

4. What is the negation of $\exists m \forall n \text{ happy}(m, n)$?

[2 marks]

5. (a) What properties must a relation satisfy to be an equivalence relation? **[3 marks]**

(b) Let $R \subset \mathbb{Z} \times \mathbb{Z}$ be the relation on integers given by

$$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x + y \text{ is even}\}$$

i. Is R a partial order? Explain why or why not. **[2 marks]**

ii. Is R a function? Explain why or why not. **[2 marks]**

6. Prove by contraposition that if the product of two natural numbers is greater than 100, then at least one of them is greater than or equal to 10. [5 marks]

7. Prove by contradiction that the sum of two positive numbers is positive.

[5 marks]

List of laws of logic

1. Double negation: $P \equiv \neg\neg P$
2. De Morgan's laws:
 $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$
 $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
3. $P \rightarrow Q \equiv \neg P \vee Q$
4. Commutative laws:
 $P \wedge Q \equiv Q \wedge P$
 $P \vee Q \equiv Q \vee P$
5. Idempotent laws:
 $P \wedge P \equiv P$
 $P \vee P \equiv P$
6. Distributive laws:
 $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
 $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
7. Associative laws:
 $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
 $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
8. Contrapositive: $(P \rightarrow Q) \equiv (\neg Q \rightarrow \neg P)$
9. Tautology: if \mathbb{T} is a tautology, then
 $P \vee \mathbb{T} \equiv \mathbb{T}$
 $P \wedge \mathbb{T} \equiv P$
10. Contradiction: if \mathbb{F} is a contradiction, then
 $P \vee \mathbb{F} \equiv P$
 $P \wedge \mathbb{F} \equiv \mathbb{F}$

Some rules of inference

- *Modus ponens.*

$$\frac{P \quad P \rightarrow Q}{Q}$$

- *Modus tollens.*

$$\frac{P \rightarrow Q \quad \neg Q}{\neg P}$$

- *Or-elimination.*

$$\frac{P \vee Q \quad \neg P}{Q}$$

- *And-elimination.*

$$\frac{P \wedge Q}{P}$$

- *Transitivity.*

$$\frac{P \rightarrow Q \quad Q \rightarrow R}{P \rightarrow R}$$

- *Or-introduction.*

$$\frac{P}{P \vee Q}$$

- *Contrapositive.*

$$\frac{P \rightarrow Q}{\neg Q \rightarrow \neg P}$$

- *Implies-introduction.*

$$\frac{Q}{P \rightarrow Q}$$