ENGR123 Test One
45 minutes. 6 questions.
30 marks total
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Please use the spaces provided in this test booklet to give your answers. Attempt all questions. Blank pages for rough work are provided toward the end. A formula sheet is on the last two pages.

1. Complete the following truth table **[6 marks]**

P	Q	R	$P \implies \neg(Q \vee \neg R)$	$(P \wedge Q) \iff (R \implies Q)$
0	0	0	1 0 1 1	0 0 1
0	0	1	1 1 0 0	0 1 0
0	1	0	1 0 1 1	0 0 1
0	1	1	1 0 1 0	0 0 1
1	0	0	0 0 1 1	0 0 1
1	0	1	1 1 0 0	0 1 0
1	1	0	0 0 1 1	1 1 1
1	1	1	0 0 1 0	1 1 1
			4 3 2 1	1 3 2

2. Consider the following (short) jumbled argument about birds, using the predicate P for parrot, G for gigantic and K for Kea:

   (1) All parrots are gigantic;
   (2) Nothing gigantic is fearful;
   (3) All kea are parrots.

   (a) Rewrite each statement using predicates. **[3 marks]**

   (b) State the contrapositive version of (2). **[2 marks]**

   (c) Derive a conclusion, and order all the statements so that the conclusion follows logically from the premises. **[2 marks]**

a)

(1) $\forall x\,(P(x) \to G(x))$

(2) $\forall y\,(G(\overset{y}{\cancel{x}}) \to \neg F(\overset{y}{\cancel{x}}))$ OR $\forall y\,(F(\overset{y}{\cancel{x}}) \to \neg G(\overset{y}{\cancel{x}}))$

(3) $\forall z\,(K(z) \to P(z))$

b) alterative in (2)

c)

(3)

(2) Should be 3, 2, 1

(1)

$\forall a\,(K(a) \to \neg F(a))$

No kea are fearful.

3. Determine the truth values of the following statements, provide a brief explanation in each case. **[4 marks]**

   (a) $(\exists a \in \mathbb{N})\ (\forall b \in \mathbb{N})\ (a^2 > b)$

   (b) $(\forall b \in \mathbb{N})\ (\exists a \in \mathbb{N})\ (a^2 > b)$

(a) False. Suppose $b = a^2 + 1$. Clearly $a^2 > a^2 + 1$ is false. So no a exists that works for <u>all</u> b.

(b) True. In general, let $a = b + 1$. $(b+1)^2 > b$ is true when $b^2 + b + 1 > 0$ which is true for all $b \in \mathbb{R}$.

4. What is the negation of **[2 marks]**

$$(\exists a \in \mathbb{N})\ (\forall b \in \mathbb{N})\ (a^2 > b)$$

$$\neg (\exists a \in \mathbb{N})(\forall b \in \mathbb{N})(a^2 > b)$$
$$\equiv (\forall a \in \mathbb{N}) \neg (\forall b \in \mathbb{N})(a^2 > b)$$
$$\equiv (\forall a \in \mathbb{N})(\exists b \in \mathbb{N}) \neg (a^2 > b)$$
$$\equiv (\forall a \in \mathbb{N})(\exists b \in \mathbb{N})(a^2 \leq b)$$

5. (a) What properties must a relation satisfy to be a partial order? **[3 marks]**

(b) Let R be the relation on the set of movie actors, where $(a, b) \in R$ iff they have acted in a movie together.

Show that R is reflexive and symmetric, but not transitive. **[3 marks]**

(c) Explain why $\{1, 2\}$, $\{2, 3, 4\}$, $\{4, 5, 6\}$ is not a partition of $\{1, 2, 3, 4, 5, 6\}$ **[2 marks]**

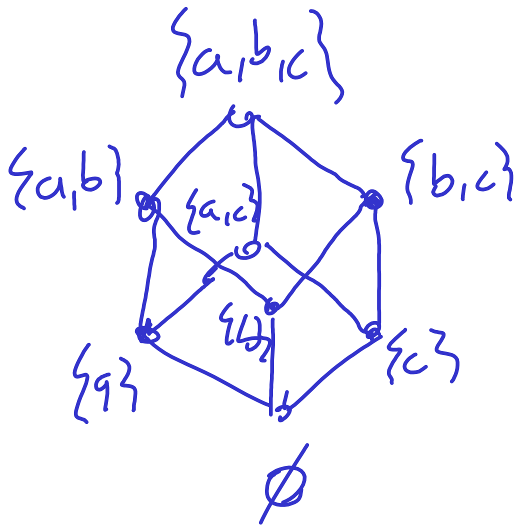
(a) Reflexive, Antisymmetric & Transitive.

(b) Reflexive: movie actors have all acted in a movie with themself.
Symmetric: If a & b have acted together, so have b & a!
Not transitive: Could be different movies.

(c) The element 2 appears in two cells $\{1,2\}$ and $\{2,3,4\}$ when it should appear in precisely one cell.

6. Let $X = \mathcal{P}(\{a, b, c\})$ be the powerset of $\{a, b, c\}$.

Draw a Hasse diagram of the partial order R on X, where $(A, B) \in R$ iff $A \subseteq B$. **[3 marks]**



Rough working page

List of laws of logic

1. Double negation: $P \equiv \neg\neg P$
2. De Morgan's laws:
   $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$
   $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
3. $P \to Q \equiv \neg P \vee Q$
4. Commutative laws:
   $P \wedge Q \equiv Q \wedge P$
   $P \vee Q \equiv Q \vee P$
5. Idempotent laws:
   $P \wedge P \equiv P$
   $P \vee P \equiv P$
6. Distributive laws:
   $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
   $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
7. Associative laws:
   $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
   $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
8. Contrapositive: $(P \to Q) \equiv (\neg Q \to \neg P)$
9. Tautology: if $\mathbb{T}$ is a tautology, then
   $P \vee \mathbb{T} \equiv \mathbb{T}$
   $P \wedge \mathbb{T} \equiv P$
10. Contradiction: if $\mathbb{F}$ is a contradiction, then
    $P \vee \mathbb{F} \equiv P$
    $P \wedge \mathbb{F} \equiv \mathbb{F}$

Some rules of inference

- *Modus ponens.*

$$\begin{array}{l} P \\ P \to Q \\ \hline Q \end{array}$$

- *Modus tollens.*

$$\begin{array}{l} P \to Q \\ \neg Q \\ \hline \neg P \end{array}$$

- *Transitivity.*

$$\begin{array}{l} P \to Q \\ Q \to R \\ \hline P \to R \end{array}$$

- *Contrapositive.*

$$\begin{array}{l} P \to Q \\ \hline \neg Q \to \neg P \end{array}$$

Quantifiers

- *Universal* All P's are Q's

$$\forall x(P(x) \to Q(x))$$

- *Universal* Some P's are Q's

$$\exists x(P(x) \land Q(x))$$

- *Negating quantifiers*

$$\neg\forall x[R(x)] \equiv \exists x[\neg R(x)]$$

$$\neg\exists x[R(x)] \equiv \forall x[\neg R(x)]$$