

**ENGR123 Test One**  
50 minutes. 5 questions.  
**40 marks total**  
9th August 2021

<b>Family name:</b>
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<b>First names:</b>
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<b>ID Number:</b>
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This is a *closed book* test.

Attempt all questions.

A formula sheet is included over the page.

Questions start on page 3.

Question	Label	Out of	Marks
1	Propositions	10	
2	Proofs	4	
3	Relations	15	
4	Induction	6	
5	Eulerian walks	5	

## List of laws of logic

1. Double negation:  $P \equiv \neg\neg P$
2. De Morgan's laws:  
 $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$   
 $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
3.  $P \rightarrow Q \equiv \neg P \vee Q$
4. Commutative laws:  
 $P \wedge Q \equiv Q \wedge P$   
 $P \vee Q \equiv Q \vee P$
5. Idempotent laws:  
 $P \wedge P \equiv P$   
 $P \vee P \equiv P$
6. Distributive laws:  
 $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$   
 $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
7. Associative laws:  
 $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$   
 $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
8. Contrapositive:  $(P \rightarrow Q) \equiv (\neg Q \rightarrow \neg P)$
9. Tautology: if  $\mathbb{T}$  is a tautology, then  
 $P \vee \mathbb{T} \equiv \mathbb{T}$   
 $P \wedge \mathbb{T} \equiv P$
10. Contradiction: if  $\mathbb{F}$  is a contradiction, then  
 $P \vee \mathbb{F} \equiv P$   
 $P \wedge \mathbb{F} \equiv \mathbb{F}$

## Some rules of inference

- *Modus ponens.*

$$\frac{P \quad P \rightarrow Q}{Q}$$

- *Modus tollens.*

$$\frac{P \rightarrow Q \quad \neg Q}{\neg P}$$

- *Transitivity.*

$$\frac{P \rightarrow Q \quad Q \rightarrow R}{P \rightarrow R}$$

- *Contrapositive.*

$$\frac{P \rightarrow Q}{\neg Q \rightarrow \neg P}$$

## Quantifiers

- *Universal* All P's are Q's

$$\forall x(P(x) \rightarrow Q(x))$$

- *Existential* Some P's are Q's

$$\exists x(P(x) \wedge Q(x))$$

- *Negating quantifiers*

$$\neg\forall x[R(x)] \equiv \exists x[\neg R(x)]$$

$$\neg\exists x[R(x)] \equiv \forall x[\neg R(x)]$$