ENGR123 Test One
50 minutes. 5 questions.
40 marks total
9th August 2021

Family name:
First names:
ID Number:

This is a *closed book* test.

Attempt all questions.

A formula sheet is included over the page.

Questions start on page 3.

Question	Label	Out of	Marks
1	Propositions	10	
2	Proofs	4	
3	Relations	15	
4	Induction	6	
5	Eulerian walks	5	

List of laws of logic

1. Double negation: $P \equiv \neg\neg P$
2. De Morgan's laws:
   $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$
   $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
3. $P \to Q \equiv \neg P \vee Q$
4. Commutative laws:
   $P \wedge Q \equiv Q \wedge P$
   $P \vee Q \equiv Q \vee P$
5. Idempotent laws:
   $P \wedge P \equiv P$
   $P \vee P \equiv P$
6. Distributive laws:
   $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
   $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
7. Associative laws:
   $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
   $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
8. Contrapositive: $(P \to Q) \equiv (\neg Q \to \neg P)$
9. Tautology: if $\mathbb{T}$ is a tautology, then
   $P \vee \mathbb{T} \equiv \mathbb{T}$
   $P \wedge \mathbb{T} \equiv P$
10. Contradiction: if $\mathbb{F}$ is a contradiction, then
    $P \vee \mathbb{F} \equiv P$
    $P \wedge \mathbb{F} \equiv \mathbb{F}$

Some rules of inference

- *Modus ponens.*

$$\frac{P \qquad P \to Q}{Q}$$

- *Modus tollens.*

$$\frac{P \to Q \qquad \neg Q}{\neg P}$$

- *Transitivity.*

$$\frac{P \to Q \qquad Q \to R}{P \to R}$$

- *Contrapositive.*

$$\frac{P \to Q}{\neg Q \to \neg P}$$

Quantifiers

- *Universal* All P's are Q's

$$\forall x(P(x) \to Q(x))$$

- *Existential* Some P's are Q's

$$\exists x(P(x) \wedge Q(x))$$

- *Negating quantifiers*

$$\neg\forall x[R(x)] \equiv \exists x[\neg R(x)]$$

$$\neg\exists x[R(x)] \equiv \forall x[\neg R(x)]$$

1. Propositions and predicates

   (a) Complete the following truth table **[6 marks]**

P	Q	R	$(Q \to R) \vee (P \to R)$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

   (b) Decide whether $(Q \to R) \vee (P \to R)$ is a tautology, a contradiction or contingent (with reasoning) **[2 marks]**

   (c) Explain why $(\forall m \in \mathbb{Z})(\exists n \in \mathbb{Z})(m \text{ divides } n)$ is true. **[2 marks]**

2. Proofs.

   Give a proof *by contrapositive* that, if the product of two numbers is even, then one of the numbers is even i.e.

   $$\text{if } mn \text{ is even, then either } m \text{ is even or } n \text{ is even}$$

   [4 marks]

3. Relations

(a) i. Suppose R is a relation on $A = \{0, 1, 2, 3, 4\}$ defined by

$$aRb \text{ iff } a \times b \text{ is even}$$

Check whether R is reflexive, symmetric, antisymmetric and/or transitive.

[4 marks]

In each case where the relation doesn't have the property, give a short reason why or an example.

ii. A. What is required for a relation S (on a set A) to be a partial order? **[2 marks]**

B. Suppose S is the divisibility relation on $\{1, 2, 3, 5, 6, 10, 15, 30\}$ (the factors of 30). Draw the Hasse diagram of S. **[4 marks]**

(b) The relation $h = \{(1, 2), (2, 1), (3, 2)\}$ is a function from $\{1, 2, 3\}$ to $\{1, 2\}$.

Determine whether h is one-to-one or onto. Make sure to state your decision clearly, and give some reasoning. **[3 marks]**

(c) Suppose f is a 1-1 function from A to B, and g is a 1-1 function from B to C. Prove that $g \circ f$ is 1-1 (from A to C). **[2 marks]**

4. Induction.

   Use induction to show that

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\cdots\left(1-\frac{1}{n}\right)=\frac{1}{n},\ \text{for } n\geq 2$$

[6 marks]

5. Eulerian walks

(a) Check that the graph G_1 (in figure 1 on page 9) is Eulerian (listing the degree's will suffice). **[1 mark]**

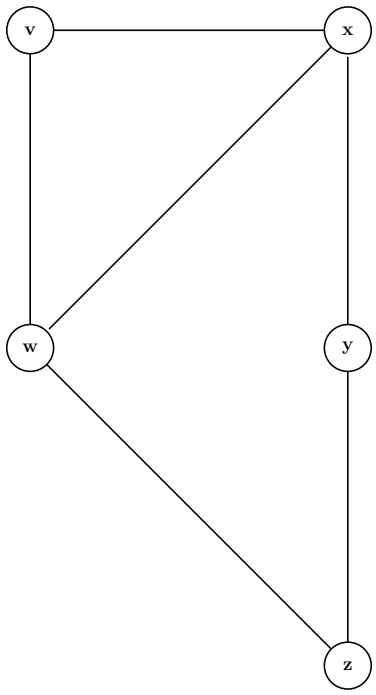

Figure 1: G_1

(b) Use Fleury's algorithm to find an eulerian walk, under the following restrictions.

- Start at vertex w.
- Where-ever possible, choose the vertex that comes first alphabetically e.g. choose y over z.
- Make sure to indicate bridges, taken or not!

[4 marks]