ENGR 101
Engineering Technology

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Victoria University of Wellington

Week 4 Lecture 6b

- Main topics
  - Introduction to Engineering Technology
  - Number system
  - Logic Gates
  - Boolean Algebra

- Course web page:

  https://ecs.wgtn.ac.nz/Courses/XMUT101_2021T1/

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Exercise 6.1

Use the Boolean rules to simplify the following expressions:

(Note: $\overline{A}$ = A' or $\overline{C}$ = C')

(a) $X = A\,B\,C + \overline{A}\,B + A\,B\,\overline{C}$

(b) $X = \overline{A}\,B\,\overline{C} + A\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

(c) $AB + \overline{A}\,C + B\,C = AB + \overline{A}\,C$

(d) $(A + B)\,(\overline{A} + C)\,(B + C) = (A + B)\,(\overline{A} + C)$

Boolean Algebra Laws

	Name of Law	Properties
1.	Identity Law	A+0=A; A+1=1; A.0=0; A.1=A
2.	Commutative Law	A.B = B.A; A+B = B+A
3.	Associative Law	A.(B.C) = A.B.C; A+(B+C) = A+B+C
4.	Idempotent Law	A.A = A; A+A = A
5.	Double Negative Law	A'' = A
6.	Complement Law	A.A' = 0; A+A' = 1
7.	Law of Union	A+1 = A; A+0 = A
8.	DeMorgan's Theorem	(AB)' = A'+B'; (A+B)' = A'.B'
9.	Distributive Law	A.(B+C) = (A.B) + (A.C); A+(BC) = (A+B).(A+C)
10.	Absorption Law	A.(A+B) = A; A+(A.B) = A
11.	Common Identities Law	A.(A'+B) = AB; A+(A'B) = A+B

Exercise 6.1(a) $x = ABC + \overline{A}B + AB\overline{C}$ = ABC + A'B + ABC'

	Boolean Law used
x = ABC + A'B + ABC'	
Common element	

Exercise 6.1(a) $x = ABC + \overline{A}B + AB\overline{C}$

	Boolean Law used
x = ABC + A'B + ABC'	3) Distributive Law A.(B+C) = (A.B) + (A.C);
Common term	

Exercise 6.1(a) $x = ABC + \overline{A}B + AB\overline{C}$

	Boolean Law used
x = ABC + A'B + ABC'	3) Distributive Law A.(B+C) = (A.B) + (A.C)
= AB(C + C') + A'B	

Parentheses (or normal brackets)

Exercise 6.1(a) $x = ABC + \overline{A}B + AB\overline{C}$

	Boolean Law used
x = ABC + A'B + ABC'	3) Distributive Law A.(B+C) = (A.B) + (A.C)
= AB(C + C') + A'B	
= AB(1) + A'B	6) Complement Law A+A' = 1

Exercise 6.1(a) $x = ABC + \overline{A}B + AB\overline{C}$

	Boolean Law used
x = ABC + A'B + ABC'	3) Distributive Law A.(B+C) = (A.B) + (A.C)
= AB(C + C') + A'B	
= AB(1) + A'B	6) Complement Law A+A' = 1
= B(A + A')	3) Distributive Law A.(B+C) = (A.B) + (A.C)
= B	

Exercise 6.1

(a) $X = ABC + \overline{A}B + AB\overline{C}$ $= B$

(b) $X = \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

(c) $AB + \overline{A}C + BC = AB + \overline{A}C$

(d) $(A + B)(\overline{A} + C)(B + C) = (A + B)(\overline{A} + C)$

Exercise 6.1(b) $x = \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

x = A’BC’ + AB’C’ + A’B’C’ + A’B’C’	Boolean Law used

Exercise 6.1(b) $x = \overline{A}\,B\,\overline{C} + A\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

x = A’BC’ + AB’C’ + A’B’C’ + A’B’C’	Boolean Law used
= A’BC’ + AB’C’ + A’B’C’ + A’B’C’	4. Idempotent Law A + A = A

Exercise 6.1(b) $x = \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

x = A'BC' + AB'C' + A'B'C' + A'B'C'	Boolean Law used
= A'BC' + AB'C' + A'B'C' + A'B'C'	4. Idempotent Law A + A = A
= A'BC' + AB'C' + A'B'C'	
= A'BC' + B'C'(A + A')	9. Distributive Law A.(B+C) = (A.B) + (A.C)

Exercise 6.1(b) $x = \overline{A} B \overline{C} + A \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C}$

x = A'BC' + AB'C' + A'B'C' + A'B'C'	Boolean Law used
= A'BC' + AB'C' + A'B'C' + A'B'C'	4. Idempotent Law A + A = A
= A'BC' + AB'C' + A'B'C'	
= A'BC' + B'C'(A + A')	9. Distributive Law A.(B+C) = (A.B) + (A.C)
= A'BC' + B'C'(1)	6. Complement Law A+A' = 1

Exercise 6.1(b) $x = \overline{A}\,B\,\overline{C} + A\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

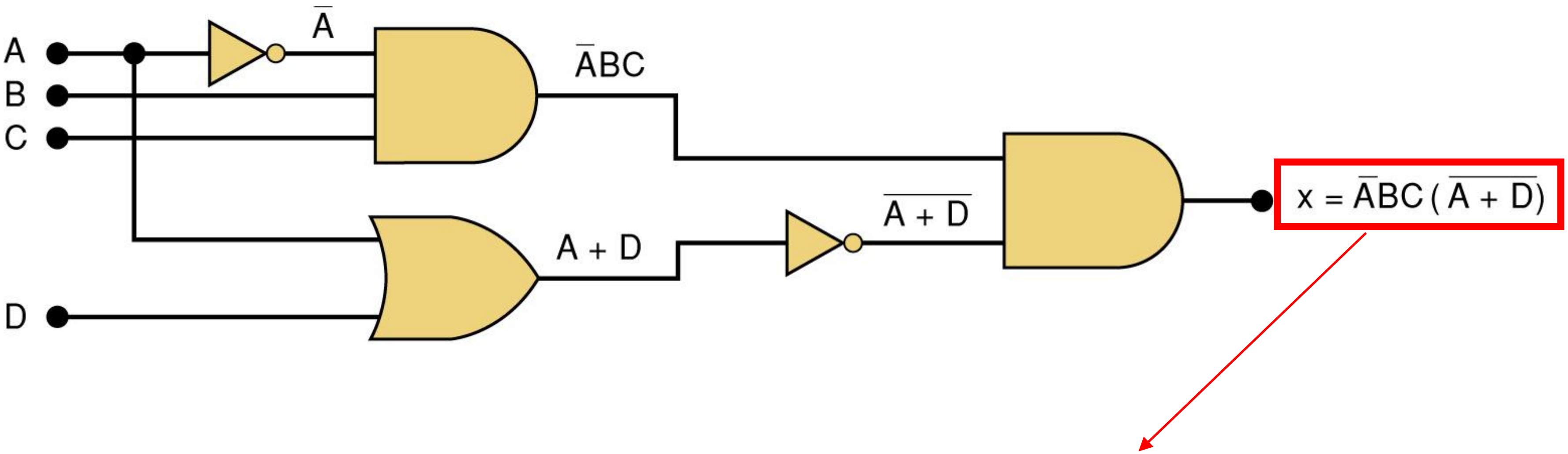
x = A'BC' + AB'C' + A'B'C' + A'B'C'	Boolean Law used	
= A'BC' + AB'C' + A'B'C' + A'B'C'	4. Idempotent Law	A + A = A
= A'BC' + AB'C' + A'B'C'		
= A'BC' + B'C'(A + A')	9. Distributive Law	A.(B+C) = (A.B) + (A.C)
= A'BC' + B'C'(1)	6. Complement Law	A+A' = 1
= A'BC' + B'C'		
= C' (A'B + B')	9. Distributive Law	A.(B+C) = (A.B) + (A.C)

Exercise 6.1(b) $x = \overline{A}B\overline{C} + A\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C}$

x = A'BC' + AB'C' + A'B'C' + A'B'C'	Boolean Law used
= A'BC' + AB'C' + A'B'C' + A'B'C'	4. Idempotent Law A + A = A
= A'BC' + AB'C' + A'B'C'	
= A'BC' + B'C'(A + A')	9. Distributive Law A.(B+C) = (A.B) + (A.C)
= A'BC' + B'C'(1)	6. Complement Law A+A' = 1
= A'BC' + B'C'	
= C' (A'B + B')	9. Distributive Law A.(B+C) = (A.B) + (A.C)
= A'C' + B'C'	11. Common Identities Law A.(A'+B) = AB; A+(A'B) = A+B

Describing Logic Circuits Algebraically

Example 1



X = A’BC (A+D)’

Minimize X = A’BC (A+D)’ using Boolean Laws

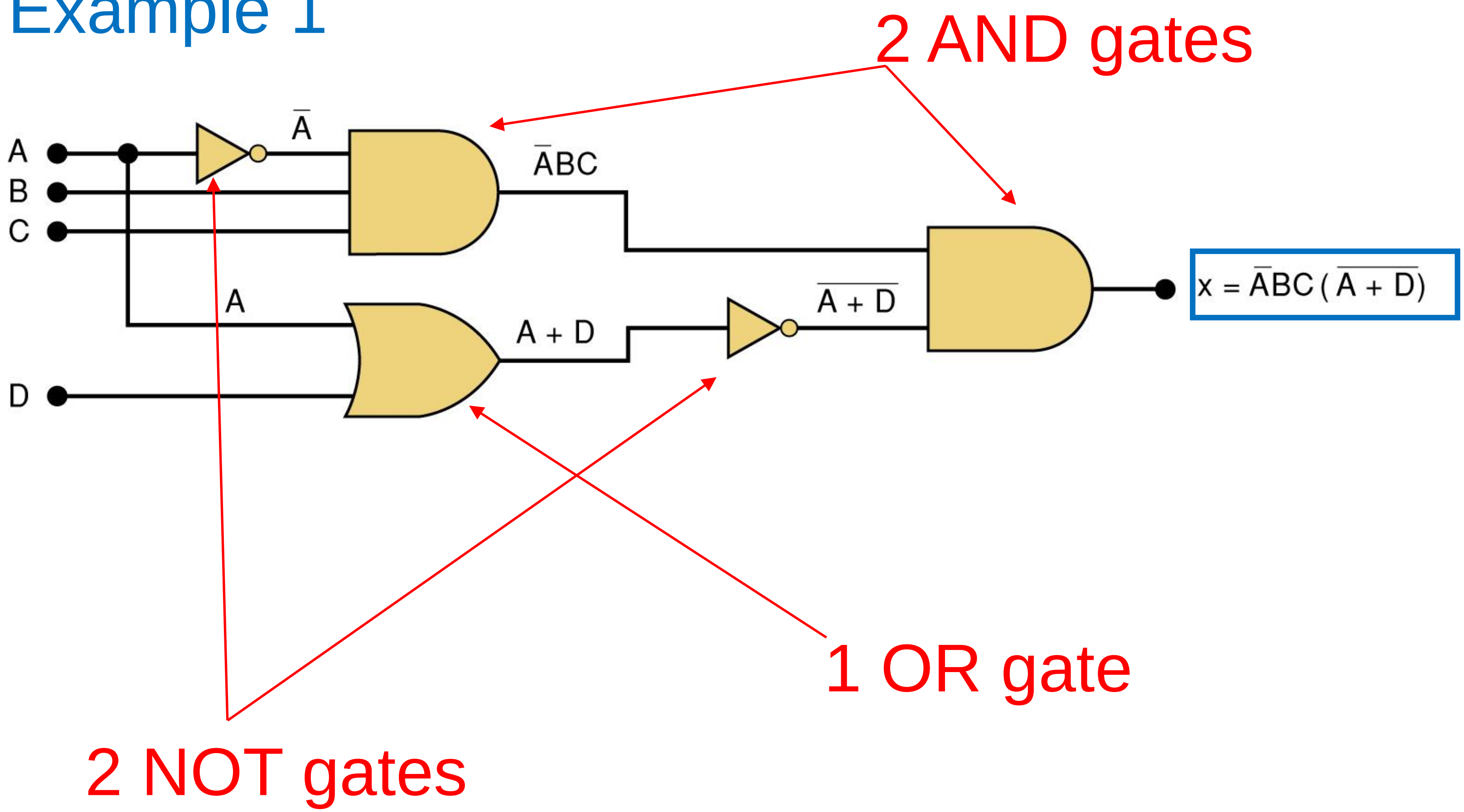
	Name of Law	Properties
1.	Identity Law	A+0=A; A+1=1; A.0=0; A.1=A
2.	Commutative Law	A.B = B.A; A+B = B+A
3.	Associative Law	A.(B.C) = A.B.C; A+(B+C) = A+B+C
4.	Idempotent Law	A.A = A; A+A = A
5.	Double Negative Law	A’’ = A
6.	Complement Law	A.A’ = 0; A+A’ = 1
7.	Law of Union	A+1 = A; A+0 = A
8.	DeMorgan’s Theorem	(AB)’ = A’+B’; (A+B)’ = A’.B’
9.	Distributive Law	A.(B+C) = (A.B) + (A.C); A+(BC) = (A+B).(A+C)
10.	Absorption Law	A.(A+B) = A; A+(A.B) = A
11.	Common Identities Law	A.(A’+B) = AB; A+(A’B) = A+B

Minimize X = A'BC (A+D)' using Boolean Laws

X = A'BC (A+D)'	Boolean Law used
= A'BC (A'D')	DeMorgan's Theorems (AB)' = A'+B'; (A+B)' = A'.B'
= A'(BCD')	Idempotent Law A.A = A
= A'BCD	Associative Law A.(B.C) = A.B.C

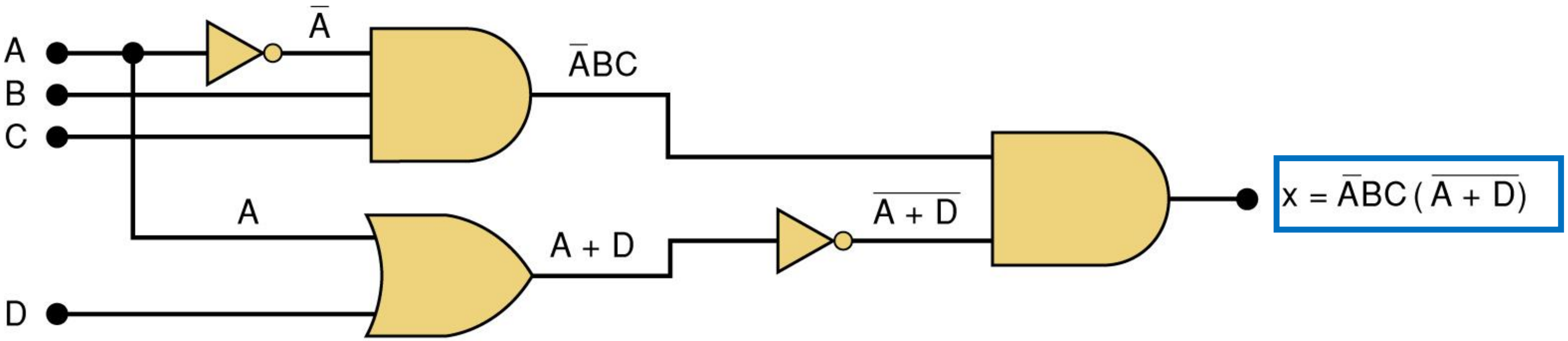
Describing Logic Circuits Algebraically

Example 1

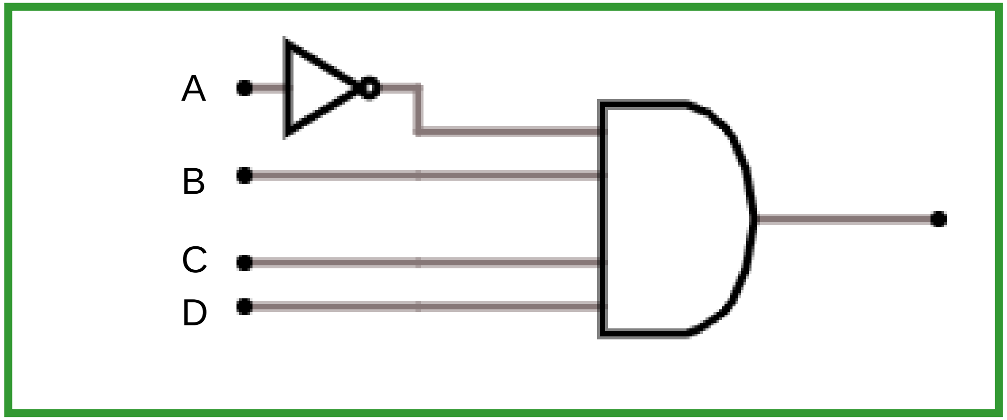


Describing Logic Circuits Algebraically

Example 1



Simplified expression: A’BCD



Exercise 6.1

Use the Boolean rules to simplify the following expressions:

(Note: $\overline{A}$ = A’ or $\overline{C}$ = C’)

(a) $X = ABC + \overline{A}B + AB\overline{C}$

(b) $X = \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

(c) $AB + \overline{A}C + BC = AB + \overline{A}C$

(d) $(A + B)(\overline{A} + C)(B + C) = (A + B)(\overline{A} + C)$

Describing Logic Circuits Algebraically

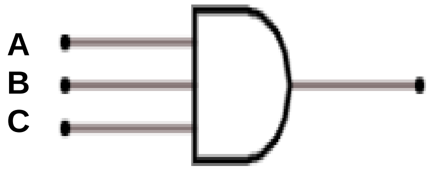
Exercise 6.1 (a): $x = ABC + \overline{A}B + AB\overline{C}$

$= ABC + A'B + ABC'$

Describing Logic Circuits Algebraically

Exercise 6.1 (a): $x = ABC + \overline{A}B + AB\overline{C}$

$= ABC + A'B + ABC'$

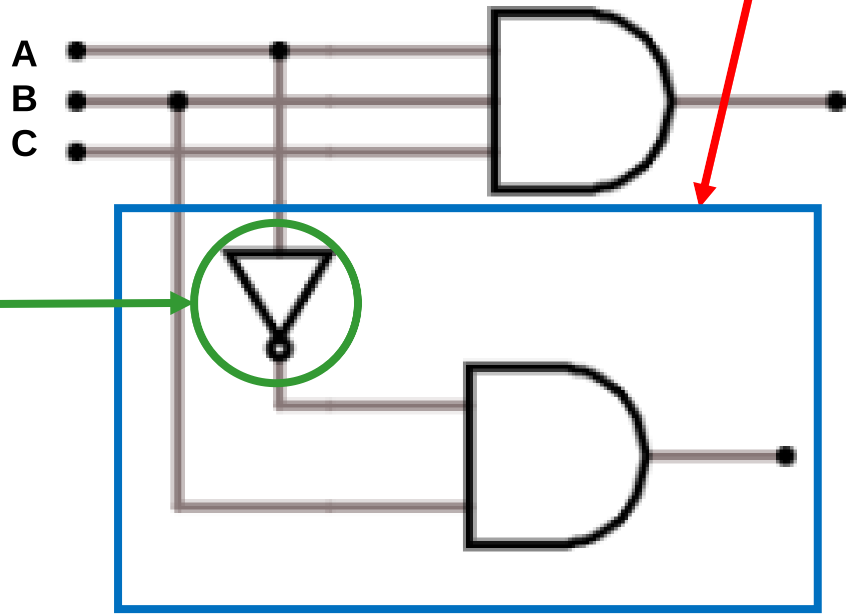


Describing Logic Circuits Algebraically

Exercise 6.1 (a): $x = ABC + \overline{A}B + AB\overline{C}$

$= ABC + A'B + ABC'$

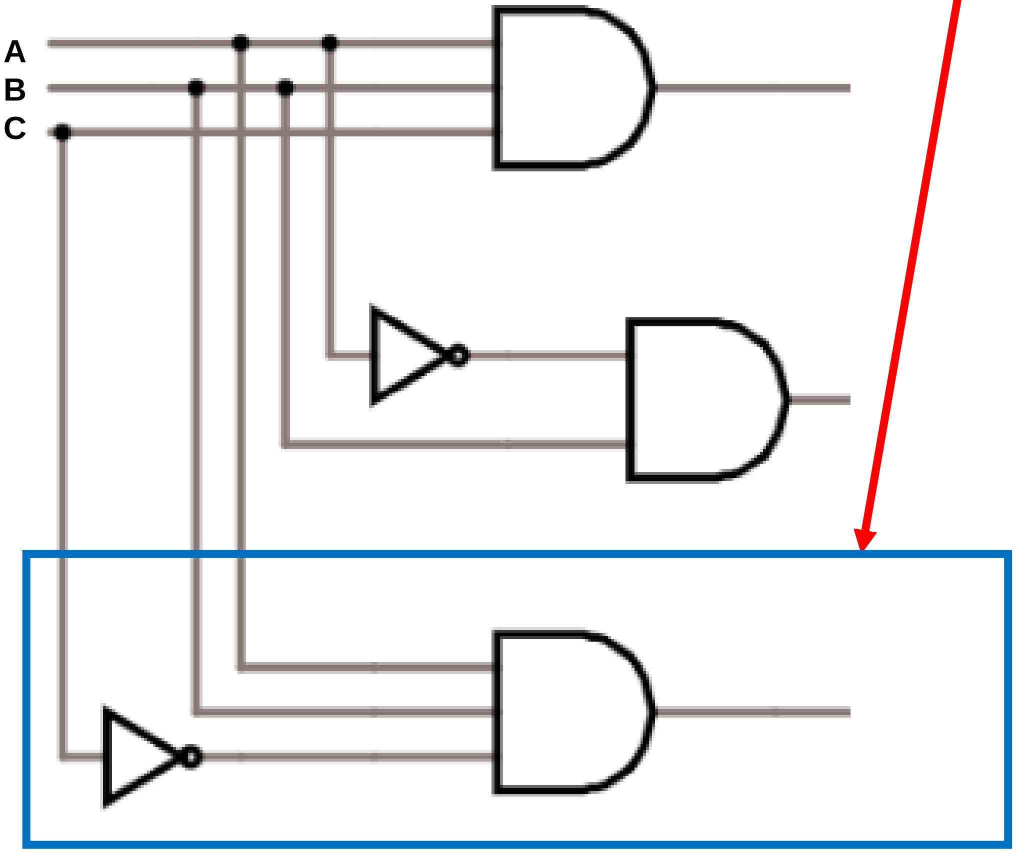
Inverter



Describing Logic Circuits Algebraically

Exercise 6.1 (a): $x = ABC + \overline{A}B + AB\overline{C}$

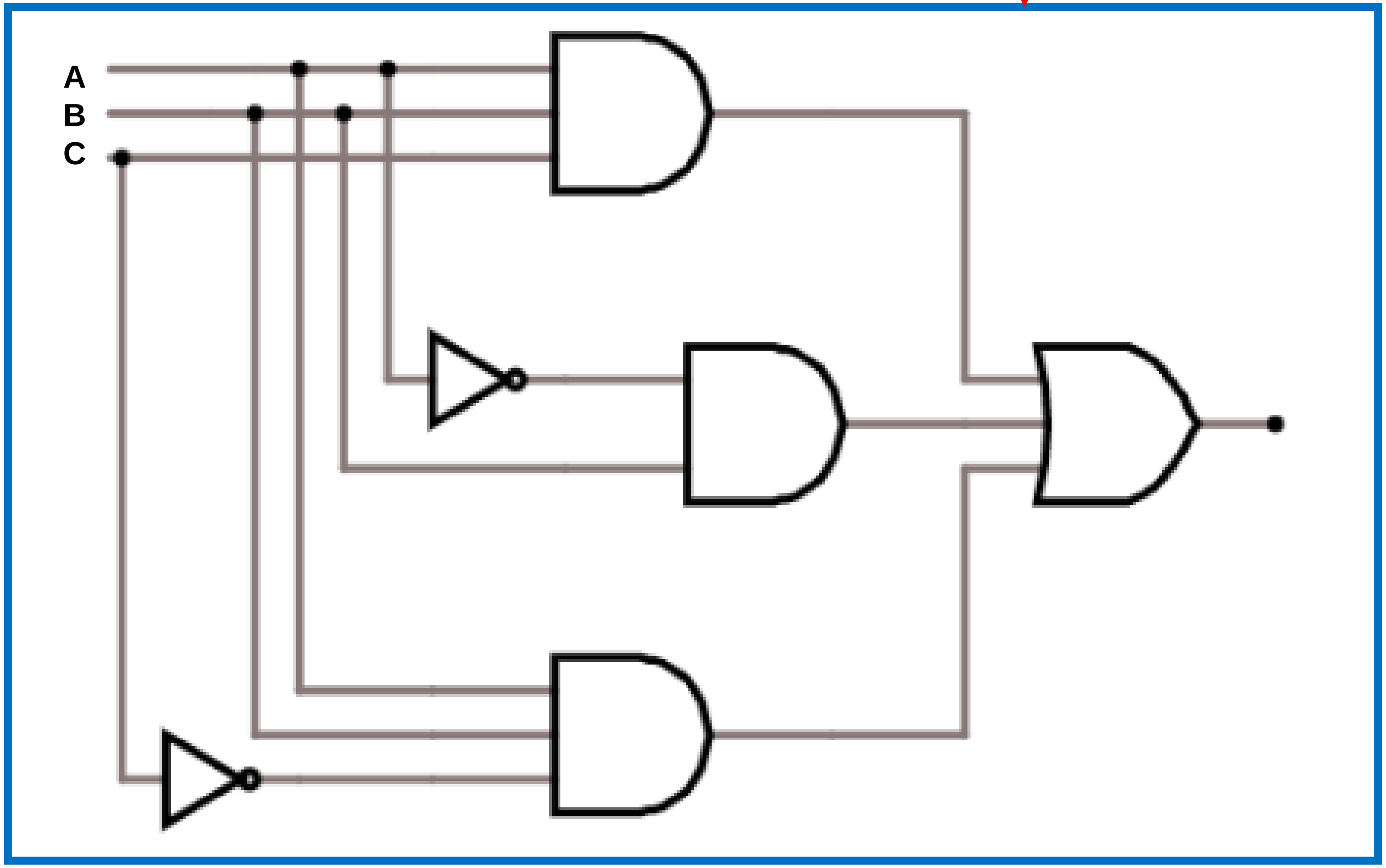
$= ABC + A'B + ABC'$ 3rd term



Describing Logic Circuits Algebraically

Exercise 6.1 (a): $x = ABC + \overline{A}B + AB\overline{C}$

$= ABC + A'B + ABC'$



Describing Logic Circuits Algebraically

Example

$$A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C$$

$$= AB'C' + A'B'C' + A'BC' + A'B'C$$

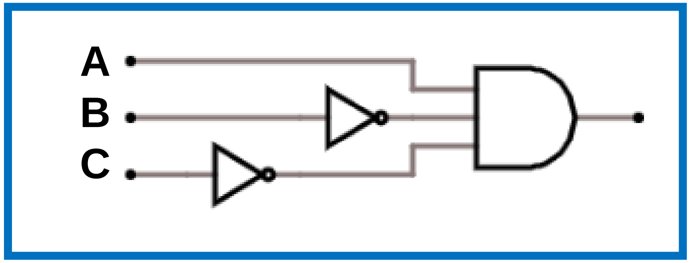
Describing Logic Circuits Algebraically

Example

$$A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C$$

$$= AB'C' + A'B'C' + A'BC' + A'B'C$$

1st term



Logic circuit

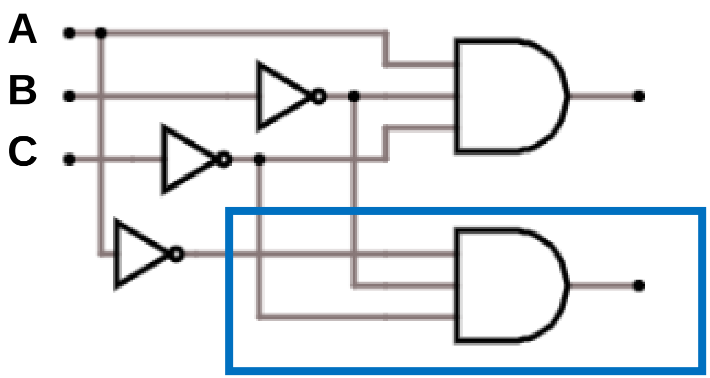
Describing Logic Circuits Algebraically

Example

$$A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C$$

$$= AB'C' + A'B'C' + A'BC' + A'B'C$$

2nd term



Logic circuit

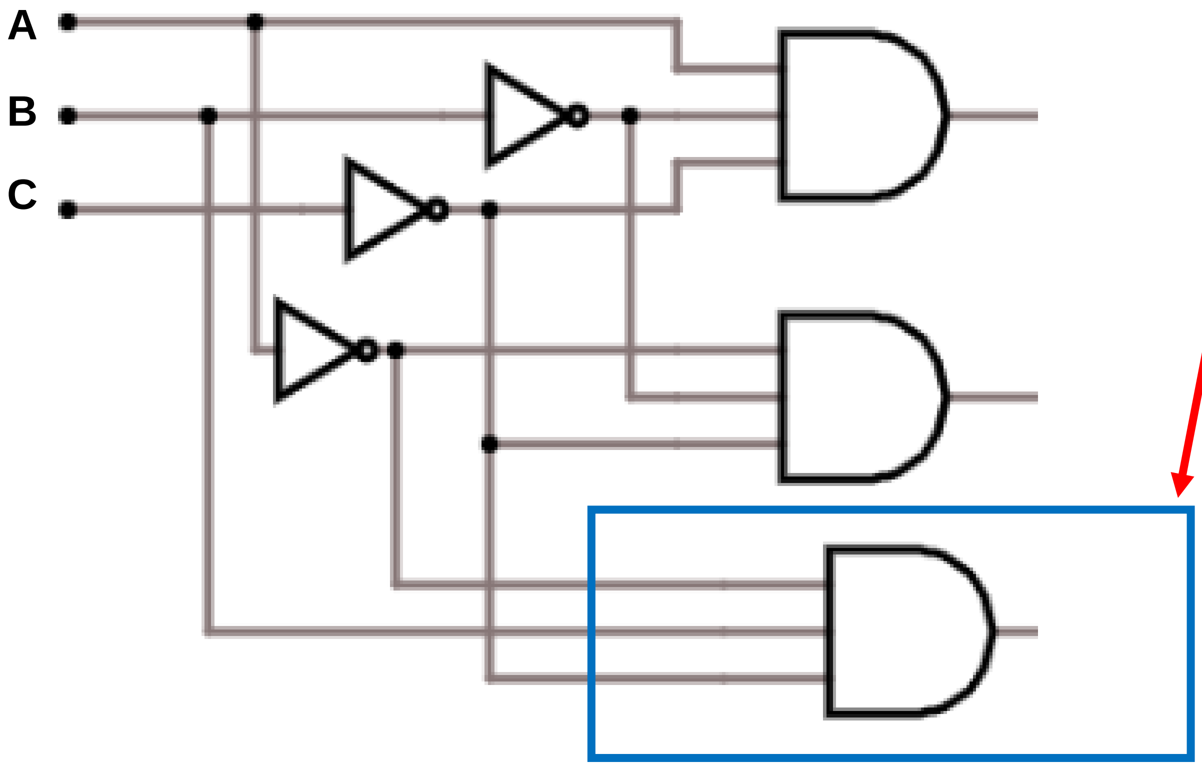
Describing Logic Circuits Algebraically

Example

$$A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C$$

$$= AB'C' + A'B'C' + A'BC' + A'B'C$$

3^{rd} term



Logic circuit

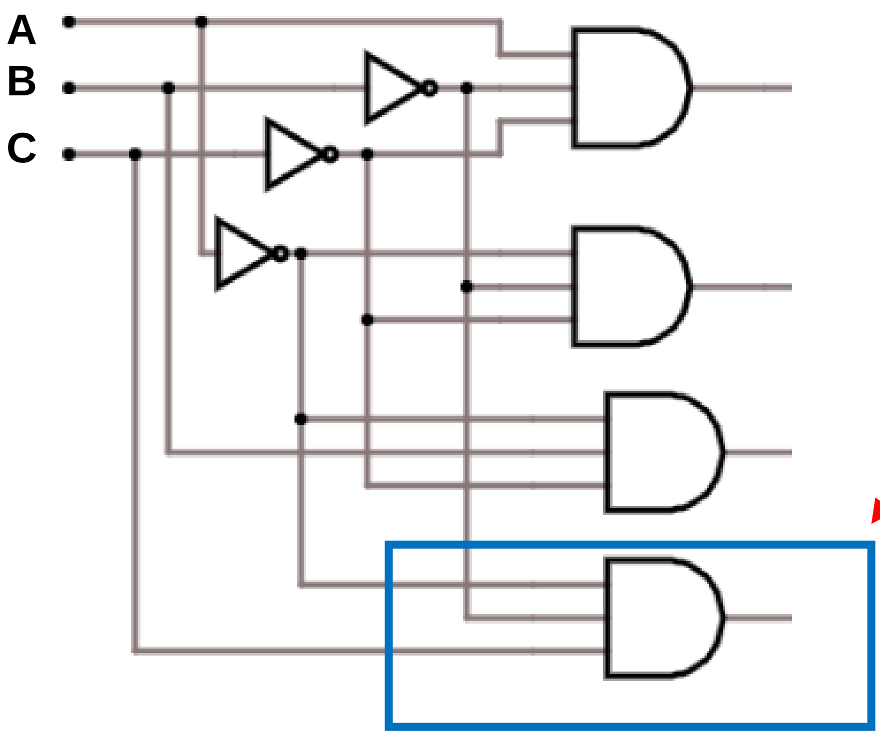
Describing Logic Circuits Algebraically

Example

$$A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C$$

$$= AB'C' + A'B'C' + A'BC' + A'B'C$$

4th term



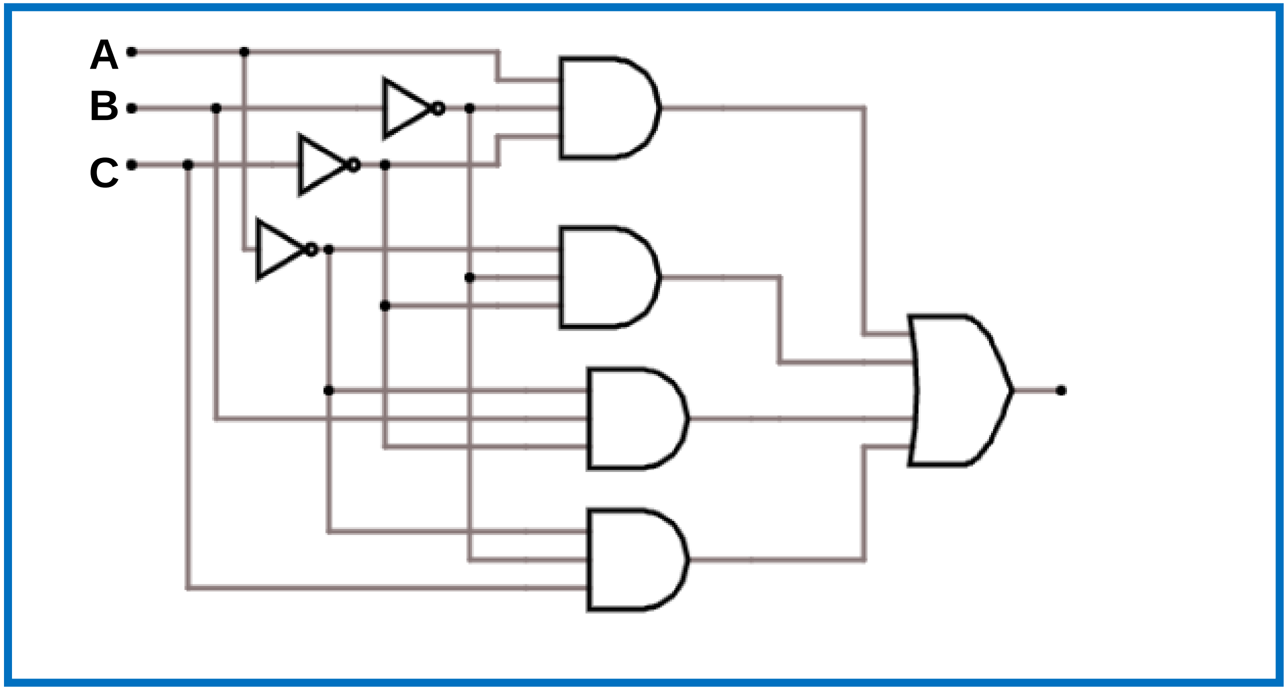
Logic circuit

Describing Logic Circuits Algebraically

Example

$$A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C$$

$$= AB'C' + A'B'C' + A'BC' + A'B'C$$

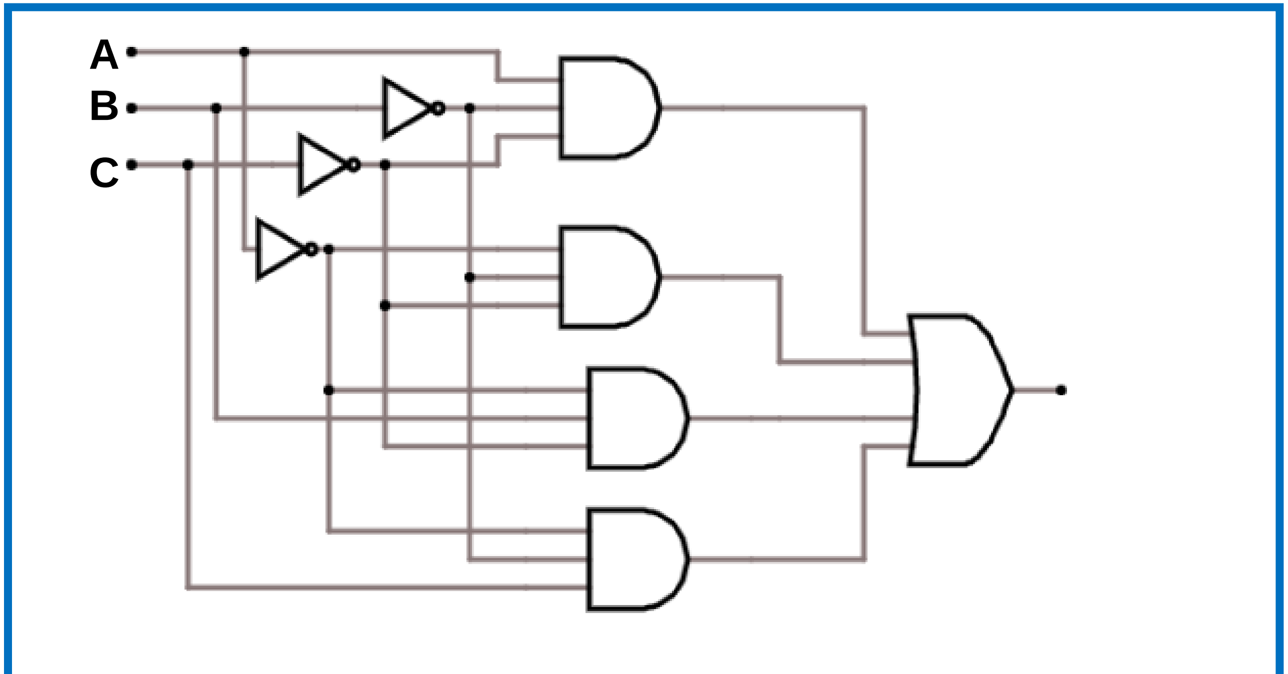


Simplify the following Boolean expression

Exercise

$$A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C$$

$$= AB'C' + A'B'C' + A'BC' + A'B'C$$



Draw the logic circuit for the simplified expression.

Week 4 Lecture 6b

- Boolean Algebra
  - Minimizing Boolean expressions
  - Describing logic circuits algebraically

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Exercise 6.2

Use the Boolean rules to simplify the following expressions:

(i) $AB + AC + ABC$

(ii) $AB + A(\overline{B} + C) + AB\overline{C}$

(iii) $\overline{A}BC + A\overline{B}C + ABC + AB\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$