ENGR 101
Engineering Technology

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Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui
CAPITAL CITY UNIVERSITY

Week 5 Lecture 9b

- Assignment 1
- Karnaugh Map
- Course web page:
  https://ecs.wgtn.ac.nz/Courses/XMUT101_2021T1/
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Assignment 1

Lecture	Week 3: 15 - 21 March	Lecture Slides (pdf)	Video (Zip) files
4	Number system conversion	Wk03Lec04a Wk03Lec04b	Wk03Lec04a.zip Wk03Lec04b.zip
5	Logic gates, symbols, truth tables and waveforms	Wk03Lec05a Wk03Lec05b	Wk03Lec05a.zip Wk03Lec05b.zip
Lecture	**Week 4: 22 - 28 March**		
6	Simplifying Boolean expressions using Boolean Algebra	Wk04Lec06a Wk04Lec06b	Wk04Lec06a.zip Wk04Lec06b.zip
7	Drawing logic circuits	Wk04Lec07a Wk04Lec07b	Wk04Lec07a.zip Wk04Lec07b.zip
Lecture	**Week 5: 29 March - 4 April**	**Lecture Slides (pdf)**	**Video (Zip) files**
8	Logic circuits	Wk05Lec08a Wk05Lec08b	Wk05Lec08a.zip Wk05Lec08b1.zip Wk05Lec08b2.zip
9	Karnaugh Maps	Wk05Lec09a	Wk05Lec09a.zip
	Assignment 1 due on Friday 9 April 2021		
Lecture	**Week 6: 5 - 11 April**		
10			
11			
Lecture	**Week 7: 12 - 18 April**	**Lecture Slides (pdf)**	**Video (Zip) files**
12			
13			
Lecture	**Week 8: 19 - 25 April**		
14			
15	Test 1 (25%) on Thursday 22 April 2021 during the lecture session		

ENGR 101 Engineering Technology

Assignment 1 **Due date: Friday, 9 April, 2021 (23:59 hrs)**

You must be type your answers using a word processor or any other similar computer program. Save the computer file containing your answers as a pdf file using the filename format: **Student Id number-A1.pdf**. For example, 1812401234-A1.pdf

1. Convert the following decimal numbers to binary numbers. Your answers must be in 8-bits format. (eg Decimal number 5 is 0000 0101 in 8-bits) Clearly show the steps that you used to obtain your answers.

   a) 48 b) 20 c) 84 d) 102 e) 68

2. Convert the following decimal numbers to octal numbers. Your answers must be in 4-digits format. (eg Decimal number 5 is 0005 in 4-digits) Clearly show the steps that you used to obtain your answers.

   a) 48 b) 20 c) 84 d) 102 e) 68

3. Convert the following decimal numbers to hexadecimal numbers. Your answers must be in 4-digits format. (eg Decimal number 10 is 000A in 4-digits) Clearly show the steps that you used to obtain your answers.

   a) 58 b) 18 c) 80 d) 122 e) 168

4. Simplify the following expression using the Boolean Laws discussed in lectures.
   a) AB + AC + ABC
   b) AB + A(B'+C) + ABC'
   c) A'BC + AB'C + ABC + ABC' + A'B'C'
   d) AB'C' + A'B'C' + A'BC' + A'B'C

Karnaugh Map (K-Map)

- An alternate approach to representing Boolean functions
- Can be used to minimize Boolean functions
- Easy conversion from truth table to K-Map to minimized SOP representation.
- Simple rules (steps) used to perform minimization
- Leads to minimized SOP representation.
  - Much faster and more efficient than previous minimization techniques with Boolean algebra.

Karnaugh Map (K-Map) Method

- The truth table values are placed in the K map as shown below.

A \ B	B'	B
A'	1	1
A	0	0

AB \ C	C'	C
A'B'		
A'B		
AB		
AB'		

Karnaugh Map (K-Map) Method

- The truth table values are placed in the K map.
- Adjacent K map square differ in only one variable both horizontally and vertically.

Karnaugh Map (K-Map) Method

- The truth table values are placed in the K map.
- Adjacent K map square differ in only one variable both horizontally and vertically.
- The pattern from top to bottom and left to right must be in the form $\overline{A}\overline{B}, \overline{A}B, AB, A\overline{B}$

2 inputs:

left to right

A \ B	B'	B
A'	1	1
A	0	0

top to bottom

4 inputs:

left to right

AB \ CD	C'D'	C'D	CD	CD'
A'B'				
A'B				
AB				
AB'				

top to bottom

Karnaugh Map (K-Map) Method

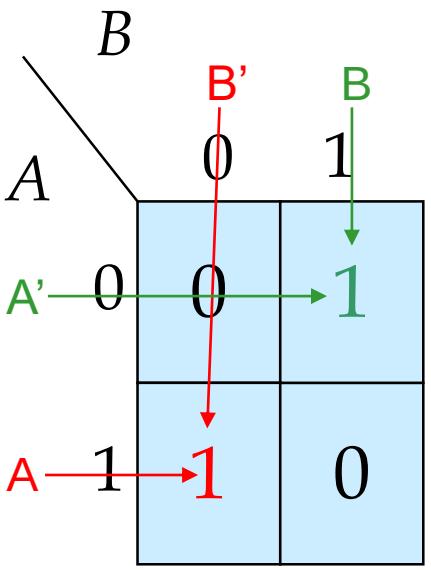
- The truth table values are placed in the K map as shown in on next slide.
- Adjacent K map square differ in only one variable both horizontally and vertically.
- The pattern from top to bottom and left to right must be in the form $\overline{A}\overline{B}, \overline{A}B, AB, A\overline{B}$

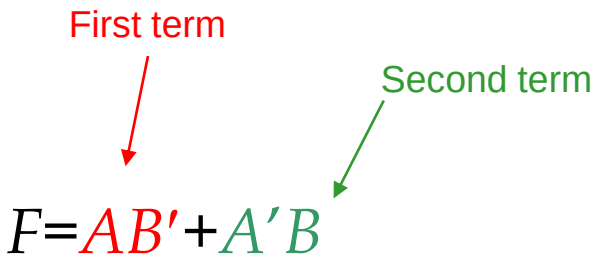
- A SOP expression can be obtained by OR-ing all squares that contain a 1.

Karnaugh Maps

- A Karnaugh map is a graphical tool for assisting in the general simplification procedure.
- **Two variable maps.**

Example 1:

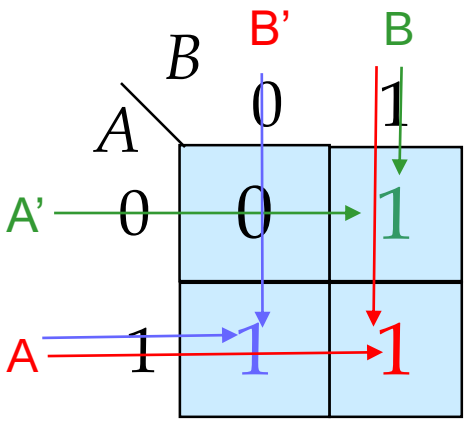


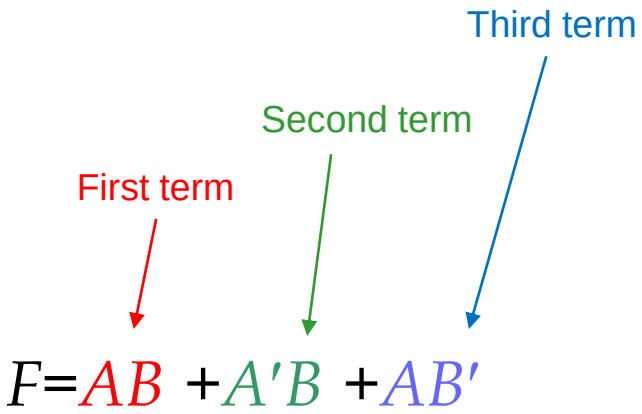


Karnaugh Maps

- A Karnaugh map is a graphical tool for assisting in the general simplification procedure.
- **Two variable maps.**

Example 2:





Karnaugh Maps

- **Three variable maps.**

Inputs

Output

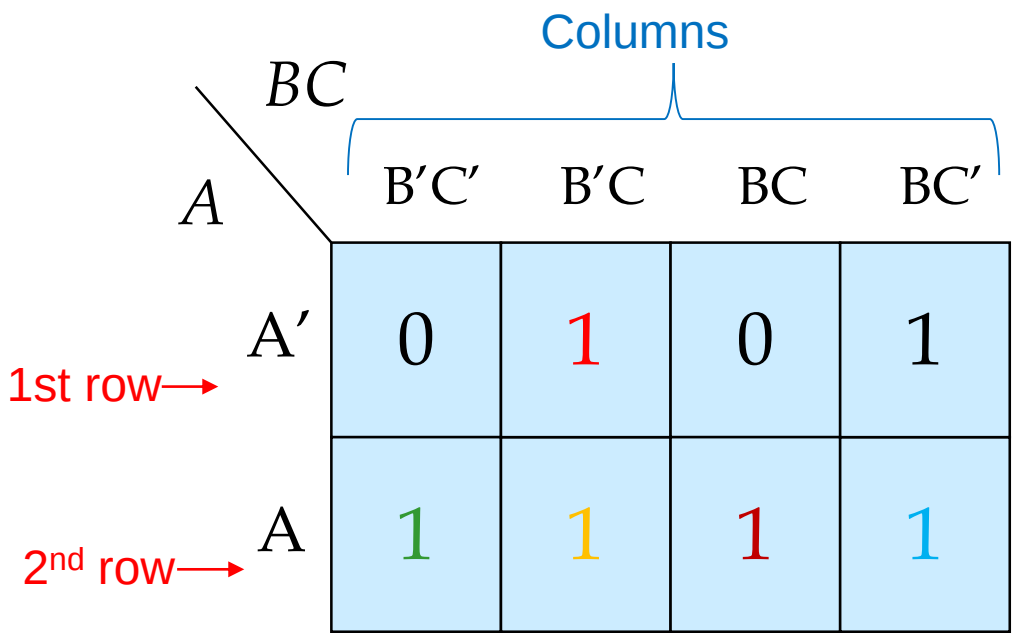
	A	B	C	F
	0	0	0	0
3rd row →	0	0	1	1
4th row →	0	1	0	1
	0	1	1	0
6th row →	1	0	0	1
7th row →	1	0	1	1
8th row →	1	1	0	1
9th row →	1	1	1	1

$$F = A'B'C + A'BC' + AB'C' + AB'C + ABC' + ABC$$

Sum of Products expression (SOP)

Karnaugh Maps

- **Three variable maps.**



A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$F = A'B'C + A'BC' + AB'C' + AB'C + ABC' + ABC$$

First term, Second term, Third term, 4th term, 5th term, 6th term

Rules for K-Maps

- We can reduce functions by circling 1's in the K-map
- Each circle represents min-term reduction
- Following circling, we can deduce minimized and-or form.

Rules to consider

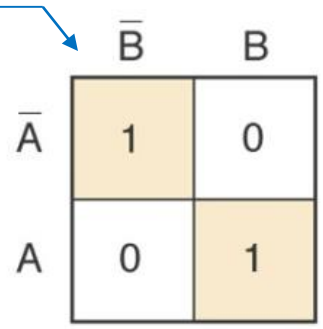
- Every cell containing a 1 must be included at least once.
- The largest possible "power of 2 rectangle" must be enclosed.
- The 1's must be enclosed in the smallest possible number of rectangles.

K-Maps and truth tables for (a) two variables.

A	B	X
0	0	1 → $\bar{A}\bar{B}$
0	1	0
1	0	0
1	1	1 → AB

$x = \bar{A}\bar{B} + AB$

(a)



K-Maps and truth tables for (b) three variables.

A	B	C	X
0	0	0	1 → $\bar{A}\bar{B}\bar{C}$
0	0	1	1 → $\bar{A}\bar{B}C$
0	1	0	1 → $\bar{A}B\bar{C}$
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1 → $AB\bar{C}$
1	1	1	0

$$X = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C}$$

(b)

	$\bar{C}$	C
$\bar{A}\bar{B}$	1	1
$\bar{A}B$	1	0
AB	1	0
$A\bar{B}$	0	0

Sum of product expression (SOP)

K-Maps and truth tables for (b) three variables.

A	B	C	X	
0	0	0	1	$\rightarrow \bar{A}\bar{B}\bar{C}$
0	0	1	1	$\rightarrow \bar{A}\bar{B}C$
0	1	0	1	$\rightarrow \bar{A}B\bar{C}$
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	1	$\rightarrow AB\bar{C}$
1	1	1	0	

$$X = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C}$$

(b)

	$\bar{C}$	C
$\bar{A}\bar{B}$	1	1
$\bar{A}B$	1	0
AB	1	0
$A\bar{B}$	0	0

K-Maps and truth tables for (c) four variables.

A	B	C	D	X	
0	0	0	0	0	
0	0	0	1	1	$\rightarrow \bar{A}\bar{B}\bar{C}D$
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	1	$\rightarrow \bar{A}B\bar{C}D$
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	1	$\rightarrow AB\bar{C}D$
1	1	1	0	0	
1	1	1	1	1	$\rightarrow ABCD$

$$X = \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + AB\bar{C}D + ABCD$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	0
$\bar{A}B$	0	1	0	0
AB	0	1	1	0
$A\bar{B}$	0	0	0	0

Sum of product expression (SOP)

K-Maps and truth tables for (c) four variables.

A	B	C	D	X	
0	0	0	0	0	
0	0	0	1	1	$\rightarrow \bar{A}\bar{B}\bar{C}D$
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	1	$\rightarrow \bar{A}B\bar{C}D$
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	1	$\rightarrow AB\bar{C}D$
1	1	1	0	0	
1	1	1	1	1	$\rightarrow ABCD$

$$X = \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + AB\bar{C}D + ABCD$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	0
$\bar{A}B$	0	1	0	0
AB	0	1	1	0
$A\bar{B}$	0	0	0	0

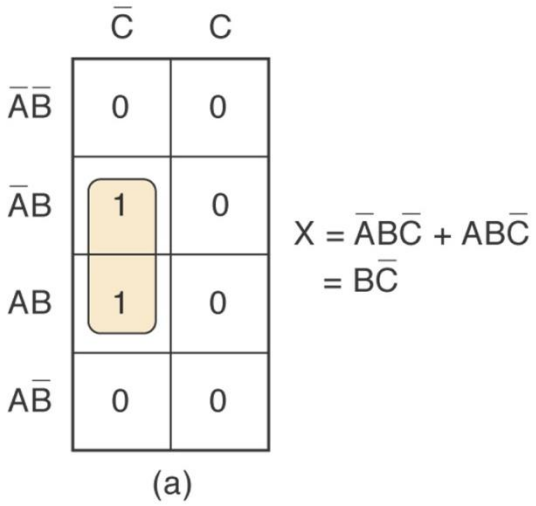
Karnaugh Map Method

- Loop adjacent groups of 2, 4, or 8 that contain 1’s will result in further simplification.

- When the largest possible groups have been looped, only the common terms are placed in the final expression.

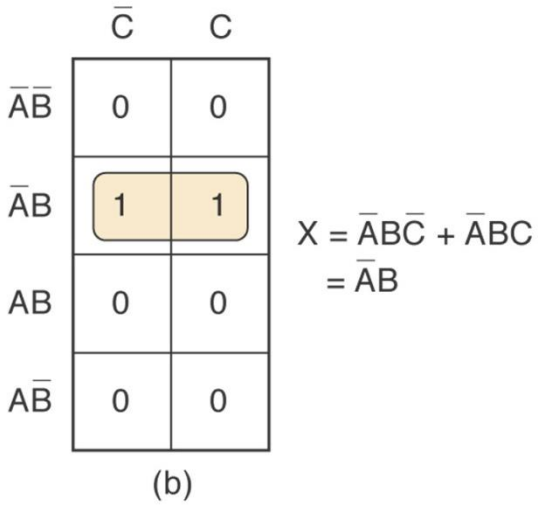
- Looping may also be wrapped between top, bottom, and sides.

Looping pairs of adjacent 1's – One variable is eliminated.



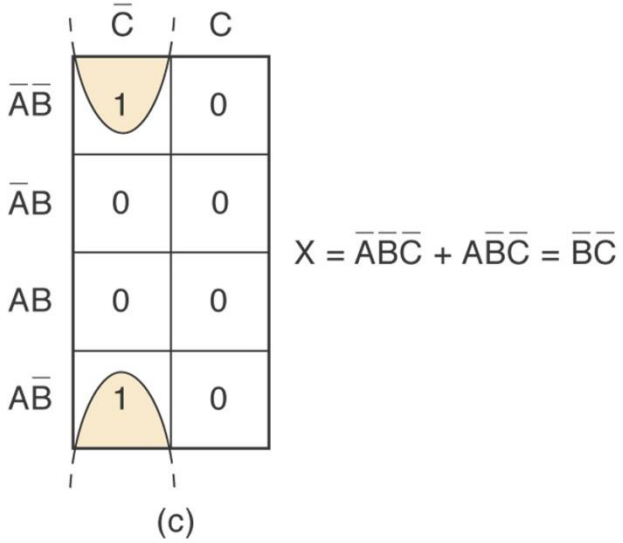
(a)

$X = \bar{A}B\bar{C} + AB\bar{C}$
$= B\bar{C}$



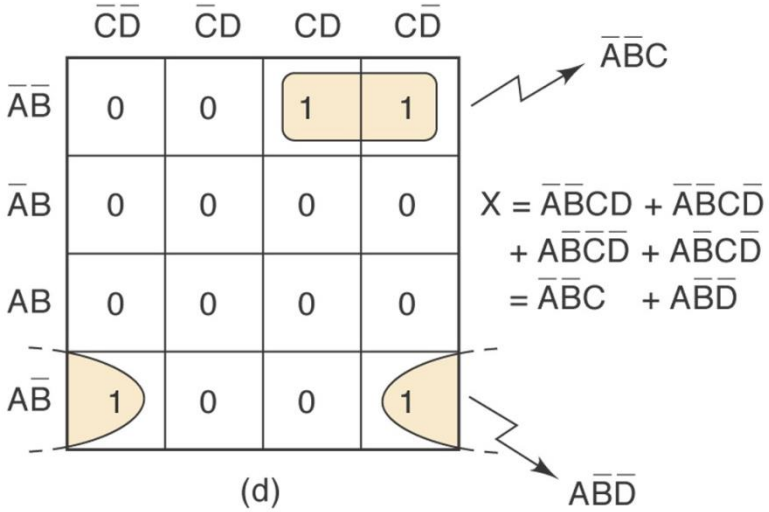
(b)

$X = \bar{A}B\bar{C} + \bar{A}BC$
$= \bar{A}B$



(c)

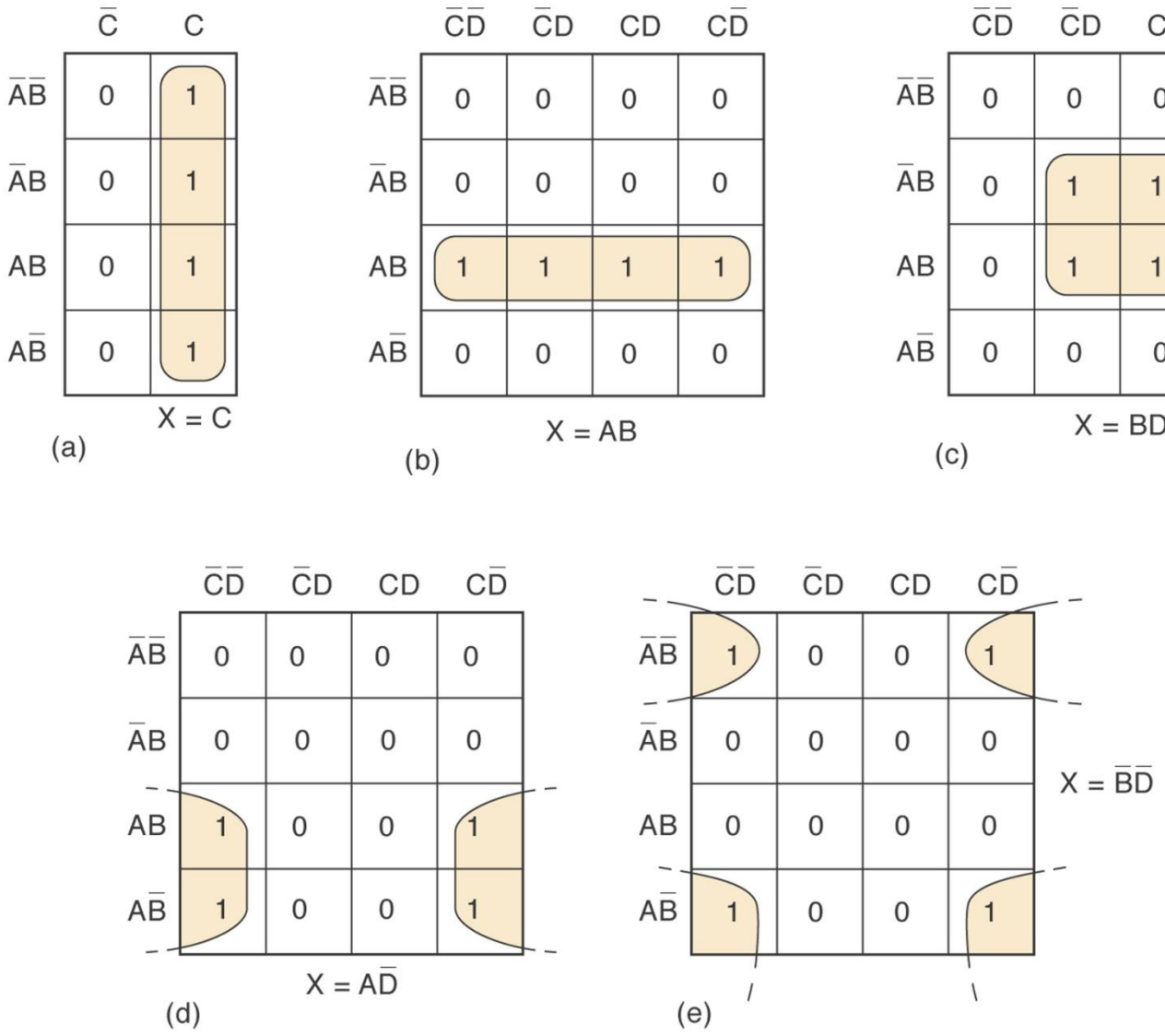
$X = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} = \bar{B}\bar{C}$



(d)

$X = \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D}$
$+ A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}$
$= \bar{A}\bar{B}C + A\bar{B}\bar{D}$

Looping groups of four adjacent 1's – Two variables are eliminated.



Steps for the Karnaugh Map Method

1. Construct the K-Map & place 1s as indicated in the truth table or the Boolean expression.
2. Loop 1s that are not adjacent to any other 1s.
3. Loop 1s that are in pairs
4. Loop 1s in octets even if they have already been looped.
5. Loop quads that have one or more 1s not already looped.
6. Loop any pairs necessary to include 1s not already looped.
7. Form the OR sum of terms generated by each loop.

Example 1:

Use a K-map to simplify:

$$Y = \overline{C}(\overline{A}\overline{B}\overline{D} + D) + A\overline{B}C + \overline{D}$$

$$y = \overline{C}(\overline{A}\overline{B}\overline{D} + D) + A\overline{B}C + \overline{D}$$

$$y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{C}D + A\overline{B}C + \overline{D} = A'B'C'D'+C'D+AB'C+D'$$

$$y = \overline{C}(\overline{A}\overline{B}\overline{D} + D) + A\overline{B}C + \overline{D}$$

$$y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{C}D + A\overline{B}C + \overline{D} = A'B'C'D'+C'D+AB'C+D'$$

Step 1:
Draw the
K-Map

CD / AB	C'D'	C'D	CD	CD'
A'B'				
A'B				
AB				
AB'				

$$y = \overline{C}(\overline{A}\overline{B}\overline{D} + D) + A\overline{B}C + \overline{D}$$

$$y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{C}D + A\overline{B}C + \overline{D} = A'B'C'D' + C'D + AB'C + D'$$

Step 2:
Insert 1 according to each term in the Boolean expression

CD / AB	C'D'	C'D	CD	CD'
A'B'	1			
A'B				
AB				
AB'				

$$y = \overline{C}(\overline{A}\overline{B}\overline{D} + D) + A\overline{B}C + \overline{D}$$

$$y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{C}D + A\overline{B}C + \overline{D} = A'B'C'D' + C'D + AB'C + D'$$

Step 2:
Insert 1 according to each term in the Boolean expression

CD / AB	C'D'	C'D	CD	CD'
A'B'	1	1		
A'B		1		
AB		1		
AB'		1		

$$y = \overline{C}(\overline{A}\overline{B}\overline{D} + D) + A\overline{B}C + \overline{D}$$

$$y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{C}D + A\overline{B}C + \overline{D} = A'B'C'D' + C'D + AB'C + D'$$

Step 2:
Insert 1 according to each term in the Boolean expression

CD / AB	C'D'	C'D	CD	CD'
A'B'	1	1		
A'B		1		
AB		1		
AB'		1	1	1

$$y = \overline{C}(\overline{A}\overline{B}\overline{D} + D) + A\overline{B}C + \overline{D}$$

$$y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{C}D + A\overline{B}C + \overline{D} = A'B'C'D'+C'D+AB'C+D'$$

Step 2:
Insert 1 according to each term in the Boolean expression

CD / AB	C'D'	C'D	CD	CD'
A'B'	1	1		1
A'B	1	1		1
AB	1	1		1
AB'	1	1	1	1

$$y = \overline{C}(\overline{A}\overline{B}\overline{D} + D) + A\overline{B}C + \overline{D}$$

$$y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{C}D + A\overline{B}C + \overline{D} = A'B'C'D'+C'D+AB'C+D'$$

Step 3:
Loop 1s in
a pair,
or in a quad,
or in an octet

CD / AB	C'D'	C'D	CD	CD'
A'B'	1	1		1
A'B	1	1		1
AB	1	1		1
AB'	1	1	1	1

C'

$$y = \overline{C}(\overline{A}\overline{B}\overline{D} + D) + A\overline{B}C + \overline{D}$$

$$y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{C}D + A\overline{B}C + \overline{D} = A'B'C'D'+C'D+AB'C+D'$$

Step 3:
Loop 1s in
a pair,
or in a quad,
or in an octet

CD / AB	C'D'	C'D	CD	CD'
A'B'	1	1		1
A'B	1	1		1
AB	1	1		1
AB'	1	1	1	1

C'

D'

$$y = \overline{C}(\overline{A}\overline{B}\overline{D} + D) + A\overline{B}C + \overline{D}$$

$$y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{C}D + A\overline{B}C + \overline{D} = A'B'C'D'+C'D+AB'C+D'$$

Step 3:
Loop 1s in
a pair,
or in a quad,
or in an octet

CD / AB	C'D'	C'D	CD	CD'
A'B'	1	1		1
A'B	1	1		1
AB	1	1		1
AB'	1	1	1	1

AB'

C'

D'

$$y = \overline{C}(\overline{A}\overline{B}\overline{D} + D) + A\overline{B}C + \overline{D}$$

$$y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{C}D + A\overline{B}C + \overline{D}$$ = A'B'C'D'+C'D+AB'C+D'

= AB' + C' + D'

Step 4:
Write the OR sum of terms

CD / AB	C'D'	C'D	CD	CD'
A'B'	1	1		1
A'B	1	1		1
AB	1	1		1
AB'	1	1	1	1

AB'

C'

D'

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