

ENGR 101

Engineering Technology

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Victoria
UNIVERSITY OF WELLINGTON
*Te Whare Wānanga
o te Ūpoko o te Ika a Māui*



CAPITAL CITY UNIVERSITY

Week 6 Lecture 10a

- Karnaugh Map (K-Map)

- Course web page:

https://ecs.wgtn.ac.nz/Courses/XMUT101_2021T1/

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Rules for K-Maps

- We can reduce functions by **circling** 1's in the K-map
- Each circle represents min-term reduction
- Following circling, we can deduce minimized and-or form.

Rules to consider

- Every cell containing a 1 must be included at least once.
- The largest possible “power of 2 rectangle” must be enclosed.
- The 1's must be enclosed in the smallest possible number of rectangles.

Steps for the Karnaugh Map Method

1. Construct the K-Map & place 1s as indicated in the truth table or the Boolean expression.
2. Loop 1s that **are not adjacent to any other 1s**.
3. Loop 1s that are in **pairs**
4. Loop 1s in **octets** even if they have already been looped.
5. Loop **quads** that have one or more 1s not already looped.
6. Loop any **pairs** necessary to include 1st not already looped.
7. Form the OR **sum of terms** generated by each loop.

Example 3: Use a K-map to simplify the following

$$X = A'B'CD + A'BCD + ABCD + AB'CD + ABC'D' + ABC'D + ABCD'$$

Example 3: Use a K-map to simplify:

$$X = A'B'CD + A'BCD + ABCD + AB'CD + ABC'D' + ABC'D + ABCD'$$

Step 1:
Draw the
K-Map

column

AB \ CD	C'D'	C'D	CD	CD'
A'B'				
A'B				
AB				
AB'				

row

Example 3: Use a K-map to simplify:

$$X = A'B'CD + A'BCD + ABCD + AB'CD + ABC'D' + ABC'D + ABCD'$$

↑
First term

Step 2:
Insert 1
according
to each
term in the
Boolean
expression

column

AB \ CD	CD	C'D'	C'D	CD	CD'
A'B'	1				
A'B					
AB					
AB'					

row

Example 3: Use a K-map to simplify:

$$X = A'B'CD + A'BCD + ABCD + AB'CD + ABC'D' + ABC'D + ABCD'$$

↑
second term

Step 2:
Insert 1
according
to each
term in the
Boolean
expression

AB \ CD	CD	C'D'	C'D	CD	CD'
A'B'				1	
A'B				1	
AB					
AB'					

Example 3: Use a K-map to simplify:

$$X = A'B'CD + A'BCD + \mathbf{ABCD} + AB'CD + ABC'D' + ABC'D + ABCD'$$

↑
third term

Step 2:
Insert 1
according
to each
term in the
Boolean
expression

AB \ CD	CD	C'D'	C'D	CD	CD'
A'B'				1	
A'B				1	
AB				1	
AB'					

Example 3: Use a K-map to simplify:

$$X = A'B'CD + A'BCD + ABCD + \text{AB'CD} + ABC'D' + ABC'D + ABCD'$$

↑
4th term

Step 2:
Insert 1
according
to each
term in the
Boolean
expression

AB \ CD	CD	C'D'	C'D	CD	CD'
A'B'				1	
A'B				1	
AB				1	
AB'				1	

Example 3: Use a K-map to simplify:

$$X = A'B'CD + A'BCD + ABCD + AB'CD + \mathbf{ABC'D'} + ABC'D + ABCD'$$

↑
5th term

Step 2:
Insert 1
according
to each
term in the
Boolean
expression

AB \ CD	C'D'	C'D	CD	CD'
A'B'			1	
A'B			1	
AB	1		1	
AB'			1	

Example 3: Use a K-map to simplify:

$$X = A'B'CD + A'BCD + ABCD + AB'CD + ABC'D' + \mathbf{ABC'D} + ABCD'$$

↑
6th term

Step 2:
Insert 1
according
to each
term in the
Boolean
expression

AB \ CD	CD	C'D'	C'D	CD	CD'
A'B'				1	
A'B				1	
AB		1	1	1	
AB'				1	

Example 3: Use a K-map to simplify:

$$X = A'B'CD + A'BCD + ABCD + AB'CD + ABC'D' + ABC'D + \mathbf{ABCD'}$$

↑
7th term

Step 2:
Insert 1
according
to each
term in the
Boolean
expression

AB \ CD	CD	C'D'	C'D	CD	CD'
A'B'				1	
A'B				1	
AB		1	1	1	1
AB'				1	

Example 3: Use a K-map to simplify:

$$X = A'B'CD + A'BCD + ABCD + AB'CD + ABC'D' + ABC'D + ABCD'$$

Steps 3-6:
Loop 1s in
a pair,
or in a quad,
or in an octet

AB \ CD	CD	C'D'	C'D	CD	CD'
A'B'				1	
A'B				1	
AB		1	1	1	1
AB'				1	

CD

AB

Example 3: Use a K-map to simplify:

$$X = A'B'CD + A'BCD + ABCD + AB'CD + ABC'D' + ABC'D + ABCD'$$

$$= AB + CD$$

Step 7:
Form the
OR sum of
terms

AB \ CD	CD	C'D'	C'D	CD	CD'
A'B'				1	
A'B				1	
AB		1	1	1	1
AB'				1	

CD

AB

Exercise 10.1: Use a K-map to simplify the following

$$X = ABC + A'B + ABC'$$

**10 minutes
to work this one!!**

Steps for the Karnaugh Map Method

1. Construct the K-Map & place 1s as indicated in the truth table or the Boolean expression.

Exercise 10.1: Use a K-map to simplify the following

$$X = ABC + A'B + ABC'$$

Step 1:
Draw the
K-Map

column

AB \ C	C'	C
A'B'		
A'B		
AB		
AB'		

row

1 minute to draw the K-map ...

Exercise 10.1: Use a K-map to simplify the following

$$X = ABC + A'B + ABC'$$

Step 2:
Insert 1
according
to each
term in the
Boolean
expression

AB \ C	C'	C
A'B'		
A'B		
AB		1
AB'		

column

row

2 minutes to insert 1 for 2nd & 3rd terms

Exercise 10.1: Use a K-map to simplify the following

$$X = ABC + A'B + ABC'$$

Step 2:
Insert 1
according
to each
term in the
Boolean
expression

AB \ C	C'	C
A'B'		
A'B	1	1
AB	1	1
AB'		

column

row

Steps for the Karnaugh Map Method

1. Construct the K-Map & place 1s as indicated in the truth table or the Boolean expression.
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7. Form the OR **sum of terms** generated by each loop.

Exercise 10.1: Use a K-map to simplify the following

$$X = ABC + A'B + ABC'$$

Steps 3-6:
Loop 1s in
a pair,
or in a quad,
or in an octet

AB \ C	C'	C
A'B'		
A'B	1	1
AB	1	1
AB'		

column

row

1 minute to write the expression for the 1s

Exercise 10.1: Use a K-map to simplify the following

$$X = ABC + A'B + ABC'$$

Step 7:
Form the
OR sum of
terms

AB \ C	C'	C
A'B'		
A'B	1	1
AB	1	1
AB'		

Original
expression

B

Exercise 6.1(a) $x = A B C + \bar{A} B + A B \bar{C}$

	Boolean Law used
$x = ABC + A'B + ABC'$	3) Distributive Law $A.(B+C) = (A.B) + (A.C)$
$= AB(C + C') + A'B$	
$= AB(1) + A'B$	6) Complement Law $A+A' = 1$
$= B(A + A')$	3) Distributive Law $A.(B+C) = (A.B) + (A.C)$
$= B$	

Exercise 10.2: Use a K-map to simplify the following

$$x = A'BC' + AB'C' + A'B'C' + A'B'C'$$

**10 minutes
to work this one!!**

Steps for the Karnaugh Map Method

1. Construct the K-Map & place 1s as indicated in the truth table or the Boolean expression.

Exercise 10.2: Use a K-map to simplify the following

$$x = A'BC' + AB'C' + A'B'C' + A'B'C'$$

**10 minutes
to work this one!!**

Exercise 10.2: Use a K-map to simplify the following

$$x = A'BC' + AB'C' + A'B'C' + A'B'C'$$

Step 1:
Draw the
K-Map

column

AB \ C	C'	C
A'B'		
A'B		
AB		
AB'		

row

Exercise 10.2: Use a K-map to simplify the following

$$x = A'BC' + AB'C' + A'B'C' + A'B'C'$$

Step 2:
Insert 1
according
to each
term in the
Boolean
expression

column

AB \ C	C'	C
A'B'		
A'B		
AB		
AB'		

row

4 minutes to insert 1 for all terms

Exercise 10.2: Use a K-map to simplify the following

$$X = A'BC' + AB'C' + A'B'C' + A'B'C'$$

Steps 3-6:
Loop 1s in
a pair,
or in a quad,
or in an octet

column

AB \ C	C'	C
A'B'	1	
A'B	1	
AB		
AB'	1	

row

2 minutes to loop adjacent 1s

Exercise 10.2: Use a K-map to simplify the following

$$X = A'BC' + AB'C' + A'B'C' + A'B'C'$$

Steps 3-6:
Loop 1s in
a pair,
or in a quad,
or in an octet

AB \ C	C'	C
A'B'	1	
A'B	1	
AB		
AB'	1	

1 minute to write the terms

Exercise 10.2: Use a K-map to simplify the following

$$X = A'BC' + AB'C' + A'B'C' + A'B'C'$$

Steps 3-6:
Loop 1s in
a pair,
or in a quad,
or in an octet

AB \ C	C'	C
A'B'	1	
A'B	1	
AB		
AB'	1	

$$A'C' + B'C'$$

Exercise 6.1(b) $x = \overline{A} B \overline{C} + A \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} C$

$x = A'BC' + AB'C' + A'B'C' + A'B'C'$	Boolean Law used
$= A'BC' + AB'C' + A'B'C' + A'B'C'$	4. Idempotent Law $A + A = A$
$= A'BC' + AB'C' + A'B'C'$	
$= A'BC' + B'C'(A + A')$	9. Distributive Law $A.(B+C) = (A.B) + (A.C)$
$= A'BC' + B'C'(1)$	6. Complement Law $A+A' = 1$
$= A'BC' + B'C'$	
$= C' (A'B + B')$	9. Distributive Law $A.(B+C) = (A.B) + (A.C)$
$= A'C' + B'C'$	11. Common Identities Law $A.(A'+B) = AB$; $A+(A'B) = A+B$

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