ENGR 101
Engineering Technology

Dr. Kerese Manueli

School of Engineering and Computer Science
Victoria University of Wellington

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui
CAPITAL CITY UNIVERSITY

Week 6 Lecture 10b

- Karnaugh Map (K-Map)
- Assignment 1 – submit before midnight Friday
- Course web page:

  https://ecs.wgtn.ac.nz/Courses/XMUT101_2021T1/

- kerese@ecs.vuw.ac.nz

Example 4 – 4 Variables

$$\text{Out} = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + \bar{A}BCD + ABCD + ABC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}$$

= A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

+ ABCD’ + AB’C’D’ + AB’CD’

Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

+ ABCD’ + AB’C’D’ + AB’CD’

Step 1:
Draw the
K-Map

AB \ CD	C’D’	C’D	CD	CD’
A’B’				
A’B				
AB				
AB’				

Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

+ ABCD’ + AB’C’D’ + AB’CD’

Step 2:
Insert 1 according to each term in the Boolean expression

AB \ CD	C’D’	C’D	CD	CD’
A’B’				
A’B				
AB				
AB’				

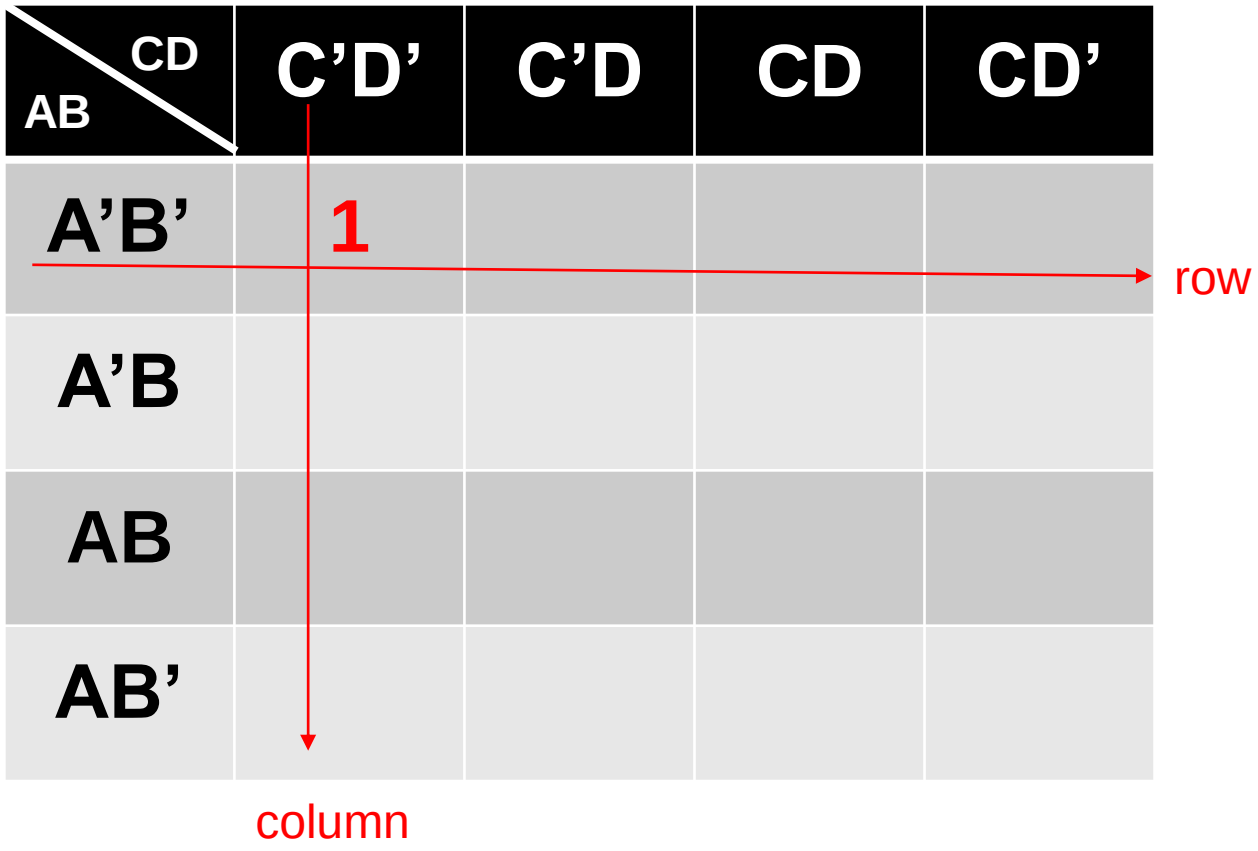
Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

+ ABCD’ + AB’C’D’ + AB’CD’

First term

Step 2:
Insert 1 according to each term in the Boolean expression



Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

Second term

+ ABCD’ + AB’C’D’ + AB’CD’

Step 2:
Insert 1 according to each term in the Boolean expression

AB \ CD	C’D’	C’D	CD	CD’
A’B’	1	1		
A’B				
AB				
AB’				

row

column

Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

+ ABCD’ + AB’C’D’ + AB’CD’

Third term

Step 2:
Insert 1 according to each term in the Boolean expression

AB \ CD	C’D’	C’D	CD	CD’
A’B’	1	1		
A’B		1		
AB				
AB’				

row

column

Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

+ ABCD’ + AB’C’D’ + AB’CD’

4th term

Step 2:
Insert 1 according to each term in the Boolean expression

AB \ CD	C’D’	C’D	CD	CD’
A’B’	1	1		
A’B		1	**1**	
AB				
AB’				

row

column

Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

+ ABCD’ + AB’C’D’ + AB’CD’

5th term

Step 2:
Insert 1 according to each term in the Boolean expression

AB \ CD	C’D’	C’D	CD	CD’
A’B’	1	1		
A’B		1	1	
AB			**1**	
AB’				

row

column

Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

+ ABCD’ + AB’C’D’ + AB’CD’

6th term

Step 2:
Insert 1 according to each term in the Boolean expression

AB \ CD	C’D’	C’D	CD	CD’
A’B’	1	1		
A’B		1	1	
AB			1	**1**
AB’				

row

column

Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

+ ABCD’ + AB’C’D’ + AB’CD’

7th term

Step 2:
Insert 1 according to each term in the Boolean expression

AB \ CD	C’D’	C’D	CD	CD’
A’B’	1	1		
A’B		1	1	
AB			1	1
AB’	1			

column

row

Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

+ ABCD’ + AB’C’D’ + AB’CD’ ← 8th term

Step 2:
Insert 1 according to each term in the Boolean expression

AB \ CD	C’D’	C’D	CD	CD’
A’B’	1	1		
A’B		1	1	
AB			1	1
AB’	1			**1**

row

column

Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD
+ ABCD’ + AB’C’D’ + AB’CD’

Steps 3-6:
Loop 1s in
a pair,
or in a quad,
or in an octet

AB \ CD	C’D’	C’D	CD	CD’
A’B’	1	1		
A’B		1	1	
AB			1	1
AB’	1			1

Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

+ ABCD’ + AB’C’D’ + AB’CD’

Step 7:
Form the OR sum of terms

AB \ CD	C’D’	C’D	CD	CD’
A’B’	1	1		
A’B		1	1	
AB			1	1
AB’	1			1

Out = A’C’D + BCD + ACD’ + B’C’D’

Example 4 – 4 Variables

Out = A’B’C’D’ + A’B’C’D + A’BC’D + A’BCD + ABCD

+ ABCD’ + AB’C’D’ + AB’CD’

Step 7:
Form the OR sum of terms

AB \ CD	C’D’	C’D	CD	CD’
A’B’	1	1		
A’B		1	1	
AB			1	1
AB’	1			1

Out = A’B’C’ + A’BD + ABC + AB’D’

Exercise 10.3: Use a K-map to simplify the following

X = A'B'C'D' + A'B'CD + A'BC'D' + A'BCD + ABC'D'

+ ABC'D + ABCD + AB'C'D' + AB'CD

10 minutes

to work this one!!

Exercise 10.3: Use a K-map to simplify the following

X = A'B'C'D' + A'B'CD + A'BC'D' + A'BCD + ABC'D'

+ ABC'D + ABCD + AB'C'D' + AB'CD

Step 1:
Draw the
K-Map

AB \ CD	C'D'	C'D	CD	CD'
A'B'				
A'B				
AB				
AB'				

Exercise 10.3: Use a K-map to simplify the following

X = A'B'C'D' + A'B'CD + A'BC'D' + A'BCD + ABC'D'

+ ABC'D + ABCD + AB'C'D' + AB'CD

First term

Step 2:
Insert 1 according to each term in the Boolean expression

AB \ CD	C'D'	C'D	CD	CD'
A'B'	1			
A'B				
AB				
AB'				

row

column

7 minutes to insert 1 for all terms

Exercise 10.3: Use a K-map to simplify the following

X = A’B’C’D’ + A’B’CD + A’BC’D’ + A’BCD + ABC’D’

+ ABC’D + ABCD + AB’C’D’ + AB’CD

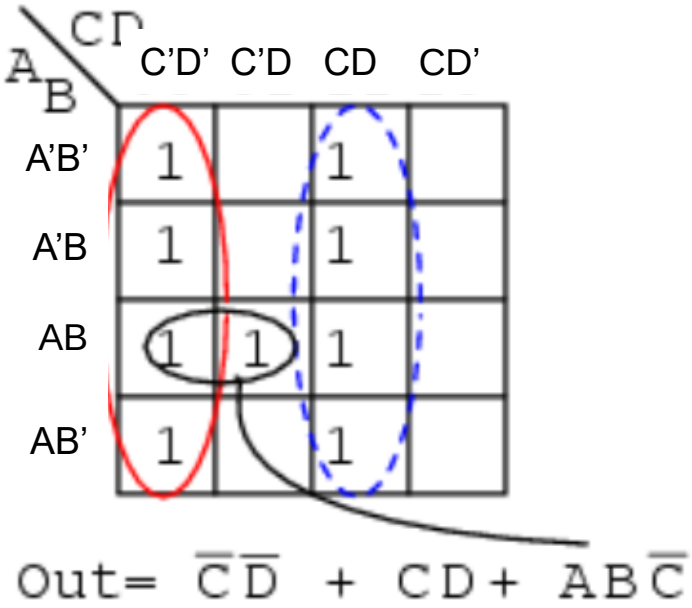
Steps 3-6:
Loop 1s in
a pair,
or in a quad,
or in an octet

CD / AB	C’D’	C’D	CD	CD’
A’B’	1		1	
A’B	1		1	
AB	1	1	1	
AB’	1		1	

Exercise 10.3: Use a K-map to simplify the following

X = A'B'C'D' + A'B'CD + A'BC'D' + A'BCD + ABC'D'

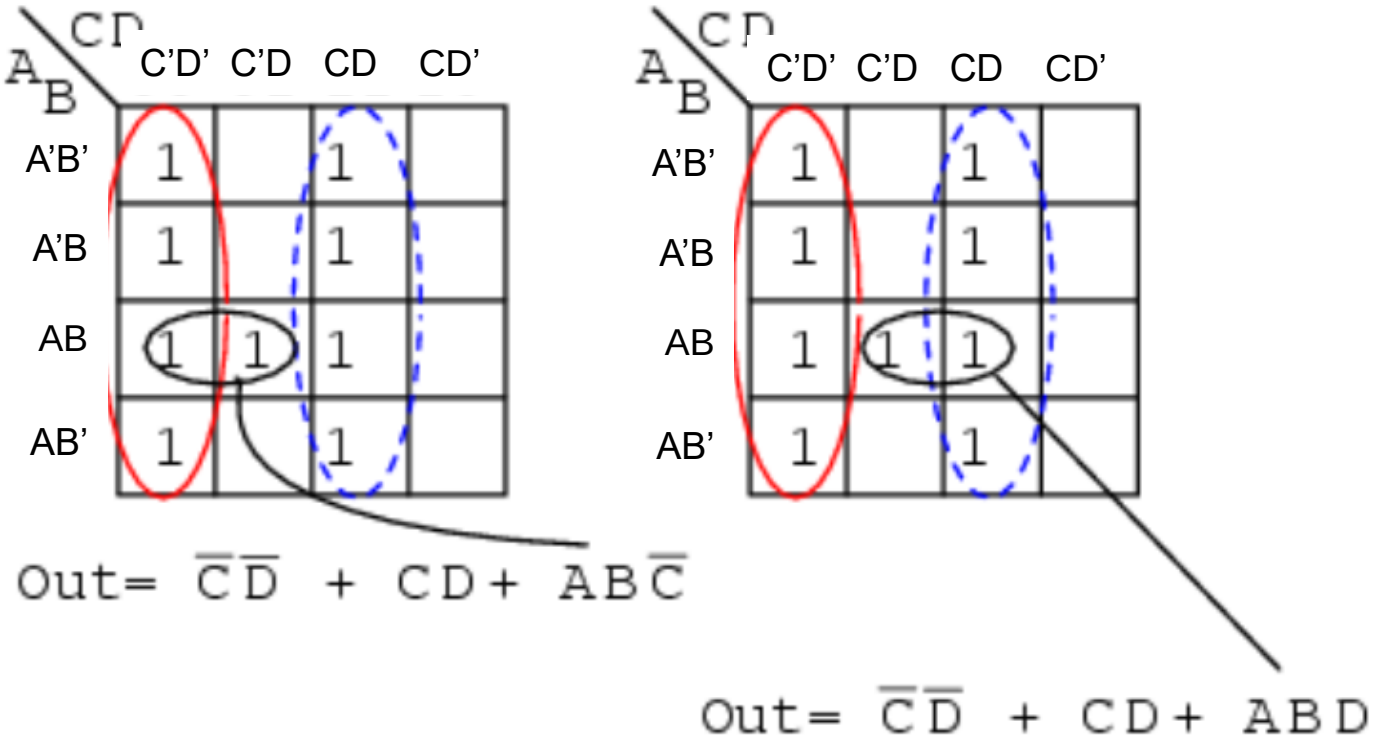
+ ABC'D + ABCD + AB'C'D' + AB'CD



Exercise 10.3: Use a K-map to simplify the following

X = A'B'C'D' + A'B'CD + A'BC'D' + A'BCD + ABC'D'

+ ABC'D + ABCD + AB'C'D' + AB'CD



Don’t care Output Conditions

Can be changed 0/1 so that the simplest expression can be obtained from the K-Map. Typically occur when we know certain input conditions are impossible.

Truth Table

A	B	C	z	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	x	"don't
1	0	0	x	care"
1	0	1	1	
1	1	0	1	
1	1	1	1	

(a)

Don’t care Output Conditions

Can be changed 0/1 so that the simplest expression can be obtained from the K-Map. Typically occur when we know certain input conditions are impossible.

Truth Table

A	B	C	z	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	x	"don't care"
1	0	0	x	
1	0	1	1	
1	1	0	1	
1	1	1	1	

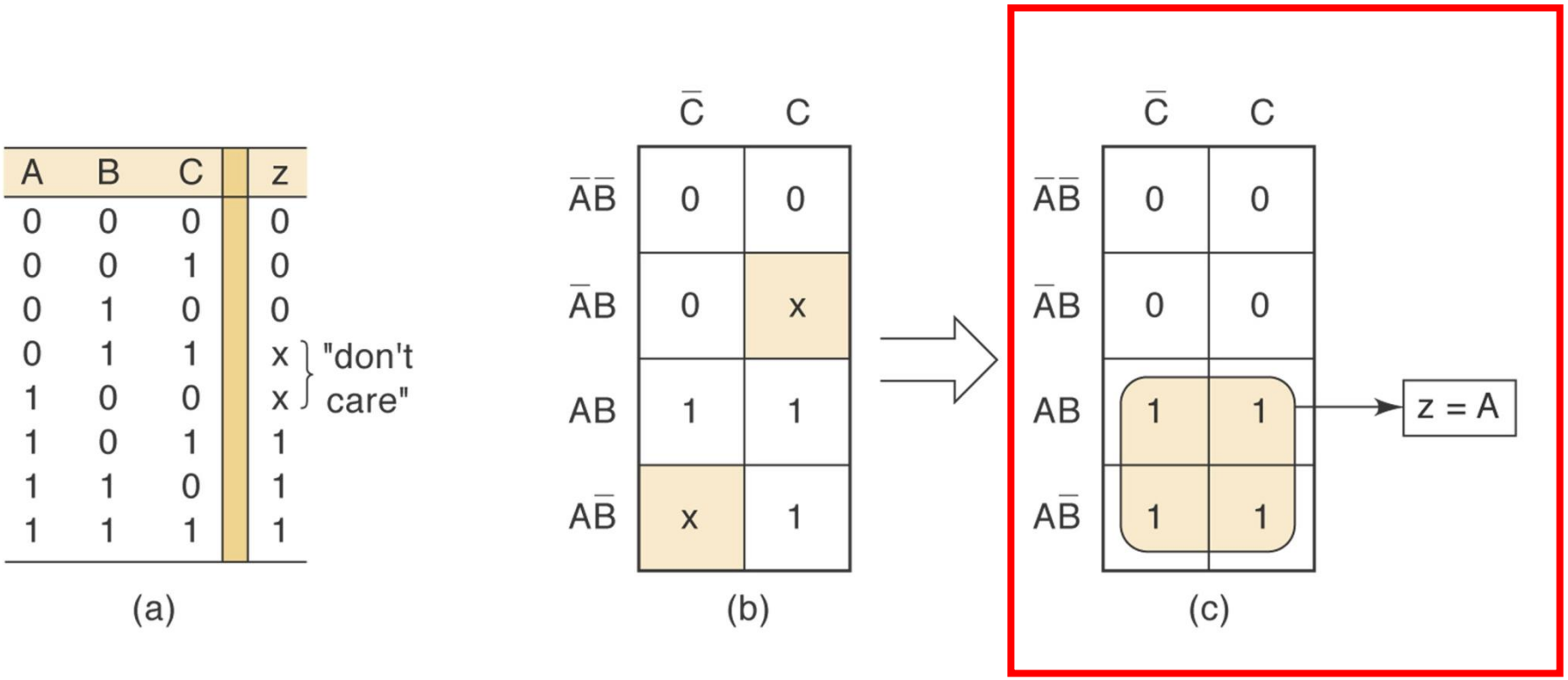
(a)

	$\bar{C}$	C
$\bar{A}\bar{B}$	0	0
$\bar{A}B$	0	x
AB	1	1
$A\bar{B}$	x	1

(b)

Don’t care Output Conditions

Can be changed 0/1 so that the simplest expression can be obtained from the K-map. Typically occur when we know certain input conditions are impossible.



A	B	C	z	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	x	"don't
1	0	0	x	care"
1	0	1	1	
1	1	0	1	
1	1	1	1	

(a)

	$\bar{C}$	C
$\bar{A}\bar{B}$	0	0
$\bar{A}B$	0	x
AB	1	1
$A\bar{B}$	x	1

(b)

	$\bar{C}$	C
$\bar{A}\bar{B}$	0	0
$\bar{A}B$	0	0
AB	1	1
$A\bar{B}$	1	1

z = A

(c)

Problem:

Design a logic circuit for an elevator in a three storey building.

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The circuit output is the “Door Open” signal, should be normally 0 but changes to 1 when the door is to open

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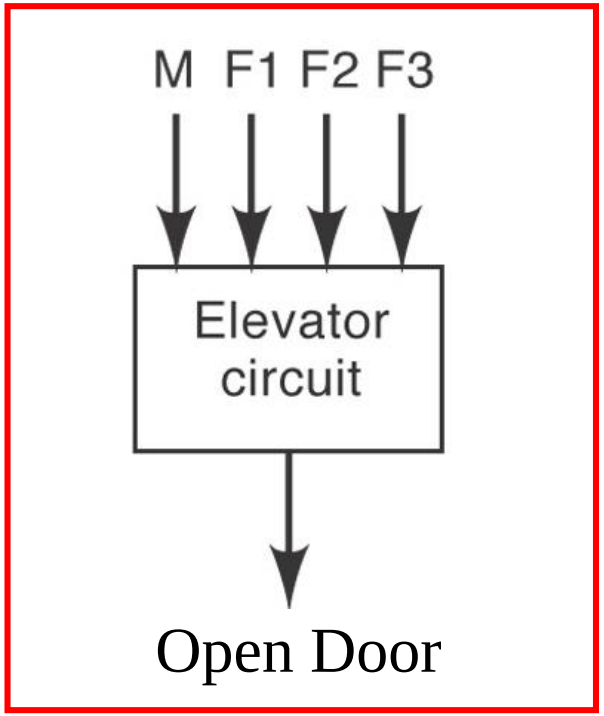
M = Logic signal indicating if the elevator is moving (M = 1) or stationary (M = 0)

F1, F2 and F3 are the floor level signals, normally 0 but gets to 1 when a particular floor is reached.

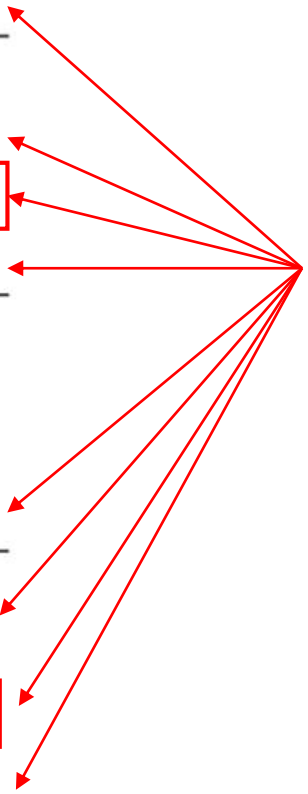
The circuit output is the "Open Door" signal, should be normally 0 but changes to 1 when the door is to open.



Inputs

Output

M	F1	F2	F3	OPEN
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	X
0	1	0	0	1
0	1	0	1	X
0	1	1	0	X
0	1	1	1	X
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	X
1	1	0	0	0
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X



Can only be on one floor at a time, only one floor input can be HI. The other floor inputs are then don't care conditions.

Use X to indicate the don't care conditions.

M	F1	F2	F3	OPEN
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	X
0	1	0	0	1
0	1	0	1	X
0	1	1	0	X
0	1	1	1	X
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	X
1	1	0	0	0
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Can only be on one floor at a time, only one floor input can be HI. The other floor inputs are then don't care conditions.

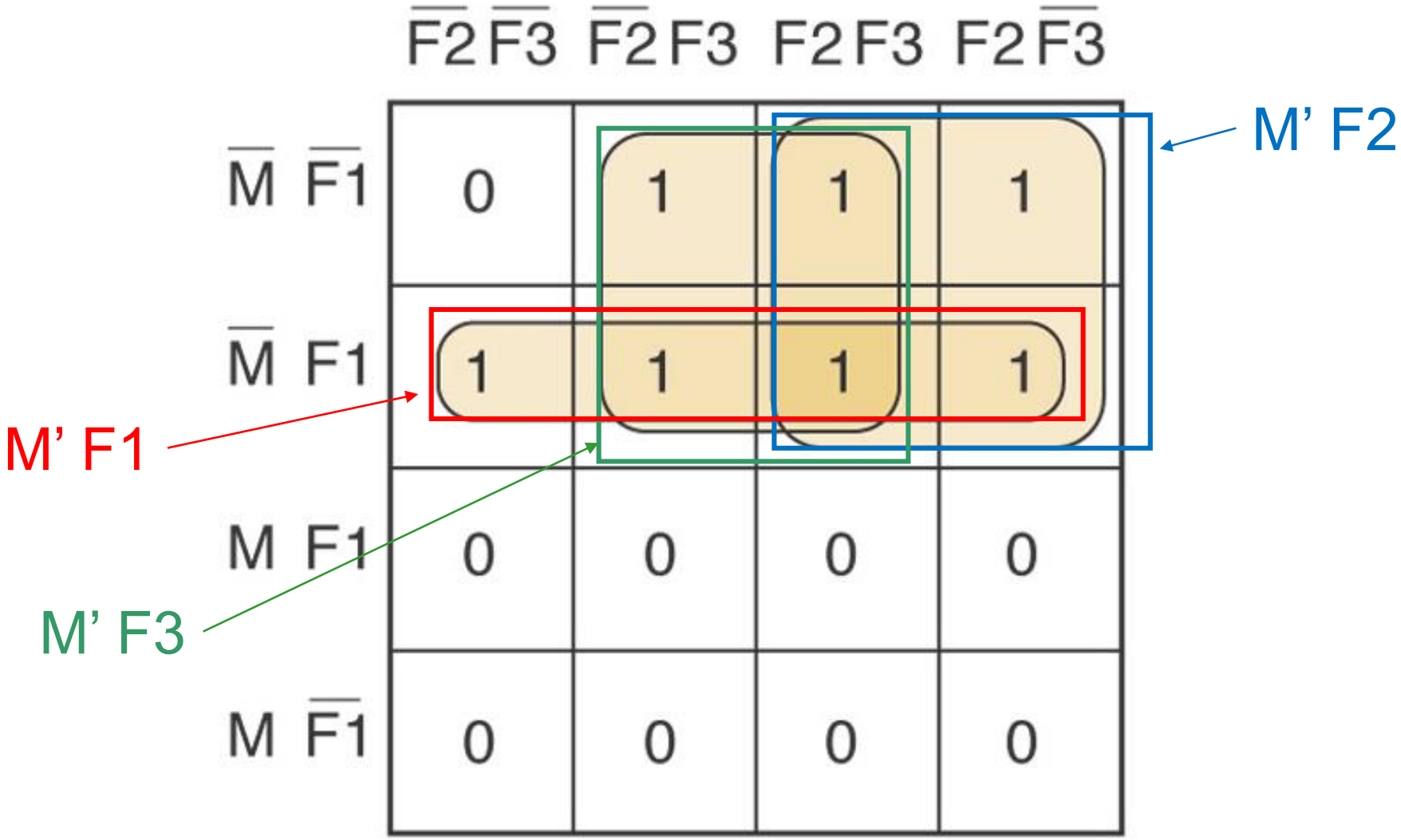
Use X to indicate the don't care conditions.

$$\text{Open Door} = \overline{M}\overline{F1}\overline{F2}F3 + \overline{M}F1\overline{F2}F3 + \overline{M}F1F2\overline{F3}$$

Open Door = $\overline{M}\,\overline{F1}\,\overline{F2}\,\overline{F3} + \overline{M}F1\,\overline{F2}\,\overline{F3} + \overline{M}F1F2\,\overline{F3}$

	$\overline{F2}\,\overline{F3}$	$\overline{F2}F3$	$F2F3$	$F2\,\overline{F3}$
$\overline{M}\,\overline{F1}$	0	1	X	1
$\overline{M}\,F1$	1	X	X	X
$M\,F1$	0	X	X	X
$M\,\overline{F1}$	0	0	X	0

Open Door = $\overline{M}F1\overline{F2}\overline{F3} + \overline{M}F1\overline{F2}F3 + \overline{M}F1F2\overline{F3}$



Open Door = M' F1 + M' F2 + M' F3

Week 6 Lecture 10b

- Karnaugh Map (K-Map)

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