ENGR (XMUT) 101
Engineering Technology

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Week 9 Lecture 3

- Main topics
  - Introduction to Engineering Technology
  - **Number system**
  - Logic Gates
  - Boolean Algebra

Number system

- What is a number system?
  - A way or style of writing symbols to represent numbers.
  - A mathematical notation for representing numbers of a given set.
  - Provides a unique representation of every number in the system

Number system

- The value of any digit in a number can be determined by:
  - the digit
  - its position in the number
  - the base of the number system

Types of Number system

Four types of number system

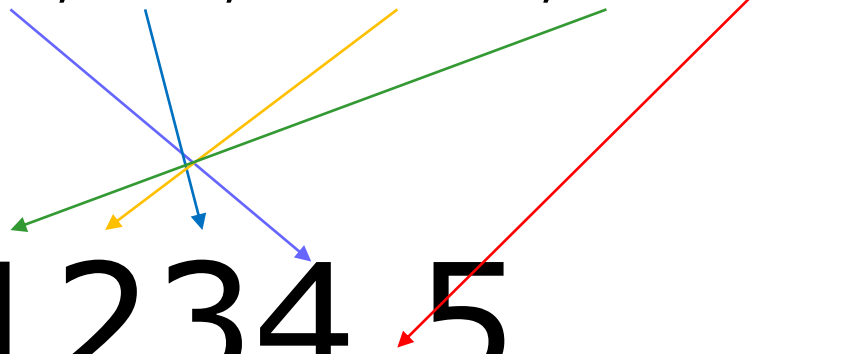
1. Decimal number system
2. Binary number system
3. Octal number system
4. Hexadecimal number system

Decimal number system

- Base 10 number system
  - 10 digits
  - 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9
  - Positions successive to the left of the decimal point represent units, tens, hundreds, thousands and so on.

Decimal number system

- Base 10 number system
  - 10 digits
  - 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9
  - Positions successive to the left of the decimal point represent units, tens, hundreds, thousands and so on
  - Example: 1234.5



Decimal number system

- Every position shows a particular power of the base (10)
- Example: 6789

Decimal number system

- Example: 6789

first digit

last digit

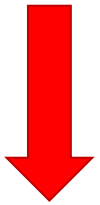
Row 1		Thousands	Hundreds	Tens	Units

Decimal number system

- Example: 6789

first digit

last digit



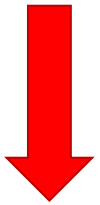
	Thousands	Hundreds	Tens	Units
Power	3	2	1	0

Decimal number system

- Example: 6789

first digit

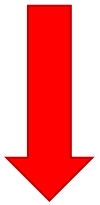
last digit



	Thousands	Hundreds	Tens	Units
Power	3	2	1	0
Base 10	10^3	10^2	10^1	10^0

Decimal number system

- Example: 6789



	Thousands	Hundreds	Tens	Units
Power	3	2	1	0
Base 10	10^3	10^2	10^1	10^0
Example	6	7	8	9
	6×10^3	7×10^2	8×10^1	9×10^0
	6 x 1000	7 x 100	8 x 10	9 x 1
	6000	700	80	9

Decimal number system

- Example: 6789

	Thousands	Hundreds	Tens	Units
Power	3	2	1	0
Base 10	10^3	10^2	10^1	10^0
Example	6	7	8	9
	6×10^3	7×10^2	8×10^1	9×10^0
	6 x 1000	7 x 100	8 x 10	9 x 1
	6000	700	80	9

Binary number system

- Base 2 number system
  - 2 digits
  - 0 and 1

Binary number system

- Base 2 number system
  - 2 digits
  - 0 and 1

- Rewind – decimal (Base 10) number system

  0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ……

Binary number system

- Base 2 number system
  - 2 digits
  - 0 and 1

- Rewind – decimal (Base 10) number system
  - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,
  - 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, ……

Binary number system

- Base 2 number system
  - 2 digits
  - 0 and 1

Compare decimal (Base 10) number system

- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, …
- 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24,…, 99, …
- 100, 101, 102, 103, 104, 105, 106,…, 999…

… to binary (Base 2) number system

- 0, 1, …
- 10, 11, …
- 100, 101,110, 111, …
- 1000, 1001, 1010, 1011, 1100, 1101,1110, 1111 …

Binary number system

Decimal	Binary
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111
16	1 0000

Decimal	Binary
17	1 0001
18	1 0010
19	1 0011
20	1 0100
21	1 0101
22	1 0110
23	1 0111
24	1 1000
25	1 1001
26	1 1010
27	1 1011
28	1 1100
29	1 1101
30	1 1110
31	1 1111
32	10 0000
33	10 0001

Octal number system

- Base 8 number system
  - 8 digits
  - 0, 1, 2, 3, 4, 5, 6, and 7

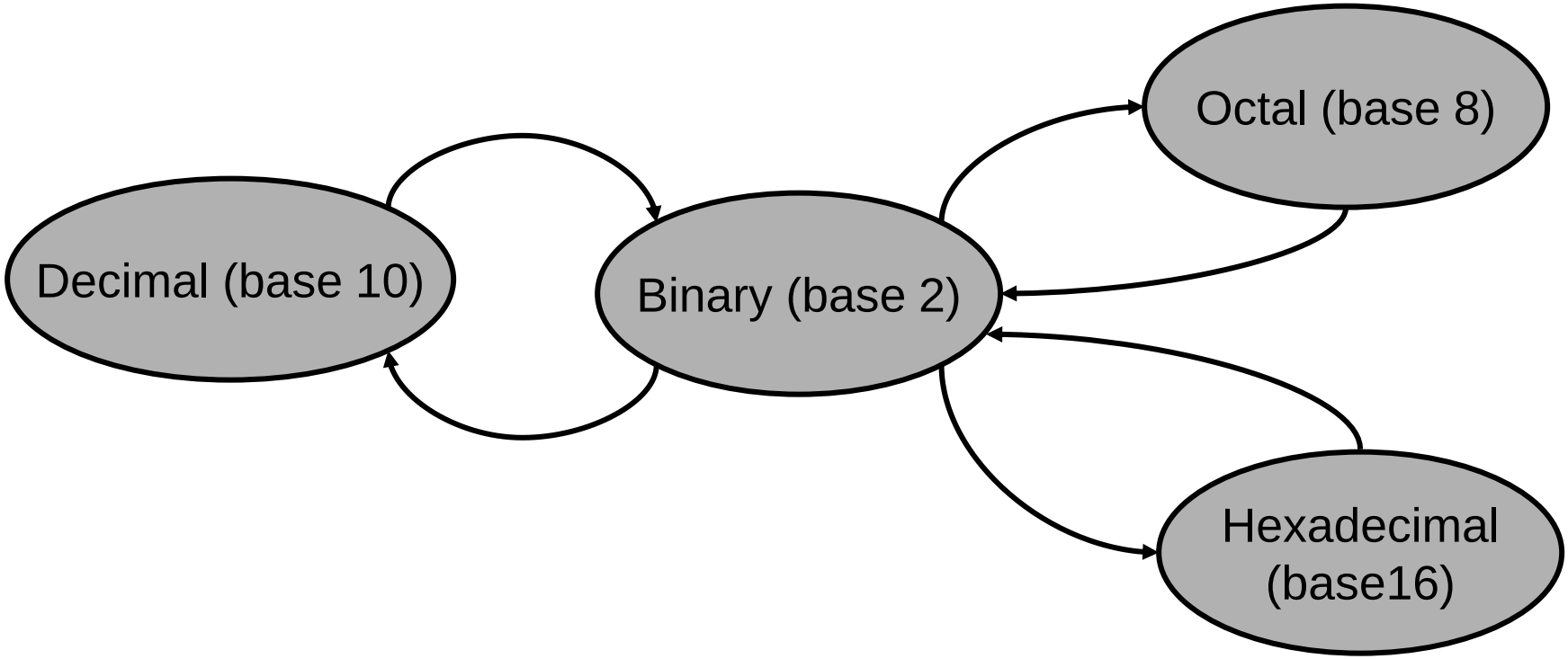
  - 0, 1, 2, 3, 4, 5, 6, 7,…
  - 10, 11, 12, 13, 14, 15, 16, **17, 20**, 21, 22…., 77…
  - 100, 101, 102, 103, 104, 105, **106, 107**, 110,

Hexadecimal number system

- Base 16 number system
  - 16 digits
  - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Conversion Between Number Systems



- **Learn to convert between bases.**

Convert *from* Decimal *to* Another Base

For each digit position:

1. **Divide decimal number by the base** (e.g. 2 for binary**)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Convert an Integer *from* Decimal *to* Another Base

For each digit position:

1. **Divide decimal number by the base (e.g. 2)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Example (a) $(13)_{10}$:

decimal number

subscript

Convert an Integer *from* Decimal *to* Another Base

For each digit position:

1. **Divide decimal number by the base (e.g. 2)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Example for $(13)_{10}$:

	Integer Quotient		**Remainder**	**Coefficient**
13/2 =	6	+	1	$a_0 = 1$

Convert an Integer *from* Decimal *to* Another Base

For each digit position:

1. **Divide decimal number by the base (e.g. 2)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Example for $(13)_{10}$:

	Integer Quotient		Remainder	Coefficient
13/2 =	6	+	1	$a_0 = 1$
6/2 =	3	+	0	$a_1 = 0$

Convert an Integer *from* Decimal *to* Another Base

For each digit position:

1. **Divide decimal number by the base (e.g. 2)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Example for $(13)_{10}$:

	Integer Quotient		Remainder	Coefficient
13/2 =	6	+	1	$a_0 = 1$
6/2 =	3	+	0	$a_1 = 0$
3/2 =	1	+	1	$a_2 = 1$

Convert an Integer *from* Decimal *to* Another Base

For each digit position:

1. **Divide decimal number by the base (e.g. 2)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Example for $(13)_{10}$:

	Integer Quotient		Remainder	Coefficient
13/2 =	6	+	1	$a_0 = 1$
6/2 =	3	+	0	$a_1 = 0$
3/2 =	1	+	1	$a_2 = 1$
1/2 =	0	+	1	$a_3 = 1$

Convert an Integer *from* Decimal *to* Another Base

For each digit position:

1. **Divide decimal number by the base (e.g. 2)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Example for $(13)_{10}$:

	Integer Quotient		Remainder	Coefficient
13/2 =	6	+	1	$a_0 = 1$
6/2 =	3	+	0	$a_1 = 0$
3/2 =	1	+	1	$a_2 = 1$
1/2 =	0	+	1	$a_3 = 1$

Answer $(13)_{10} = (a_3\, a_2\, a_1\, a_0)_2 = (1101)_2$ ← subscript

Convert an Integer *from* Decimal *to* Another Base

For each digit position:

1. **Divide decimal number by the base (e.g. 2)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Exercise 2

Convert the following decimal numbers to binary:

a) 17
b) 32
c) 85
d) 114

5 minutes to **convert** these 4 **decimal numbers** to **binary**!!

Convert an Integer *from* Decimal *to* Another Base

For each digit position:

1. **Divide decimal number by the base (e.g. 2)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Exercise 2

Convert the following decimal numbers to binary:

a) $(17)_{10} = (10001)_2$

b) $(32)_{10} = (100000)_2$

c) $(85)_{10} = (1010101)_2$

d) $(114)_{10} = (1110010)_2$

Convert an Integer *from* Decimal *to* Octal

For each digit position:

1. **Divide decimal number by the base (8)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Convert an Integer *from* Decimal *to* Octal

For each digit position:

1. **Divide decimal number by the base (8)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Example for $(175)_{10}$:

	Integer Quotient		Remainder	Coefficient
175/8 =	21	+	7	$a_0 = 7$
21/8 =	2	+	5	$a_1 = 5$
2/8 =	0	+	2	$a_2 = 2$

Answer $(175)_{10} = (a_2\, a_1\, a_0)_8 = (257)_8$

Convert a Integer *from* Decimal *to* Another Base

For each digit position:

1. **Divide decimal number by the base (8)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Exercise 4

Convert the following decimal numbers to octal:

a) $172 \rightarrow (???)_8$

b) 32

c) 99

d) 114

5 minutes to solve these 4 decimal numbers to octal!!

Convert a Integer *from* Decimal *to* Another Base

For each digit position:

1. **Divide decimal number by the base (8)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Exercise 4

Convert the following decimal numbers to octal:

a) $(172)_{10} = (254)_8$

b) $(32)_{10} = (40)_8$

c) $(99)_{10} = (143)_8$

d) $(114)_{10} = (162)_8$

Convert a Fraction *from* Decimal *to* Another Base

For each digit position:

1. **Multiply decimal number by the base (e.g. 2)**
2. **The *integer* is the highest-order digit**
3. **Repeat first two steps until fraction becomes zero.**

Convert a Fraction *from* Decimal *to* Another Base

For each digit position:

1. **Multiply decimal number by the base (e.g. 2)**
2. **The *integer* is the highest-order digit**
3. **Repeat first two steps until fraction becomes zero.**

Example for $(0.625)_{10}$:

	Integer		Fraction	Coefficient
0.625 x 2 =	1	+	0.25	$a_{-1} = 1$
0.250 x 2 =	0	+	0.50	$a_{-2} = 0$
0.500 x 2 =	1	+	0	$a_{-3} = 1$

Answer $(0.625)_{10} = (0.a_{-1}\, a_{-2}\, a_{-3})_2 = (0.101)_2$

Convert a Fraction *from* Decimal *to* Another Base

For each digit position:

1. **Multiply decimal number by the base (e.g. 2)**
2. **The integer is the highest-order digit**
3. **Repeat first two steps until fraction becomes zero.**

Exercise 3

Convert the following fraction numbers to binary:

a) $(0.172)_{10}$

b) $(0.32)_{10}$

c) $(0.859)_{10}$

d) $(0.114)_{10}$

Convert a Fraction *from* Decimal *to* Octal

For each digit position:

1. **Multiply decimal number by the base (e.g. 8)**
2. **The *integer* is the highest-order digit**
3. **Repeat first two steps until fraction becomes zero.**

Convert a Fraction *from* Decimal *to* Octal

For each digit position:

1. **Multiply decimal number by the base (e.g. 8)**
2. **The *integer* is the highest-order digit**
3. **Repeat first two steps until fraction becomes zero.**

Example for $(0.3125)_{10}$:

	Integer		Fraction	Coefficient
0.3125 x 8 =	2	+	0.5	$a_{-1} = 2$
0.5000 x 8 =	4	+	0	$a_{-2} = 4$

Answer $(0.3125)_{10} = (0.24)_8$

Convert a fraction *from* Decimal *to* Another Base

For each digit position:

1. **Multiply decimal number by the base (e.g. 8)**
2. **The integer is the highest-order digit**
3. **Repeat first two steps until fraction becomes zero.**

Exercise 5

Convert the following decimal numbers to octal:

a) $(0.172)_{10}$

b) $(0.32)_{10}$

c) $(0.99)_{10}$

d) $(0.114)_{10}$

Convert an Integer *from* Decimal *to* Hexadecimal

For each digit position:

1. **Divide decimal number by the base** (e.g. 16**)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Convert an Integer *from* Decimal *to* Hexadecimal

For each digit position:

1. **Divide decimal number by the base (e.g. 16)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Example for $(130)_{10}$:

	Integer Quotient		Remainder	Coefficient
130/16 =	8	+	2	$a_0 = 2$
8/16 =	0	+	8	$a_1 = 8$

Answer $(130)_{10} = (a_1\, a_0)_{16} = (82)_{16}$

Convert an Integer *from* Decimal *to* Hexadecimal

For each digit position:

1. **Divide decimal number by the base (e.g. 16)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Exercise 6

Convert the following decimal numbers to hexadecimal:

a) 127
b) 35
c) 89
d) 157

5 minutes to solve these 4 decimal numbers to binary!!

Convert an Integer *from* Decimal *to* Hexadecimal

For each digit position:

1. **Divide decimal number by the base (e.g. 16)**
2. **The *remainder* is the lowest-order digit**
3. **Repeat first two steps until no *divisor* remains.**

Exercise 6

Convert the following decimal numbers to hexadecimal:

a) $(171)_{10} = (AB)_{16}$

b) $(32)_{10} = (20)_{16}$

c) $(85)_{10} = (55)_{16}$

d) $(114)_{10} = (72)_{16}$

Convert a Fraction *from* Decimal *to* Hexadecimal

For each digit position:

1. **Multiply decimal number by the base (e.g. 16)**
2. **The *integer* is the highest-order digit**
3. **Repeat first two steps until fraction becomes zero.**

Convert a Fraction *from* Decimal *to* Hexadecimal

For each digit position:

1. **Multiply decimal number by the base (e.g. 16)**
2. **The *integer* is the highest-order digit**
3. **Repeat first two steps until fraction becomes zero.**

Example for $(0.3125)_{10}$:

		Integer		Fraction	Coefficient
0.3125×16	=	5	+	0	$a_{-1} = 5$

Answer $(0.3125)_{10} = (0.5)_{16}$

Convert a Fraction *from* Decimal *to* Hexadecimal

For each digit position:

1. **Multiply decimal number by the base (e.g. 16)**
2. **The integer is the highest-order digit**
3. **Repeat first two steps until fraction becomes zero.**

Exercise 7

Convert the following decimal numbers to hexadecimal:

a) $(0.172)_{10}$

b) $(0.32)_{10}$

c) $(0.99)_{10}$

d) $(0.114)_{10}$

Week 9 Lecture 3

- Decimal number conversion
  - Binary
  - Octal
  - Hexadecimal