

ECEN202 2024

Sampling time-varying signals.

Time Varying Signals

To this point we have assumed that the input signals to our A/D measurement system are static, or changes only slowly with time. We will now consider what happens when the input signal changes relatively quickly w.r.t. the sampling rate:

Consider the A/D sampling process:

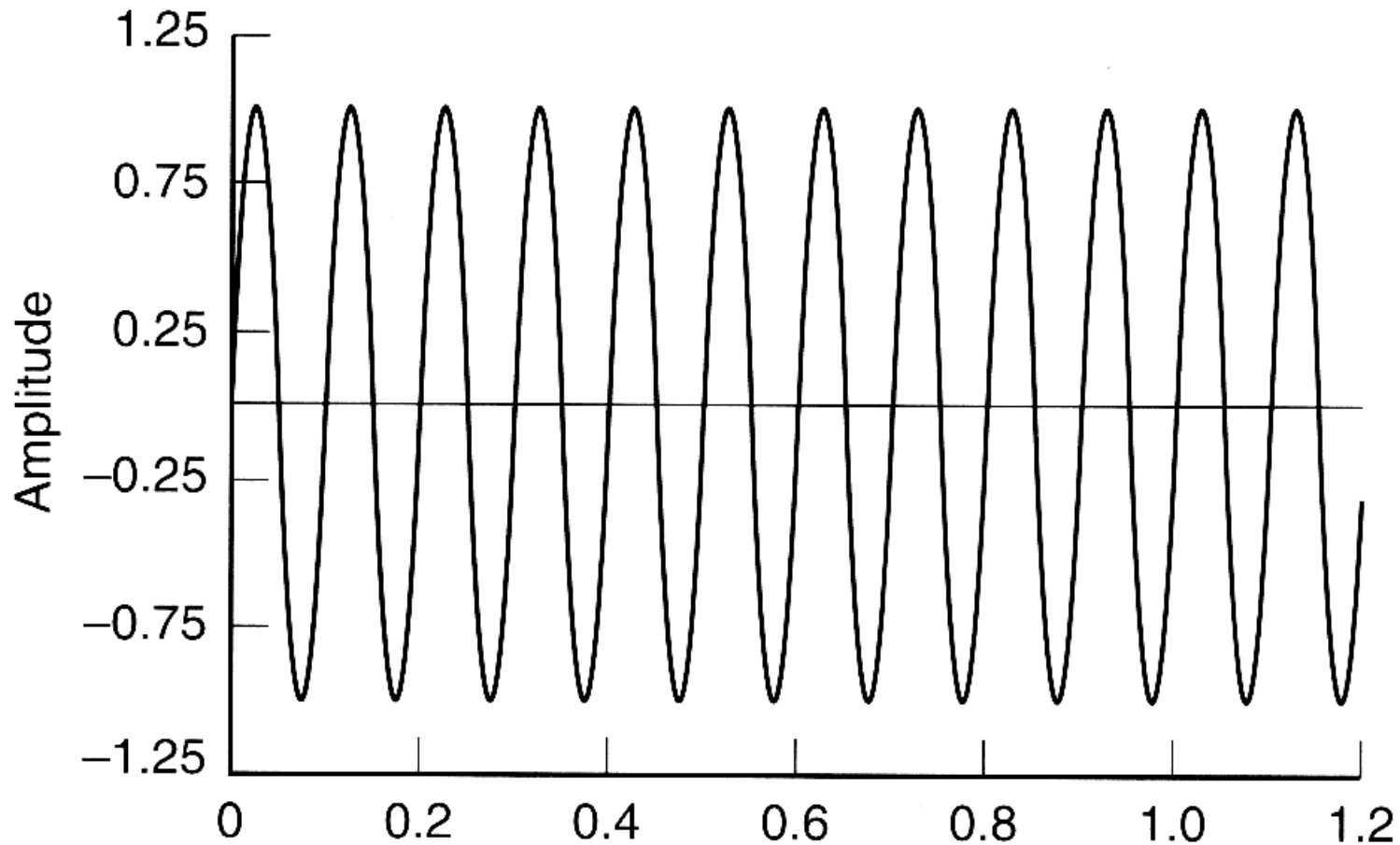
- The input voltage is sampled by the system and converted into a binary code. The number of bits in the code is determined by the A/D hardware and represents the accuracy with which the signal voltage can be represented
- However, this sampling also takes place at discrete points in time and the sampling rate determines how accurately the signal can be represented on the time scale.

Problem 1: How fast do I need to sample ?

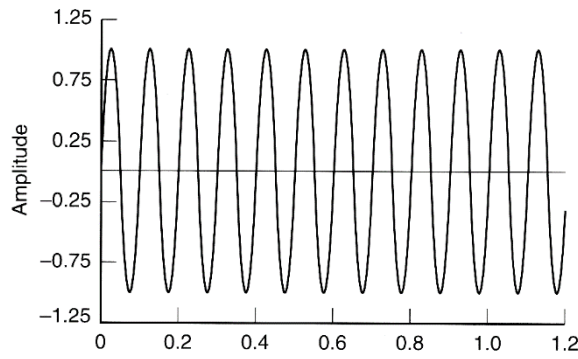
When we sample a continuous analogue signal with a A/D converter system, we are sampling this signal at discrete points in time - we thus make "snapshots" of the signal at that specific point in time. We have no record of the signal between these times and simply interpolate between the measured points to build up a picture of the signal.

However, this can lead to incorrect results and we need to take care in analysis of these signals. The degree to which the analogue signal can be faithfully represented in digital form is largely determined by the sampling rate, i.e. the number of times per second that the incoming signal is digitized.

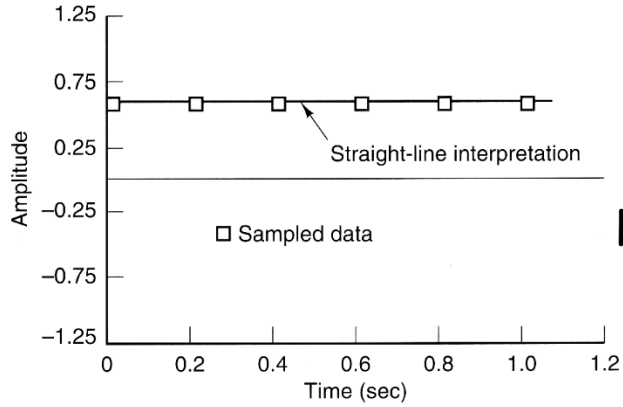
Consider an analog signal of 10 Hz:



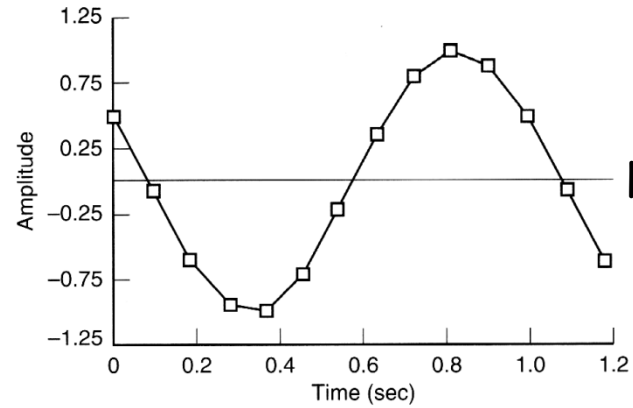
Now look at what will happen to the waveform if we sample the signal at different rates.



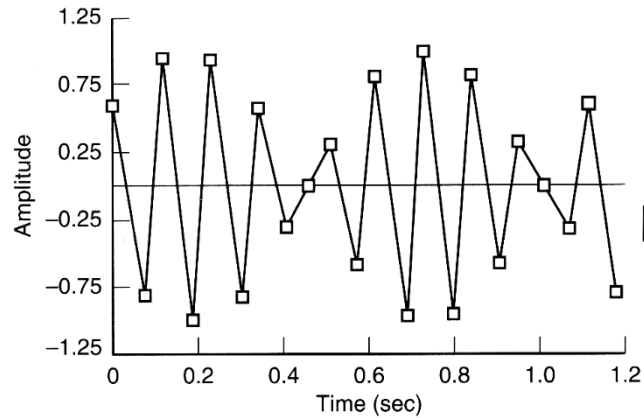
$$F_{\text{signal}} = 10 \text{ Hz}$$



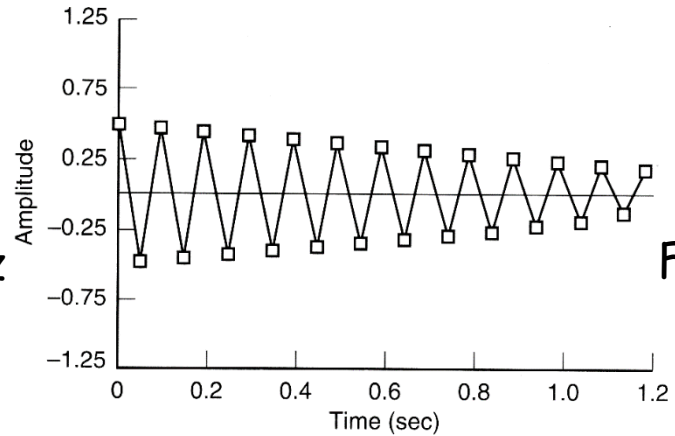
$$F_{\text{sample}} = 5 \text{ Hz}$$



$$F_{\text{sample}} = 11 \text{ Hz}$$



$$F_{\text{sample}} = 18 \text{ Hz}$$



$$F_{\text{sample}} = 21 \text{ Hz}$$

From the previous it is clear that all sampling rates below 21 Hz produced frequencies that were not present in the original signal. These false frequencies appearing in the data is known as aliasing and will occur when the signal is sampled at too low a rate.

To know what is the correct (minimum acceptable) sampling rate we look at the sampling rate theorem:

Sampling rate theorem: In order to construct the original waveform correctly, the sampling rate must be greater than twice the highest frequency component of the original signal. Also known as the Nyquist or the Nyquist-Shannon theorem.

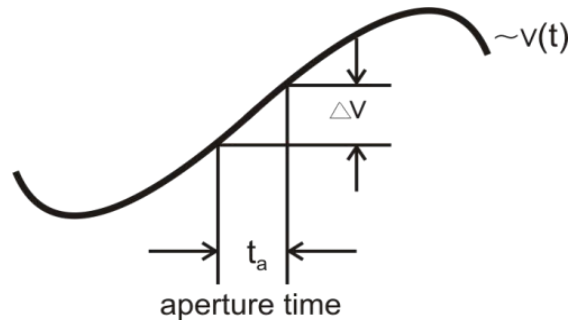
If the Nyquist-Shannon theorem is not satisfied there will be alias frequencies present in our sampled data. Alias frequencies will also be present if the signal contains high frequency components higher than the Nyquist frequency, eg. high frequency noise.

To prevent the occurrence of such alias frequencies, the incoming signal would normally be passed through a low pass filter. Such a filter will only allow frequencies lower than the Nyquist frequency to be sampled. However, a real filter does not have a sharp edge ("roll-off") it is more practical to sample at 5 to 10 times the highest frequency component.

Problem 2: By how much does the signal change during the sampling process - i.e. how good a representation of the signal is the value that corresponds to the time stamp of the signal ?

As we have seen with practical A/D converters the conversion process takes time - if we have a fast changing signal the signal will be changing while the conversion process is taking place - how much of an error does that introduce ?

The time required to acquire a sample is called the aperture time.



$$\Delta v = \frac{dv}{dt} t_a$$

Example: *sine wave $\frac{dv}{dt}$ is maximum at zero crossing*

$$\begin{aligned}\Delta v &= \frac{d}{dt} (V \sin \omega t)_{t=0} t_a \\ &= V \omega t_a\end{aligned}$$

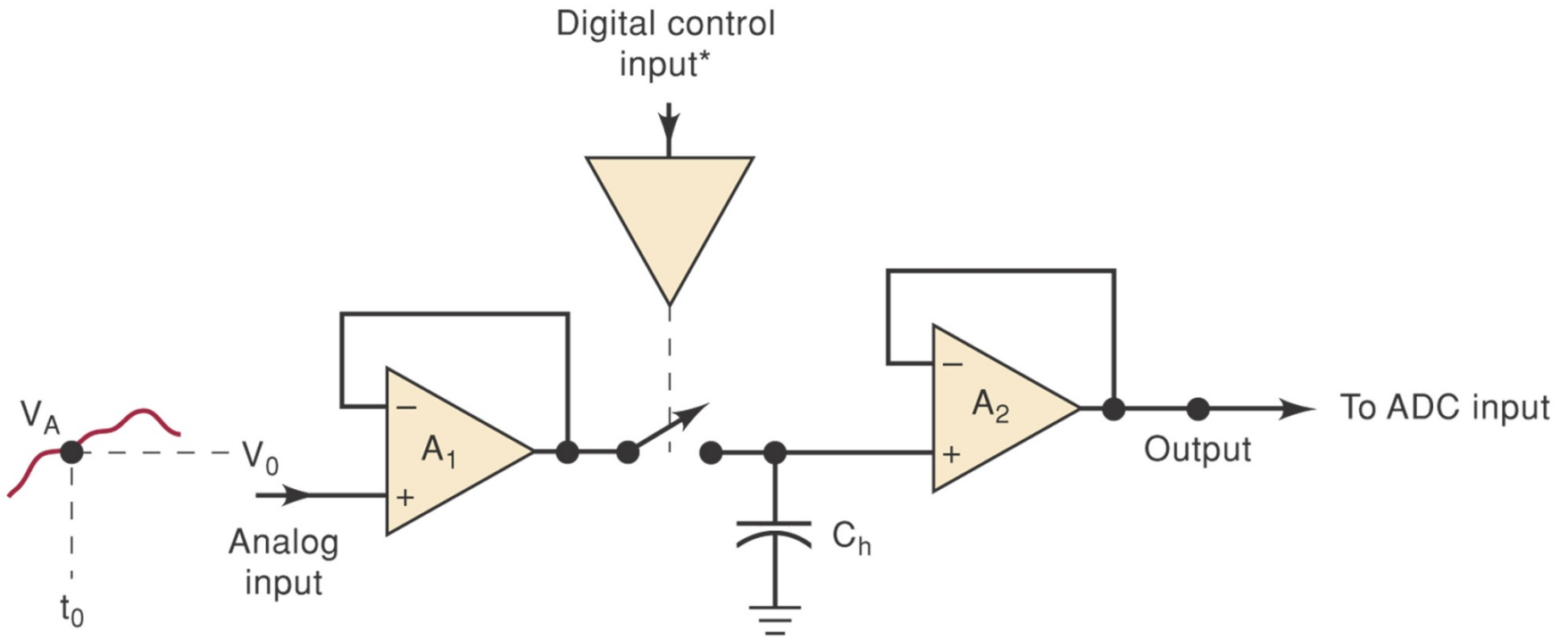
or:
$$\boxed{\frac{\Delta v}{V} = 2\pi f t_a}$$

therefore to achieve 10-bit resolution ie, 1 in 2^{10} or 0.1%:

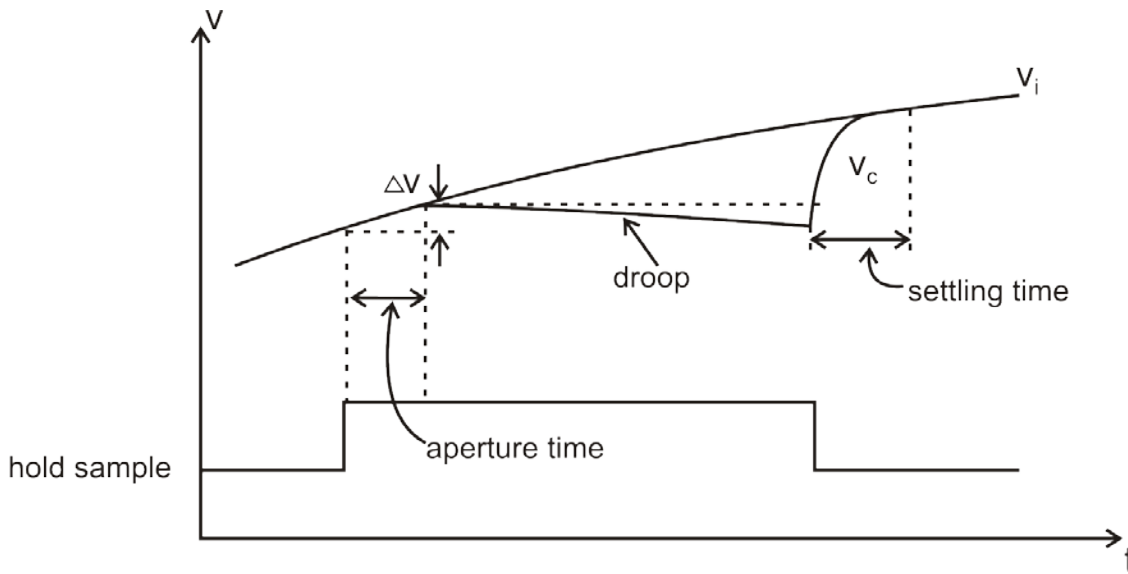
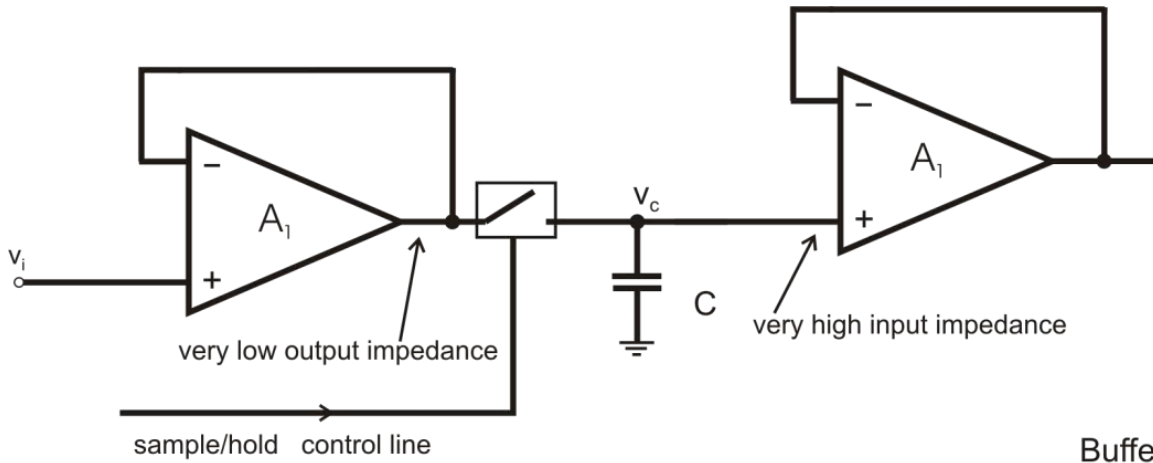
$$\begin{aligned}t_a &= \frac{\Delta V}{V} \frac{1}{2\pi f} = \frac{0.1}{100} \frac{1}{2\pi f} \\ &= 160\text{ns for } f = 1\text{kHz}\end{aligned}$$

This would require either a very fast A/D converter, or the use of a sample and hold circuit.

Sample and Hold Circuits



*Control = 1 → switch closed → sample mode
Control = 0 → switch open → hold mode

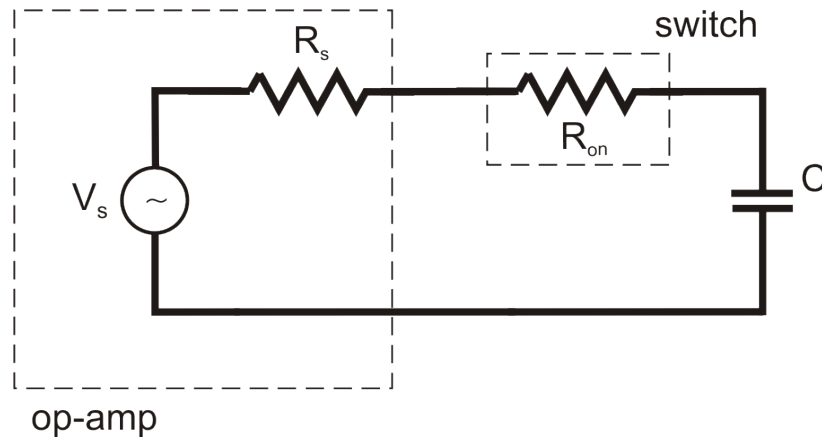


Acquisition Time: Time switch remains closed

Aperture time – delay between ‘hold’ command and capacitor ceasing to follow input.

Droop rate – $\frac{dv_c}{dt}$ during hold (determined by C and input impedance of op-amp).

Delay when switching from “hold” to “sample” is determined by:



$$\tau = (R_s + R_{on})C$$

Acquisition time (sample & hold) – determined by the maximum settling time (occurs for full scale change in voltage across C)

Aperture uncertainty – variation in the actual sampling time. This is the main parameter that limits the upper frequency that can be sampled.

A/D D/A System characteristics.

Previously: A/D converters, flash, successive approximation, staircase,

Could clearly see:

- An inherent error in the conversion process
- Error decreased with the number of bits used in representation.
- The conversion process takes time - is there also an error associated with this ?
- Trade-off between price - speed - accuracy

Will now look at some typical characteristics that define the performance of an A/D system.

Resolution

The resolution is defined as the smallest change in the input voltage that will ensure a change the digital output by one code level.

Determined by:

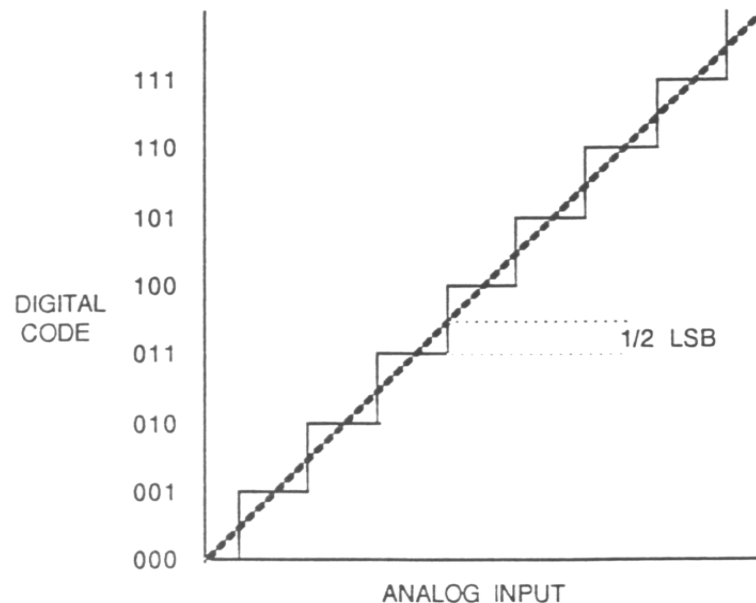
- (1) The number of A/D bits
- (2) The input range of the converter.

$$\text{Resolution} = \text{Input Voltage Range} / (2^{\text{bits}} - 1)$$

A so-called resolution error will then always be associated with an A/D measurement. This resolution error or quantization error is then defined as:

$$= \pm 0.5 \text{ LSB} = \pm 0.5 * \text{Input Range} / (2^{\text{bits}} - 1)$$

The number of bits is also often simply referred to as the resolution of the converter. However, this neglects the input range.



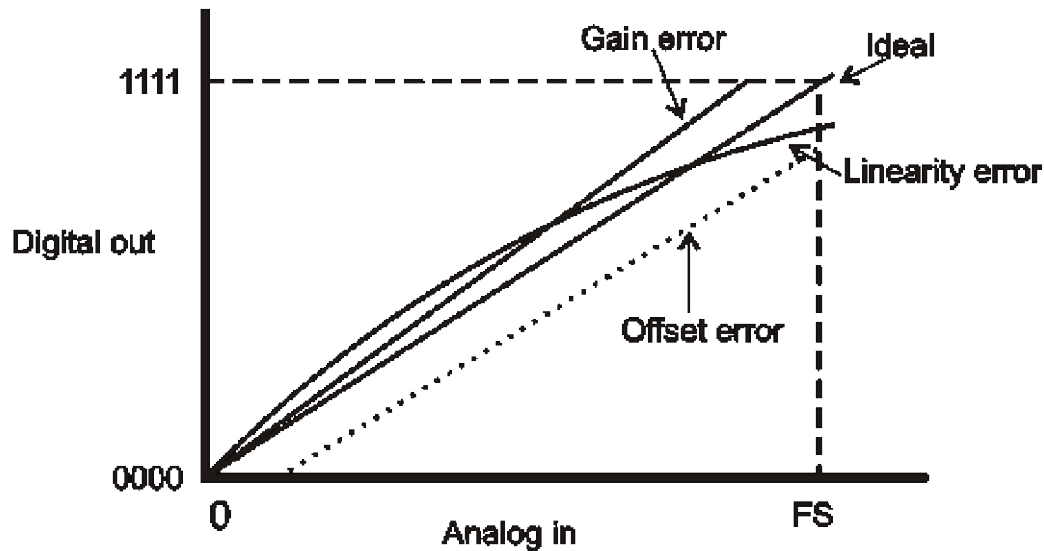
For an input Range of 10 V (0 to 10 V or -5 to +5 V):

Property	8 bits	10 bits	12 bits	14 bits	16 bits
Steps ($2^n - 1$)	255	1023	4095	16 383	65 535
Resolution	39.2 mV	9.78 mV	2.44 mV	0.61 mV	0.15 mV
Resolution as % Range	0.39 %	0.098 %	0.024 %	0.006 %	0.0015 %
Quantisation Error	19.61 mV	4.89 mV	1.22 mV	0.31 mV	0.08 mV

Some important parameters used to characterise A/D and D/A converter accuracy:

Resolution – Smallest analog change that can be distinguished by an ADC or produced by a DAC. It is the analog value of the LSB, (Q). $\text{Resolution} = V_{\text{FS}} / (2^n - 1)$

Linearity – The maximum deviation from a straight line drawn between the end points of the converter transfer function.



Offset error – Error by which the transfer function fails to pass through origin, referred to analog axis (adjustable to zero).

Gain error – Difference in slope between actual and ideal transfer function in percent. (adjustable to zero)

Differential linearity – Maximum deviation of actual bit size from theoretical.

Monotonicity – Continuously increasing input results in continuously increasing output.

Accuracy – The difference between the expected and measured output voltage or current in terms of change caused by changing the LSB. Accurate to $\pm \frac{1}{2}$ LSB

Settling time – Time required for output to settle within $\frac{1}{2}$ LSB of final value after full scale change in input.

Stability – A measure of how constant the output remains with age and with variations in temperature and power supply.

$$\begin{aligned}\text{Dynamic range} &= 20 \log_{10} \frac{\text{full scale analog}}{Q} \\ &= 20 \log_{10} \frac{(2^N - 1)Q}{Q} \\ &= 20 \log_{10} (2^N - 1)\end{aligned}$$

Input Range

Range of analog voltages over which the converter will be able to produce a meaningful digital representation of the signal. This input range can be unipolar (0 to +x V) or bipolar (-x V to +x V).

Number of bits + Range will determine effective resolution.

Take care when setting I/P range to ensure maximum resolution.

Test yourself

An 8-bit analog to digital converter has an input range of 0-10 Volt. Calculate:

(a) The resolution.

(b) The quantisation error.

(c) Assume that the converter is constructed using a flash or parallel architecture. How many comparators and resistors were needed for this construction ?

(d) The converter is constructed using a successive approximation architecture. What are the most important components ? Assume a clock cycle of $1 \mu\text{s}$ - how long will one conversion take ?

(e) Calculate what the output will be in both binary and decimal for inputs of:

(a) 1.32 V

(b) 5.95 V

(c) 10.51 V