XMUT 202
Digital Electronics

A/Prof. Pawel Dmochowski

School of Engineering and Computer Science
Victoria University of Wellington

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UNIVERSITY OF WELLINGTON
Te Whare Wānanga o te Ūpoko o te Ika a Māui
CAPITAL CITY UNIVERSITY

Review of Logic Gates

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  - An electronic component that can be used to conduct electricity based on a rule.

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  - An electronic component that can be used to conduct electricity based on a rule.
  - The output of the logic gate is the result of applying this rule to one or more inputs.

Review of Logic Gates

- Assuming we understand the concept of binary numbers, we will study ways of describing how systems using binary logic levels make decisions.

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- Boolean algebra is an important tool in describing, analyzing, designing, and implementing digital circuits.

Boolean Constants and Variables

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Boolean Constants and Variables

- Boolean algebra allows only two values; 0 and 1.
- Logic 0 can be: false, off, low, no, open switch.
- Logic 1 can be: true, on, high, yes, closed switch.
- Three basic logic operations: OR, AND, and NOT.

Truth Tables

- A truth table describes the relationship between the inputs and output of a logic circuit.

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Inputs Output

A	B	X
0	0	1
0	1	0
1	0	1
1	1	0

Truth Tables

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Number of inputs = 2

A	B	X
0	0	1
0	1	0
1	0	1
1	1	0

Number of entries $= 2^2 = 4$

Truth Tables

- The number of entries corresponds to the number of inputs.
  - A 3 inputs table will have 2^3 or 8 entries.

Number of inputs = 3

Number of entries $= 2^3 = 8$

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Truth Tables

- Truth table with 4 inputs.

Number of inputs = 4

Number of entries $= 2^4 = 16$

A	B	C	D		x
0	0	0	0		0
0	0	0	1		0
0	0	1	0		0
0	0	1	1		1
0	1	0	0		1
0	1	0	1		0
0	1	1	0		0
0	1	1	1		1
1	0	0	0		0
1	0	0	1		0
1	0	1	0		0
1	0	1	1		1
1	1	0	0		0
1	1	0	1		0
1	1	1	0		0
1	1	1	1		1

1) OR operation with OR Gate

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- Truth table for a two inputs OR gate.

OR

A	B	x = A + B
0	0	0
0	1	1
1	0	1
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0	1	X = 0 + 1 = **1**
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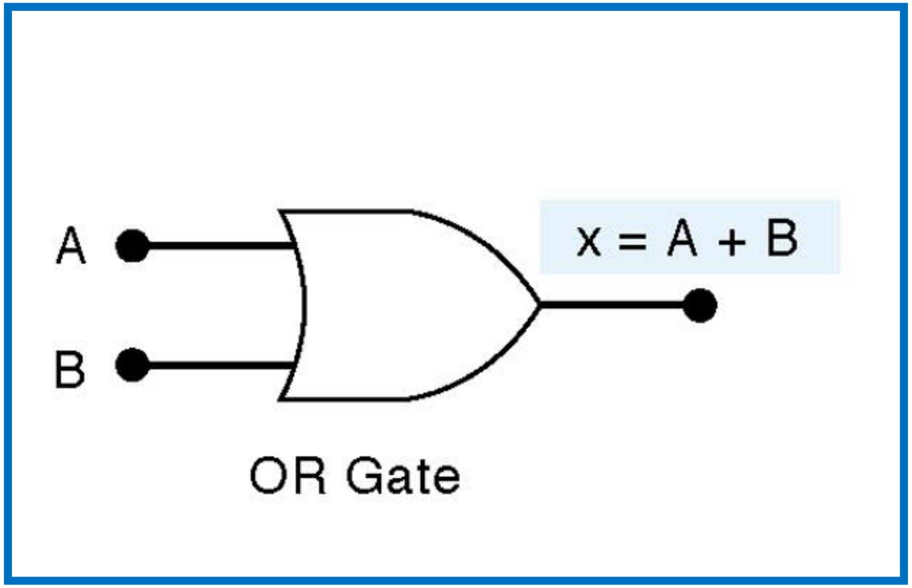
1) OR operation with OR Gate

- Truth table and circuit symbol for a two input OR gate.

OR

A	B	x = A + B
0	0	0
0	1	1
1	0	1
1	1	1



1) OR Operation With OR Gates

- The OR operation is similar to addition but when A and B are 1, the OR operation produces 1 + 1 = 1.

1) OR Operation With OR Gates

- The OR operation is similar to addition but where A and B are 1, the OR operation produces 1 + 1 = 1.
- In the Boolean expression

  $X = 1 + 1 + 1 = 1$

  We could say that x is true (1) when A is true (1) OR B is true (1) OR C is true (1).

1) OR Operation With OR Gates

- There are many examples of applications where an output function is desired when one of multiple inputs is activated.

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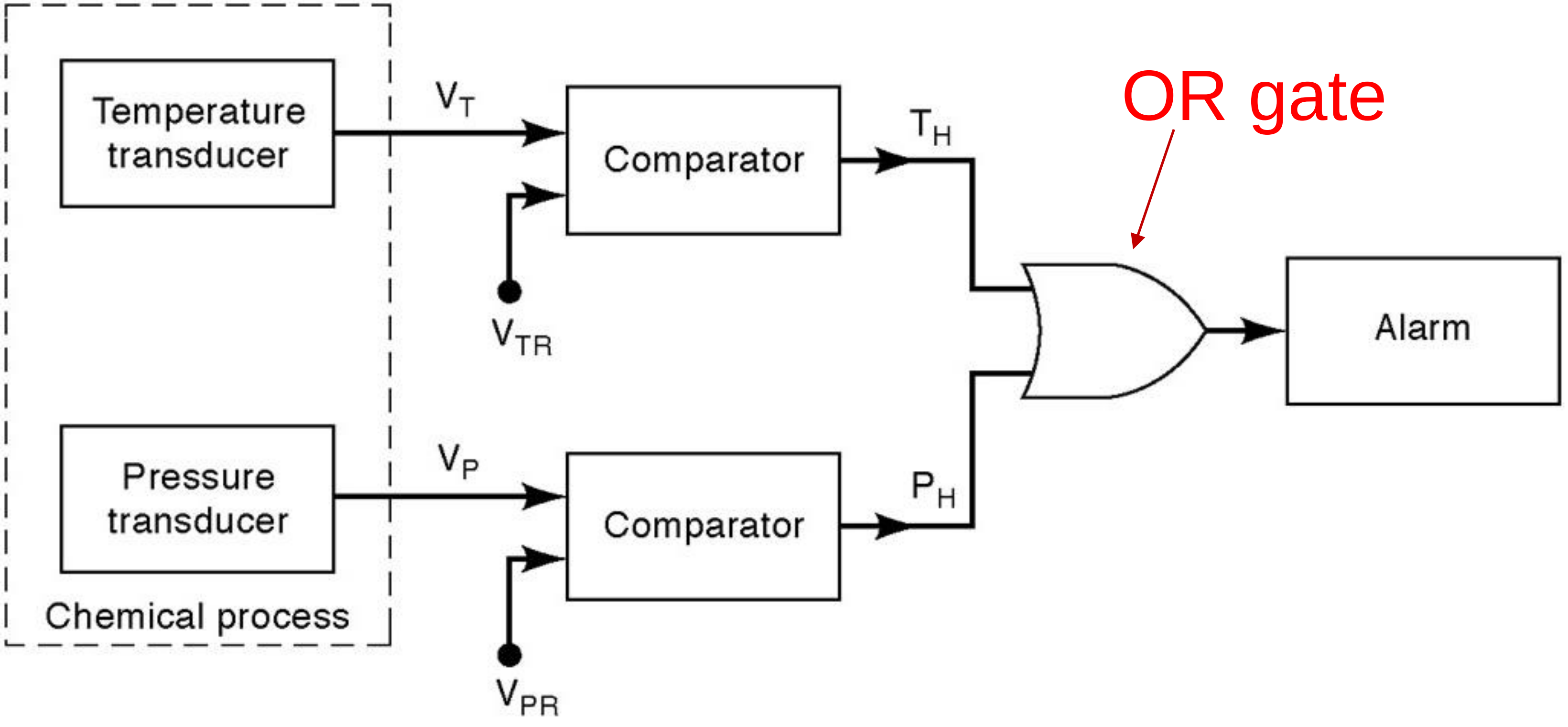
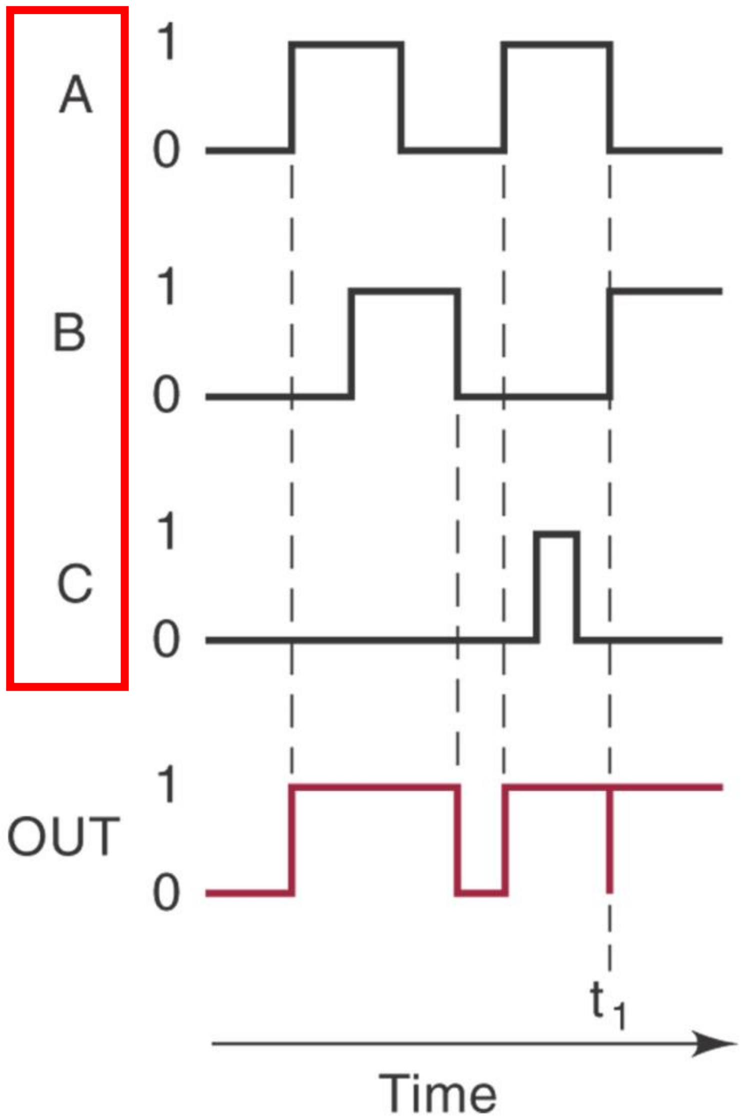
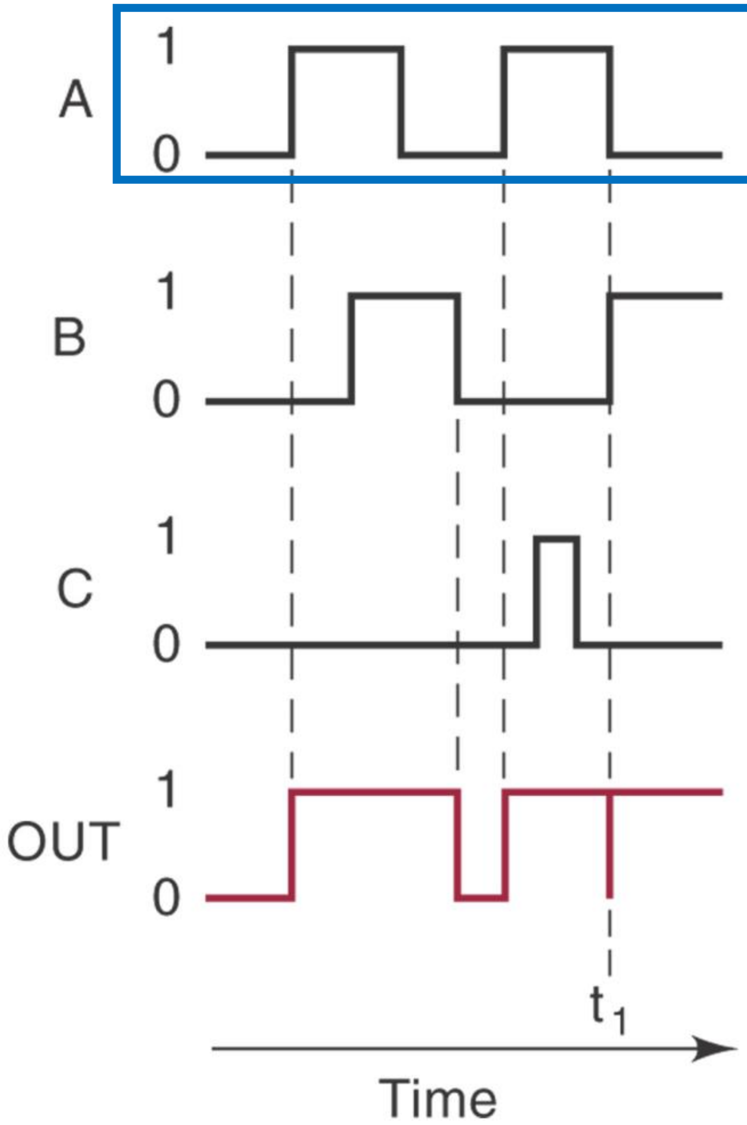


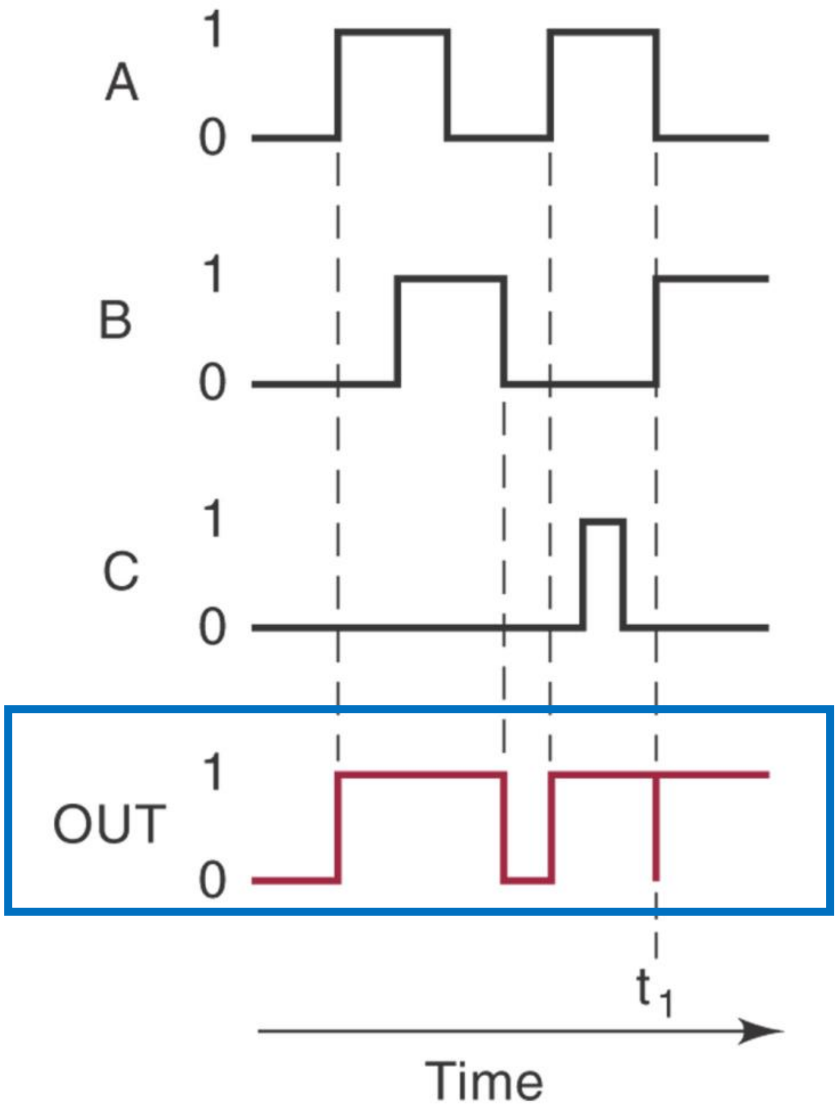
Representing Logic Functions by Timing Diagrams



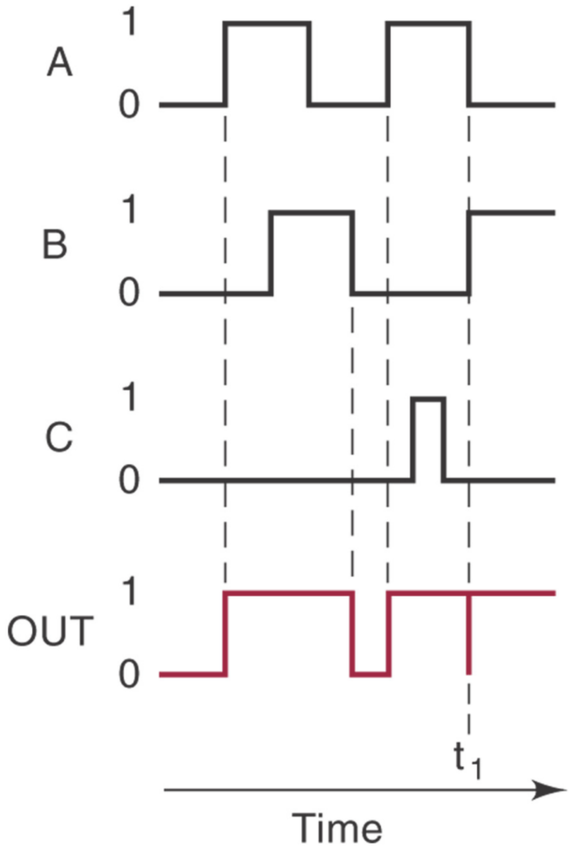
Representing Logic Functions by Timing Diagrams

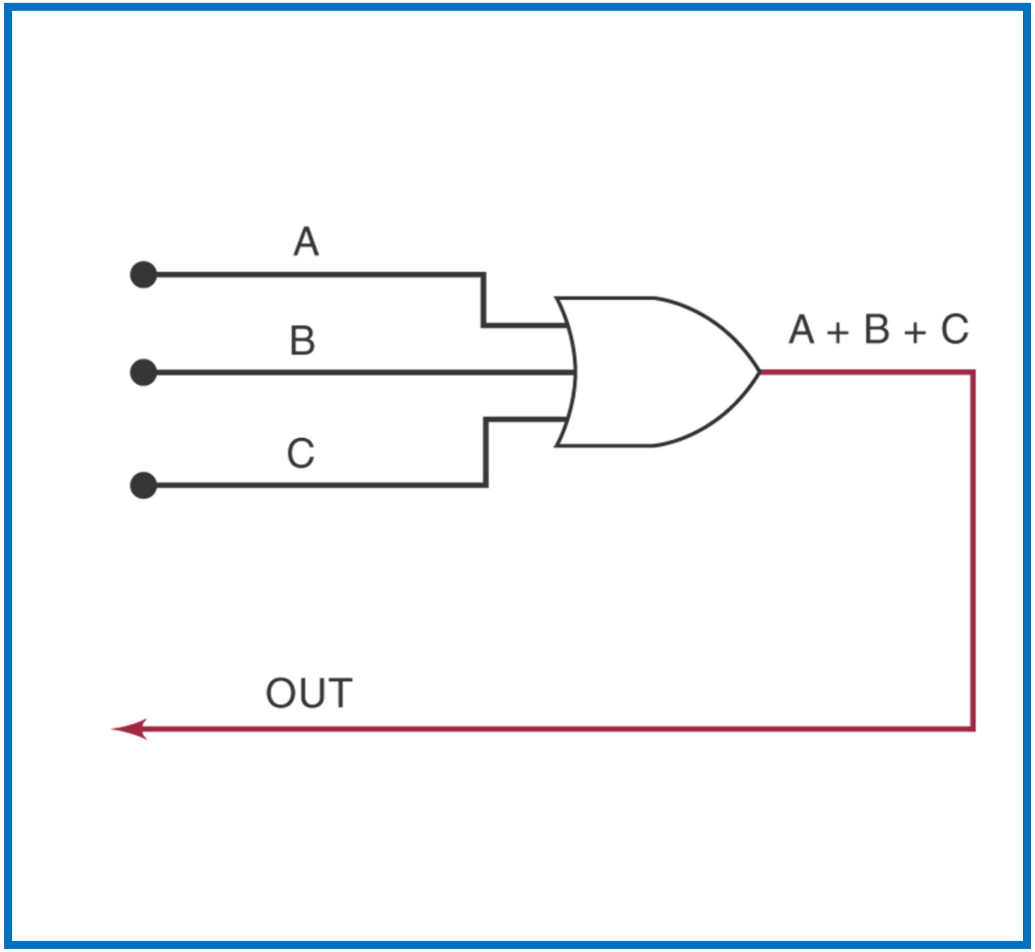


Representing Logic Functions by Timing Diagrams



Representing Logic Functions by Timing Diagrams





2) AND Operations with AND gate

- The Boolean expression for the AND operation is

$$X = A \cdot B$$

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- Truth table for a two inputs AND gate. Note the difference between OR and AND gates.

AND

A	B	$x = A \cdot B$
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0	1	0
1	0	0
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AND

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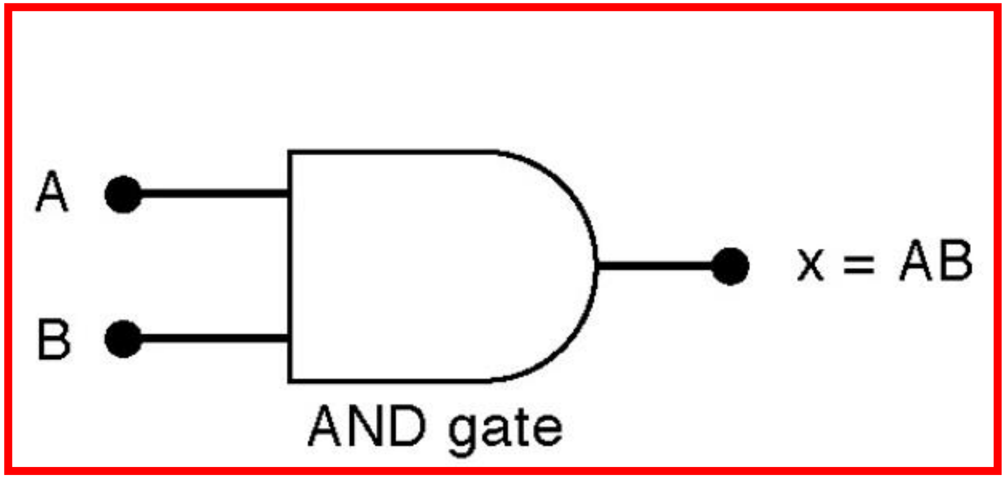
0	0	0
0	1	0
1	0	0
1	1	1



2) AND Operation With AND Gates

- The AND operation is similar to multiplication.
- In the Boolean expression

  $X = A \cdot B \cdot C$

  x will equal 1 only when A, B, and C are all 1.

3) NOT Operation

- The Boolean expression for the NOT operation is

$$x = \overline{A}$$

3) NOT Operation

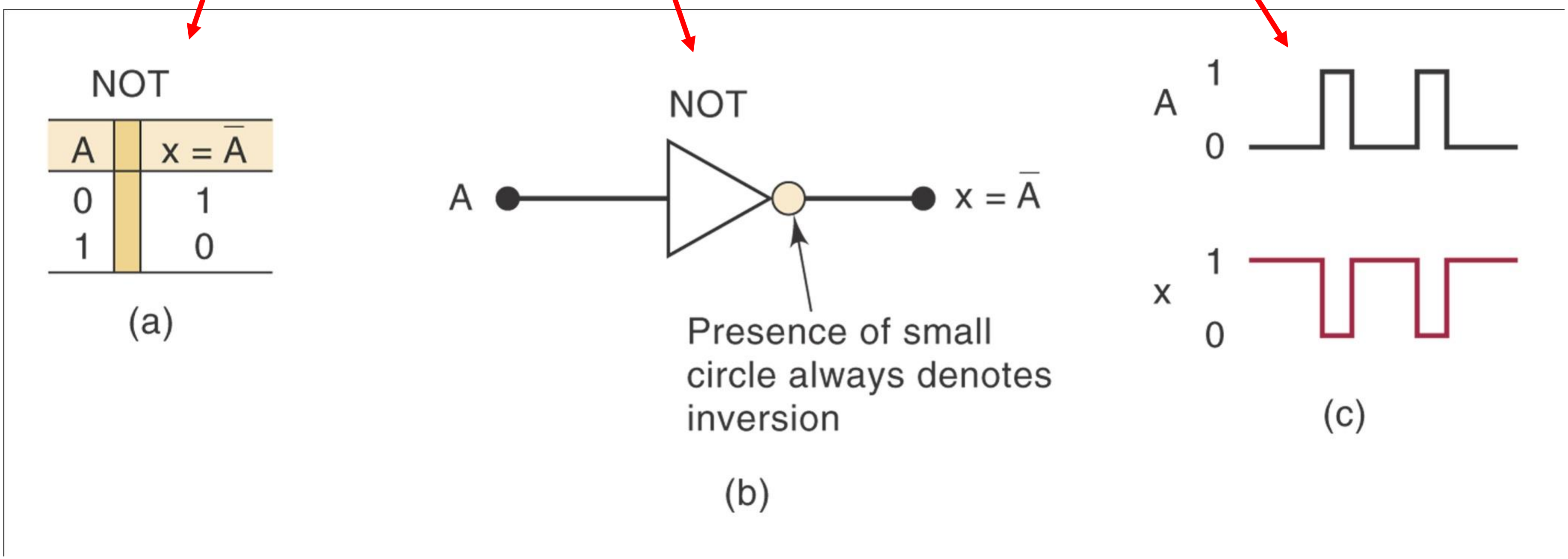
- The Boolean expression for the NOT operation is

$$x = \overline{A}$$

- This is read as:
  - x equals NOT A, or
  - x equals the inverse of A, or
  - x equals the complement of A

3) NOT Operation

- Truth table, symbol, and sample waveform for the NOT circuit.

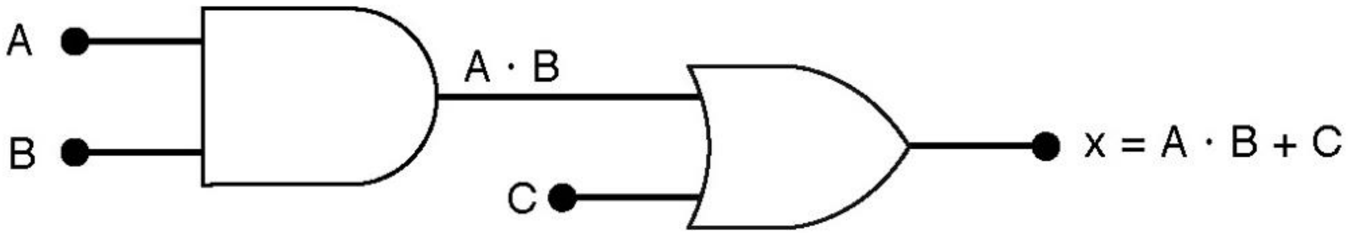


Describing Logic Circuits Algebraically

- The three basic Boolean operations (OR, AND, NOT) can describe any logic circuit.

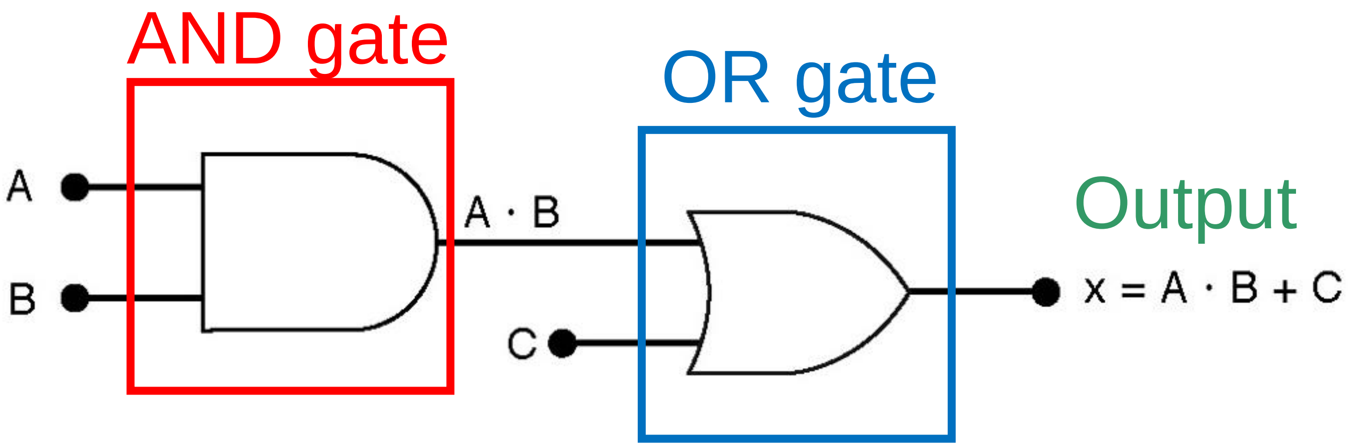
Describing Logic Circuits Algebraically

Example 1: Boolean expression for a logic circuit



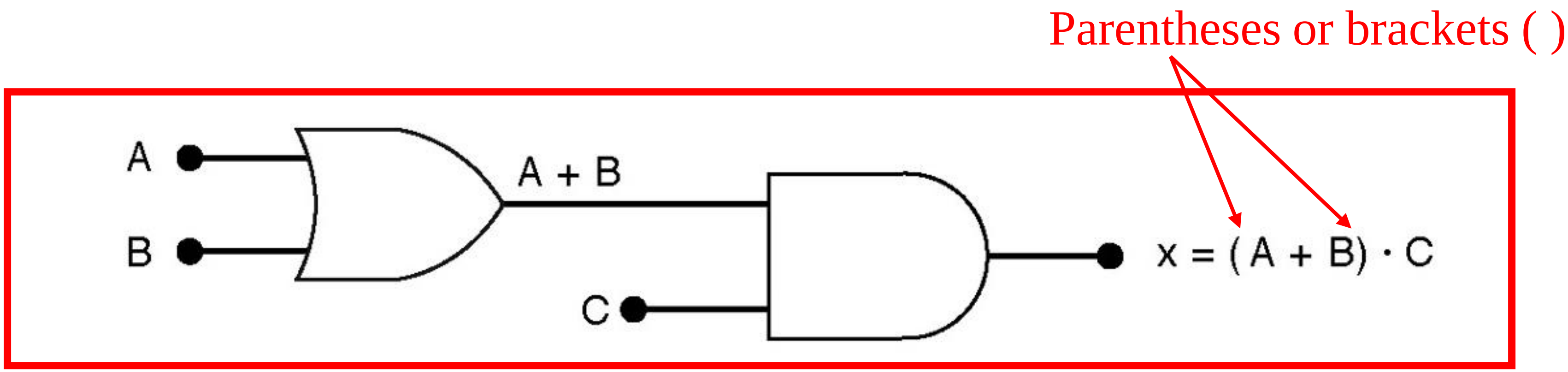
Describing Logic Circuits Algebraically

Example 1: Boolean expressions for a logic circuit:

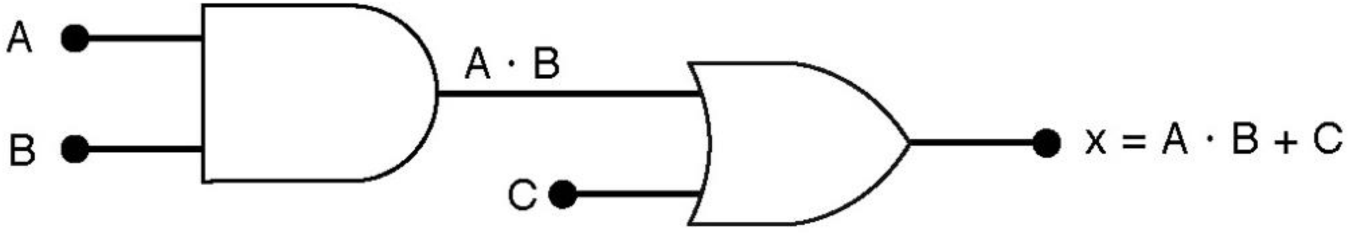


Describing Logic Circuits Algebraically

Example 2: Boolean expressions for logic circuits:

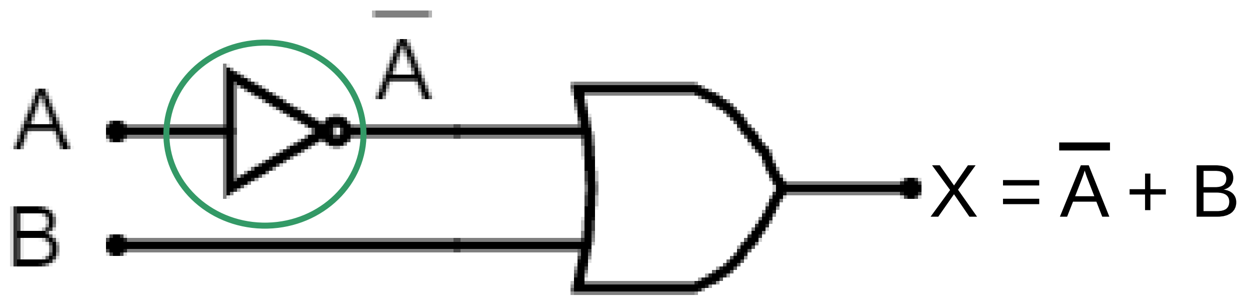


Example 1:



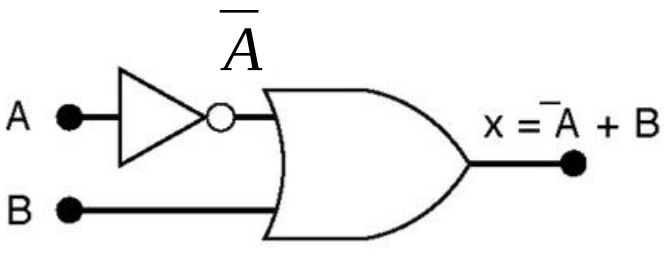
Describing Logic Circuits Algebraically

- The output of an inverter is equivalent to the input with a bar over it.
- Input A through an inverter is $\overline{A}$
- Example using inverters.

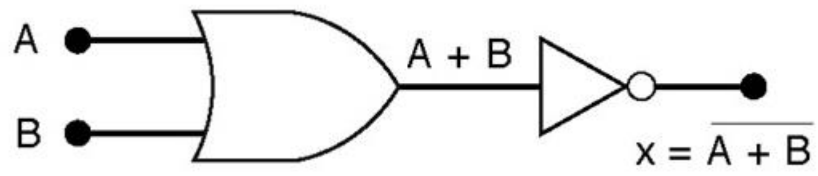


Describing Logic Circuits Algebraically

- A second example



(a)

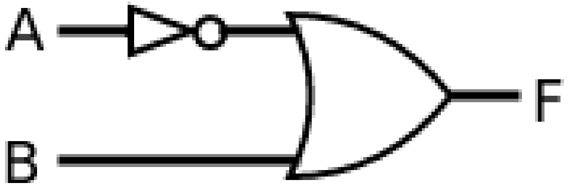


(b)

Evaluating Logic Circuit Outputs

Example 1: Complete the truth table for the given circuit.

Logic circuit



Truth table

A	B	F
0	0	
0	1	
1	0	
1	1	

Evaluating Logic Circuit Outputs

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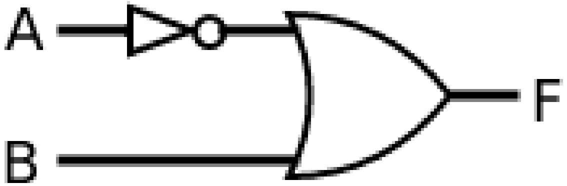
Logic circuit



Truth table

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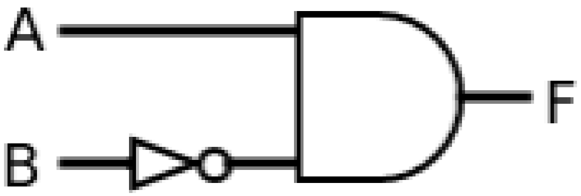
Evaluating Logic Circuit Outputs

<u>Exercise 1:</u> Complete the truth table for the given circuit.

Logic circuit



Truth table

A	B	F
0	0	**0**
0	1	**0**
1	0	**1**
1	1	**0**

Evaluating Logic Circuit Outputs

Rules for evaluating a Boolean expression:

1. Perform all inversions of single terms.
2. Perform all operations within **parenthesis**.
3. Perform AND operation before an OR operation unless parenthesis indicate otherwise.
4. If an expression has a bar over it, perform the operations inside the expression and then invert the result.

Evaluating Logic Circuit Outputs

Exercise 2:

- Evaluate the given Boolean expression by substituting values and performing the indicated operations.

$$A = 0, B = 1, C = 1, \text{ and } D = 1$$

$$x = \overline{A}BC\overline{(A + D)}$$

Evaluating Logic Circuit Outputs

- Evaluate Boolean expressions by substituting values and performing the indicated operations:

$$A = 0, B = 1, C = 1, \text{ and } D = 1$$

$$x = \overline{A}BC\overline{(A + D)}$$

Solution:

Rule 1 → $x = \overline{0} \cdot 1 \cdot 1 \cdot \overline{(0 + 1)}$

Rule 2 → $= 1 \cdot 1 \cdot 1 \cdot \overline{(0 + 1)}$

Rule 1 → $= 1 \cdot 1 \cdot 1 \cdot (\overline{1})$

Rule 3 → $= 1 \cdot 1 \cdot 1 \cdot 0$

$$x = 0$$

Evaluating Logic Circuit Outputs

Exercise 2:

- Evaluate the given Boolean expression by substituting values and performing the indicated operations.

A = 1, B = 0, C = 0 and D = 0

$$x = \overline{A}BC\overline{(A + D)}$$

You have 10 minutes to determine the answer!

Evaluating Logic Circuit Outputs

Exercise 2:

- Evaluate the given Boolean expression by substituting values and performing the indicated operations.

A = 1, B = 0, C = 0 and D = 0

$$x = \overline{A}BC\overline{(A+D)}$$

$$= \overline{1}.0.0.\overline{(1+0)}$$

$$= 0.0.0.(\overline{1})$$

$$= 0.0.0.0$$

$$= 0$$

Rule 1 → $x = \overline{0} \cdot 1 \cdot 1 \cdot \overline{(0+1)}$

Rule 2 → $x = 1 \cdot 1 \cdot 1 \cdot \overline{(0+1)}$

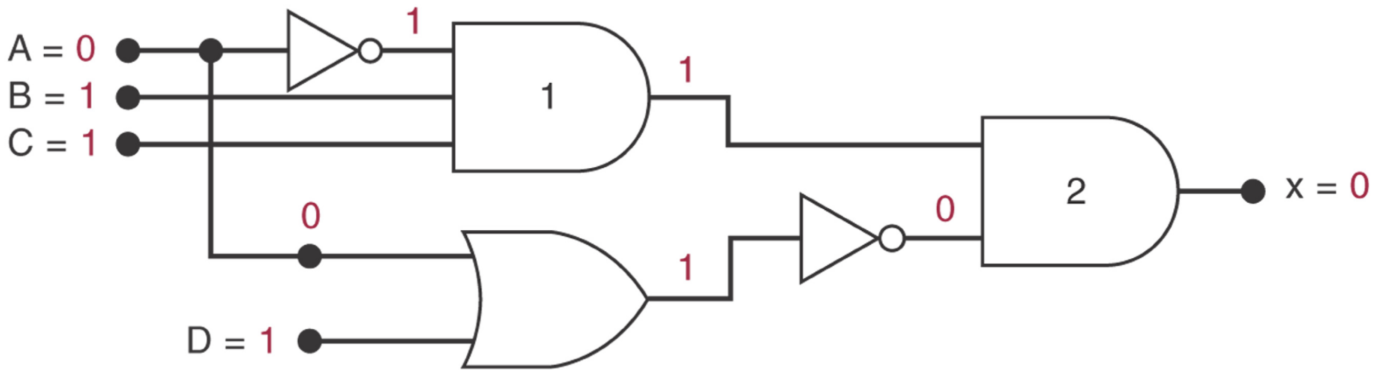
Rule 1 → $x = 1 \cdot 1 \cdot 1 \cdot (\overline{1})$

Rule 3 → $x = 1 \cdot 1 \cdot 1 \cdot 0$

$x = 0$

Evaluating Logic Circuit Outputs

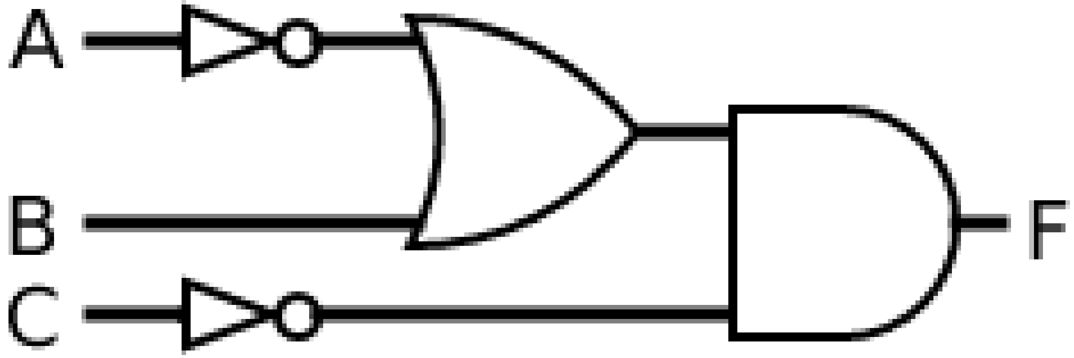
- Output logic levels can be determined directly from a circuit diagram.
- The output of each gate is noted until a final output is found.



Evaluating Logic Circuit Outputs

Exercise 3:

Evaluate the output F for the given logic circuit when A = 0; B = 1; and C = 1.

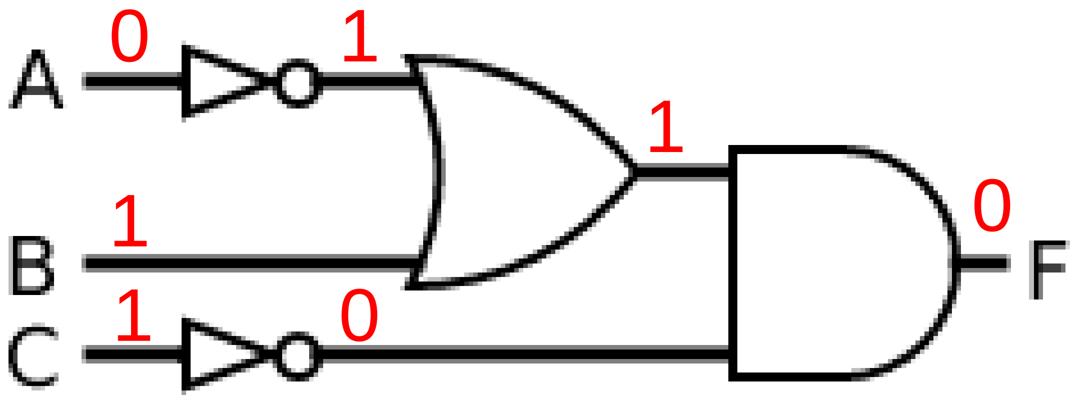


You have 5 minutes to determine the answer!

Evaluating Logic Circuit Outputs

Exercise 3:

Evaluate the output F for the given logic circuit when A = 0; B = 1; and C = 1.

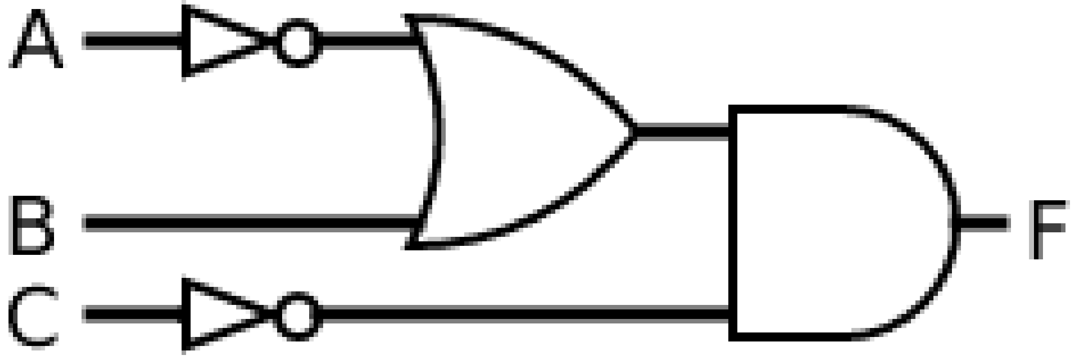


Evaluating Logic Circuit Outputs

Exercise 4 (live):

Evaluate the output F for the given logic circuit when

A = 1; B = 0; C = 0



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Evaluating Logic Circuit Outputs

Exercise 4 (live):

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- A = 1; B = 0; C = 0