#### XMUT 202 Digital Electronics

A/Prof Pawel Dmochowski

School of Engineering and Computer Science Victoria University of Wellington



CAPITAL CITY UNIVERSITY

#### Week 10 Lecture 2

- Combinatorial Logic (cont'd)
  - K Maps

#### **Boolean Algebra - Basic Rules**

- **1.** A + 0 = A **7.**  $A \cdot A = A$
- **2.** A + 1 = 1 **8.**  $A \bullet \overline{A} = 0$
- **3.**  $A \bullet 0 = 0$  **9.**  $\overline{A} = A$
- 4.  $A \bullet 1 = A$  10. A + AB = A
- **5**. A + A = A
- 6.  $A + \overline{A} = 1$

- 11.  $A + \overline{A}B = A + B$ 
  - **12.** (A + B)(A + C) = A + BC

#### **Simplification from looping:**

**<u>Pair</u>**: Looping a pair of adjacent 1's eliminate the variable that appears in complemented and uncomplemented form.

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**Quad**: Looping 4 adjacent 1's eliminate the two variables that appears in complemented and uncomplemented form.

#### Simplification from looping:

**<u>Pair</u>**: Looping a pair of adjacent 1's eliminate the variable that appears in complemented and uncomplemented form.

**Quad**: Looping 4 adjacent 1's eliminate the two variables that appears in complemented and uncomplemented form.

**Octet:** Looping 8 adjacent 1's eliminate the three variables that appears in complemented and uncomplemented form.

#### **Complete K-Map simplification process**

- 1. Construct the K map, place 1s as per the truth table.
- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs and cannot be looped into quads or octets.
- 4. Loop 1s in octets (8) even if they have already been looped.
- 5. Loop quads (4) that have one or more 1s not already looped.
- 6. Loop any pairs (2) necessary to include 1s not already looped.
- 7. Form the OR sum of terms generated by each loop.

# Simplify the following Boolean expression: $\overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}D + \overline{A}BCD + AB\overline{C}D + ABCD + A\overline{B}CD$

Simplify the following Boolean expression:

#### $\overline{ABCD} + \overline{ABCD} + \overline{ABCD} + \overline{ABCD} + \overline{ABCD} + \overline{ABCD} + \overline{ABCD}$

Sum of Product (SOP) expression

### Simplify the following Boolean expression: $\overline{ABCD} + \overline{ABCD} + \overline{ABCD} + AB\overline{CD} + A\overline{BCD} + A\overline{BCD}$

	ĊD	ĒD	CD	СD
ĀB				
ĀB				
AB				
AB				

- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs *and cannot be looped into quads or octets.*
- 4. Loop 1s in octets (8) even if they have already been looped.
- 5. Loop quads (4) that have one or more 1s not already looped.
- 6. Loop any pairs (2) necessary to *include 1s not already looped.*
- 7. Form the OR sum of terms generated by each loop.

### Simplify the following Boolean expression: $\overline{ABCD} + \overline{ABCD} + \overline{ABCD} + AB\overline{CD} + A\overline{BCD}$

	ĊD	ĒD	CD	сБ
ĀΒ				1
ĀB		1	1	
AB		1	1	
AB			1	

- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs *and cannot be looped into quads or octets.*
- 4. Loop 1s in octets (8) even if they have already been looped.
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### Simplify the following Boolean expression: $\overline{ABCD} + \overline{ABCD} + \overline{ABCD} + AB\overline{CD} + A\overline{BCD}$

	ĊD	ĊD	CD	СD
ĀΒ				
ĀB		1	1	
AB		1	1	
AB			1	

1. Construct the K map, place 1s as per the truth table.

#### 2. Loop 1s that are not adjacent to any other 1s.

- 3. Loop 1s that are in pairs *and cannot be looped into quads or octets.*
- 4. Loop 1s in octets (8) even if they have already been looped.
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## Simplify the following Boolean expression: $\overline{ABCD} + \overline{ABCD} + \overline{ABCD} + AB\overline{CD} + A\overline{BCD}$

	ĊD	ĒD	CD	СD
ĀB				
ĀB		1	1	
AB		1		
AB				

- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs and cannot be looped into quads or octets.
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- 7. Form the OR sum of terms generated by each loop.

## Simplify the following Boolean expression: $\overline{ABCD} + \overline{ABCD} + \overline{ABCD} + AB\overline{CD} + A\overline{BCD}$

	ĊD	ĒD	CD	СD
ĀB				
ĀB		1	1	
AB		1		
AB				

- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs *and cannot be looped into quads or octets.*
- 4. Loop 1s in octets (8) even if they have already been looped. (none here)
- 5. Loop quads (4) that have one or more 1s not already looped.
- 6. Loop any pairs (2) necessary to *include 1s not already looped.*
- 7. Form the OR sum of terms generated by each loop.

### Simplify the following Boolean expression: $\overline{ABCD} + \overline{ABCD} + \overline{ABCD} + AB\overline{CD} + A\overline{BCD}$

	ΖD	ĒD	CD	СD
ĀB				
ĀB		1	1	
AB		1	1	
AB				

- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs *and cannot be looped into quads or octets.*
- 4. Loop 1s in octets (8) *even if they have already been looped.*
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- 7. Form the OR sum of terms generated by each loop.

### Simplify the following Boolean expression: $\overline{ABCD} + \overline{ABCD} + \overline{ABCD} + AB\overline{CD} + A\overline{BCD}$

	ΖD	ĒD	CD	СD
ĀB				
ĀB		1	1	
AB		1	1	
AB				

- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs and cannot be looped into quads or octets.
- 4. Loop 1s in octets (8) *even if they have already been looped.*
- 5. Loop quads (4) that have one or more 1s not already looped.
- 6. Loop any pairs (2) necessary to *include 1s not already looped. (none here)*
- 7. Form the OR sum of terms generated by each loop.

# Simplify the following Boolean expression: $\overline{ABCD} + \overline{ABCD} + \overline{ABCD} + AB\overline{CD} + A\overline{BCD}$

	ĊD	ĒD	CD	СD
ĀB				
ĀB		1	1	
AB		1	1	
AB				

BD + ACD + ABCD

- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs and cannot be looped into quads or octets.
- 4. Loop 1s in octets (8) *even if they have already been looped.*
- 5. Loop quads (4) that have one or more 1s not already looped.
- 6. Loop any pairs (2) necessary to *include 1s not already looped.*
- 7. Form the OR sum of terms generated by each loop.

#### Simplify the following truth table:

Α	В	С	D	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

#### Simplify the following truth table:

	Α	В	С	D	X	
	0	0	0	0	0	
	0	0	0	1	0	
	0	0	1	0	0	
	0	0	1	1	1	
	0	1	0	0	1	
	0	1	0	1	1	
	0	1	1	0	1	
	0	1	1	1	1	
	1	0	0	0	0	
	1	0	0	1	0	
	1	0	1	0	0	
_	1	0	1	1	0	
	1	1	0	0	1	
	1	1	0	1	1	
	1	1	1	0	0	
	1	1	1	1	0	

## Simplify the following Boolean expression: $\overline{ABCD} + \overline{ABCD} + \overline$

- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs and cannot be looped into quads or octets.
- 4. Loop 1s in octets (8) even if they have already been looped.
- 5. Loop quads (4) that have one or more 1s not already looped.
- 6. Loop any pairs (2) necessary to *include 1s not already looped.*
- 7. Form the OR sum of terms generated by each loop.

# Simplify the following Boolean expression: ĀBCD + ĀBCD + ĀBCD + ĀBCD + ĀBCD + ABCD + ABCD + ABCD

	CD	<b>C</b> D	CD	CD
A B			1	
AB	1	1	1	1
AB	1	1		
AB				

- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs and cannot be looped into quads or octets.
- 4. Loop 1s in octets (8) even if they have already been looped.
- 5. Loop quads (4) that have one or more 1s not already looped.
- 6. Loop any pairs (2) necessary to *include 1s not already looped.*
- 7. Form the OR sum of terms generated by each loop.

# Simplify the following Boolean expression: ĀBCD + ĀBCD + ĀBCD + ĀBCD + ĀBCD + ABCD + ABCD + ABCD

	CD	ĊD	CD	CD
AB			1	
AB	1	1	1	1
AB	1	1		
AB				

1. Construct the K map, place 1s as per the truth table.

2. Loop 1s that are not adjacent to any other 1s (none)

- 3. Loop 1s that are in pairs and cannot be looped into quads or octets.
- 4. Loop 1s in octets (8) even if they have already been looped.
- 5. Loop quads (4) that have one or more 1s not already looped.
- 6. Loop any pairs (2) necessary to *include 1s not already looped.*
- 7. Form the OR sum of terms generated by each loop.

# Simplify the following Boolean expression: ĀBCD + ĀBCD + ĀBCD + ĀBCD + ĀBCD + ABCD + ABCD + ABCD

	CD	<b>C</b> D	CD	CD
AB			1	
AB	1	1	1	1
AB	1	1		
AB				

- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs and cannot be looped into quads or octets.
- 4. Loop 1s in octets (8) even if they have already been looped.
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- 7. Form the OR sum of terms generated by each loop.

# Simplify the following Boolean expression: $\overline{ABCD} + \overline{ABCD} + \overline$

	CD	- CD	CD	CD
ΑB			1	
AB	1	1	1	1
AB	1	1		
AB				

- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs *and cannot be looped into quads or octets.*
- 4. Loop 1s in octets (8) even if they have already been looped. (none)
- 5. Loop quads (4) that have one or more 1s not already looped.
- 6. Loop any pairs (2) necessary to *include 1s not already looped.*
- 7. Form the OR sum of terms generated by each loop.

# Simplify the following Boolean expression: ĀBCD + ĀBCD + ĀBCD + ĀBCD + ĀBCD + ABCD + ABCD + ABCD

	CD	- CD	CD	CD
AB			1	
AB	1	1	1	1
AB	1	1		
AB				

- 2. Loop 1s that are not adjacent to any other 1s.
- 3. Loop 1s that are in pairs *and cannot be looped into quads or octets.*
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# Simplify the following Boolean expression: ĀBCD + ĀBCD + ĀBCD + ĀBCD + ĀBCD + ABCD + ABCD + ABCD

	CD	- CD	CD	CD
AB			1	
AB	1	1	1	1
AB	1	1		
AB				

- 2. Loop 1s that are not adjacent to any other 1s.
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- 6. Loop any pairs (2) necessary to *include 1s not already looped (none here)*
- 7. Form the OR sum of terms generated by each loop.

# Simplify the following Boolean expression: $\overline{ABCD} + \overline{ABCD} + \overline$

	CD	<b>C</b> D	CD	CD
AB			1	
AB (	1	1	1	
AB	1	1		
AB				

 $B\overline{C} + \overline{A}CD + \overline{A}B$ 

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#### Try this one:

#### Simplify the following truth table using the K-Map method

5 minutes

Α	В	С	C D	
0	0	0	0 0	
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	1 0	
1	1	1	1	1

#### Exercise: K-Map simplification

Boolean expression derived from the truth table:

 $\overline{ABCD} + \overline{ABCD} + \overline{AB$ 

	ĊD	ĒD	CD	CD
ĀB		1		
ĀB		1	1	1
AB	1	1	1	
AB			1	

# Exercise: K-Map simplification

#### Boolean expression derived from the truth table: $\overline{ABCD} + \overline{ABCD} + \overline{ABC$

	ĊD	ĊD	CD	СD
ĀB				
ĀB			1	1
AB	1	1	1	
AB			1	

1. Construct the K map, place 1s as per the truth table.

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- 3. Loop 1s that are in pairs *and cannot be looped into quads or octets.*
- 4. Loop 1s in octets (8) even if they have already been looped.
- 5. Loop quads (4) that <u>have one or more 1s not</u> <u>already looped.</u>
- 6. Loop any pairs (2) necessary to *include 1s not already looped.*
- 7. Form the OR sum of terms generated by each loop.

#### $\overline{ACD} + \overline{ABC} + \overline{ABC} + \overline{ACD}$

#### **Don't Care Output Conditions**

Can be changed 0/1 so that the simplest expression can be obtained from the K-map. Typically occurs when we know certain input conditions are impossible.

#### **Don't care Output Conditions**

Can be changed 0/1 so that the simplest expression can be obtained from the K-map. Typically occur when we know certain input conditions are impossible.

	Α	B	С	Z
	0	0	0	0
	0	0	1	0
_	0	1	0	0
ſ	0	1	1	x
	1	0	0	x∫care"
1	1	0	1	1
	1	1	0	1
	1	1	1	1
1				

#### **Don't care Output Conditions**

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Can be changed 0/1 so that the simplest expression can be obtained from the K-map. Typically occur when we know certain input conditions are impossible.



#### **Example:** Design a logic circuit for a three storey elevator.



**Example:** Design a logic circuit for a three-storey elevator.

M = Logic signal indicating if the elevator is moving (M = 1) or stationary (M = 0)

F1, F2 and F3 are the floor level signals, normally LO but go HI when a particular floor is reached.

The circuit output (O/P) is the "Door Open" signal, should be normally LO but go HI when the door is to open



М	F1	F2	F3	OPEN
0	0	0	0	Sec. 10
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	07.0070
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	1.5100.524

M = elevator moving F1 = Floor 1F2 - Floor 2F3 – Floor 3 OPEN – elevator door opening M F1 F2 F3



М	F1	F2	F3	OPEN
0	0	0	0	area/
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	20 <b>2</b> 0
0	1	0	1	
0	1	1	0	
0	1	1	1	STANT S
1	0	0	0	1999
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

- Can only be on one floor at a time (only one floor I/P can be HI) .
- The other floor I/P's are then don't care conditions.
- Use x to indicate the don't care conditions.
- Door can't open when moving!

М	F1	F2	F3	OPEN
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	Х
0	1	0	0	1
0	1	0	1	Х
0	1	1	0	Х
0	1	1	1	Х
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	 Х
1	1	0	0	0
1	1	0	1	Х
1	1	1	0	Х
1	1	1	1	Х

- Can only be on one floor at a time (only one floor I/P can be HI) .
- The other floor I/P's are then don't care conditions.
- Use x to indicate the don't care conditions.
- Door can't open when moving!

М	F1	F2	F3	OPEN
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	Х
0	1	0	0	1
0	1	0	1	Х
0	1	1	0	Х
0	1	1	1	Х
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	Х
1	1	0	0	0
1	1	0	1	Х
1	1	1	0	Х
1	1	1	1	Х

- Can only be on one floor at a time (only one floor I/P can be HI) .
- The other floor I/P's are then don't care conditions.
- Use x to indicate the don't care conditions.
- Door can't open when moving!

#### $OPEN = \overline{M}\overline{F1}\overline{F2}F3 + \overline{M}F1\overline{F2}\overline{F3} + \overline{M}\overline{F1}F2\overline{F3}$



#### F2F3 F2F3 F2F3 F2F3

M F1	0	1	Х	1
M F1	1	Х	Х	х
M F1	0	Х	х	х
M F1	0	0	х	0



#### **Exercises**

Use the K-Map method to simplify the following:

- a) AB + A(B + C) + B(B + C)
- b)  $\overline{AB} + A(B + C) + B(B + C)$ .
- c)  $[\overline{AB}(C + BD) + \overline{A}\overline{B}]C$
- d)  $\overline{ABC} + A\overline{BC} + \overline{A}\overline{B}\overline{C} + A\overline{BC} + ABC$

#### Week 10 Lecture 2

• K-Map method