XMUT 202
Digital Electronics

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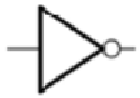
Review Part 1: Combinational Logic

- Logic Gates
- Laws of Boolean Algebra
- Truth Tables
- K-Maps
- Logic Design Problems

Logic Gates

NOT



INPUT	OUTPUT
A	
0	1
1	0

AND

INPUT		OUTPUT
A	B	
0	0	0
1	0	0
0	1	0
1	1	1

OR

INPUT		OUTPUT
A	B	
0	0	0
1	0	1
0	1	1
1	1	1

XOR

INPUT		OUTPUT
A	B	
0	0	0
1	0	1
0	1	1
1	1	0

NAND

INPUT		OUTPUT
A	B	
0	0	1
1	0	1
0	1	1
1	1	0

NOR

INPUT		OUTPUT
A	B	
0	0	1
1	0	0
0	1	0
1	1	0

XNOR

INPUT		OUTPUT
A	B	
0	0	1
1	0	0
0	1	0
1	1	1

Exercise

- Design a circuit that compares two 4-bit numbers and returns TRUE when they are equal

Evaluating Logic Circuit Outputs

Rules for evaluating a Boolean expression:

1. Perform all inversions of single terms.
2. Perform all operations within **parenthesis**.
3. Perform AND operation before an OR operation unless parenthesis indicate otherwise.
4. If an expression has a bar over it, perform the operations inside the expression and then invert the result.

Boolean Algebra Rules

1) $A + 0 = A$
2) $A + 1 = 1$
3) $A \bullet 0 = 0$
4) $A \bullet 1 = A$
5) $A + A = A$
6) $A + \overline{A} = 1$
7) $A \bullet A = A$
8) $A \bullet \overline{A} = 0$
9) $\overline{\overline{A}} = A$

1) $x + y = y + x$
2) $x \cdot y = y \cdot x$
3) $x + (y + z) = (x + y) + z = x + y + z$
4) $x(yz) = (xy)z = xyz$
5) $x(y + z) = xy + xz$
6) $(w + x)(y + z) = wy + xy + wz + xz$
7) $x + xy = x$
8) $x + \overline{x}y = x + y$
9) $\overline{x} + xy = \overline{x} + y$

De Morgan's Theorems

- A NOR gate is equivalent to an AND gate with inverted inputs.

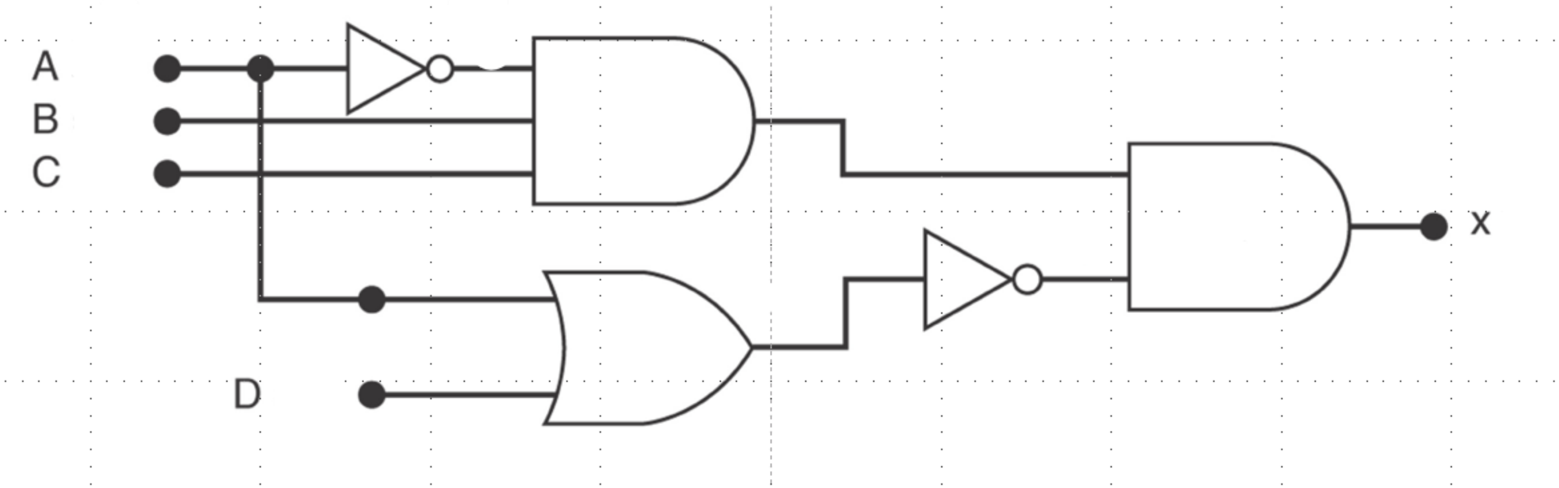
$$\overline{(x + y)} = \overline{x}.\overline{y}$$

- A NAND gate is equivalent to an OR gate with inverted inputs.

$$\overline{(x.y)} = \overline{x} + \overline{y}$$

Exercise: Logic Circuit

Write the Boolean expression for the output of the logic circuit below



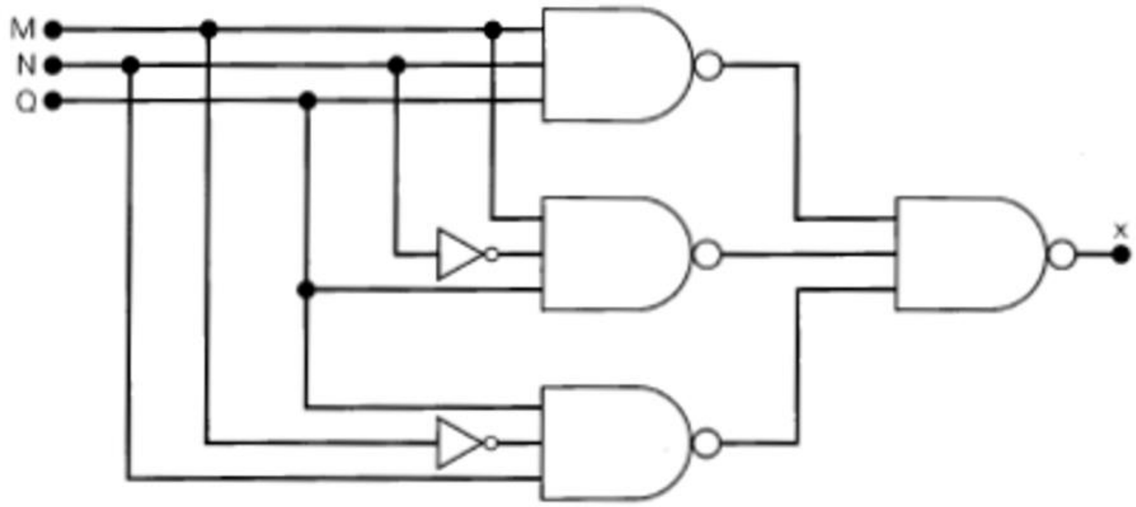
Draw the logic circuit for the following expression

$$X = (A \oplus B + \bar{C})A$$

Repeat using only AND, OR and NOT gates

Exercise: Boolean Logic Simplification

Simplify the circuit below using Boolean algebra



Exercises: Simplify using Boolean Theorems

- $X = \overline{\bar{A} + \bar{B} + \bar{C}}$
- $X = ABC + A\bar{B}(\overline{\bar{A}\bar{C}})$
- $X = A\bar{B} + A(\overline{B + C}) + B(\overline{B + C})$

K-Maps

Simplify the following Boolean expression:

$$\bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + AB\bar{C}D + ABCD + A\bar{B}CD$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				1
$\bar{A}B$		1	1	
AB		1	1	
$A\bar{B}$			1	

1. Construct the K map, place 1s as per the truth table.
2. Loop 1s that are not adjacent to any other 1s.
3. Loop 1s that are in pairs *and cannot be looped into quads or octets.*
4. Loop 1s in octets (8) *even if they have already been looped.*
5. Loop quads (4) that have one or more 1s not already looped.
6. Loop any pairs (2) necessary to *include 1s not already looped.*
7. Form the OR sum of terms generated by each loop.

$$BD + ACD + \bar{A}\bar{B}C\bar{D}$$

Exercise: K-Maps

- Simplify and write the resulting Boolean expression

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	1	1
$\bar{A}B$	1	0	0	1
AB	0	0	0	0
$A\bar{B}$	1	0	1	1

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	1	1
$\bar{A}B$	1	0	x	1
AB	0	1	x	0
$A\bar{B}$	0	x	1	1

- Draw the circuit diagram using as few gates as possible

Exercises: Simplify using K-maps

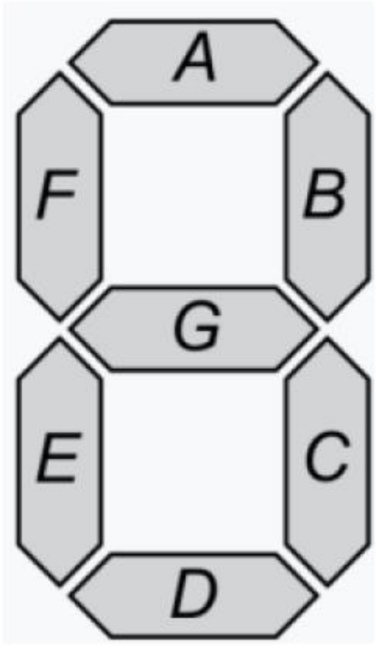
- $X = ABC + A\bar{B}(\overline{\bar{A}\bar{C}})$
- $X = A\bar{B} + A(\overline{B + C}) + B(\overline{B + C})$

Exercise: Logic Design 1

We want to design a logic circuit for a 7-segment LED display shown below.

The inputs are 4 binary digits **d**, **c**, **b**, **a**, which represent the number to be displayed on the LED. Bit **d** is the most significant bit (MSB), and **a** is the least significant bit (LSB).

Design the logic circuits to drive LED segments A, B and F.



Exercise: Logic Design 2

A BCD code is being transmitted to a remote receiver. The bits are A_3,A_2, A_1, A_0, with A_3 as the MSB (Most significant bit). The receiver circuitry includes a BCD error detector circuit that examines the received code to see if it is a legal BCD code (i.e. ≤ 1001). Design this circuit to produce a HIGH for any error condition. Simplify the expression and draw the logic diagram.

Exercise: Logic Design 3

The figure below shows a BCD counter that produces a 4-bit output representing the BCD code for the number of pulses that have been applied to the counter input. The counter resets on the tenth pulse and starts over again. The outputs will then never present a number > $1001_2 = 9_{10}$. Use K mapping to design a logic circuit that will produce a HI output whenever the count is 2,3 or 9.