XMUT 202
Digital Electronics

Felix Yan

School of Engineering and Computer Science
Victoria University of Wellington

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga o te Ūpoko o te Ika a Māui

CAPITAL CITY UNIVERSITY

Logic Gates

- We have learned about:
  - AND, OR, NOT Boolean operations
  - Logic circuit diagrams
  - Truth tables

- Today
  - NOR and NAND gates
  - Boolean Theorems (for simplifying Boolean expressions)
  - De Morgan's Theorems (for simplifying Boolean expressions)

Implementing Circuits From Boolean Expressions

- It is important to be able to draw a logic circuit from a Boolean expression.

Implementing Circuits from Boolean Expressions

- It is important to be able to draw a logic circuit from a Boolean expression.

- Simple expression: $x = A.B.C$

Implementing Circuits from Boolean Expressions

- It is important to be able to draw a logic circuit from a Boolean expression.

- Simple expression: $x = A.B.C$

- A more complex example expression:

$$y = AC + B\overline{C} + \overline{A}BC$$

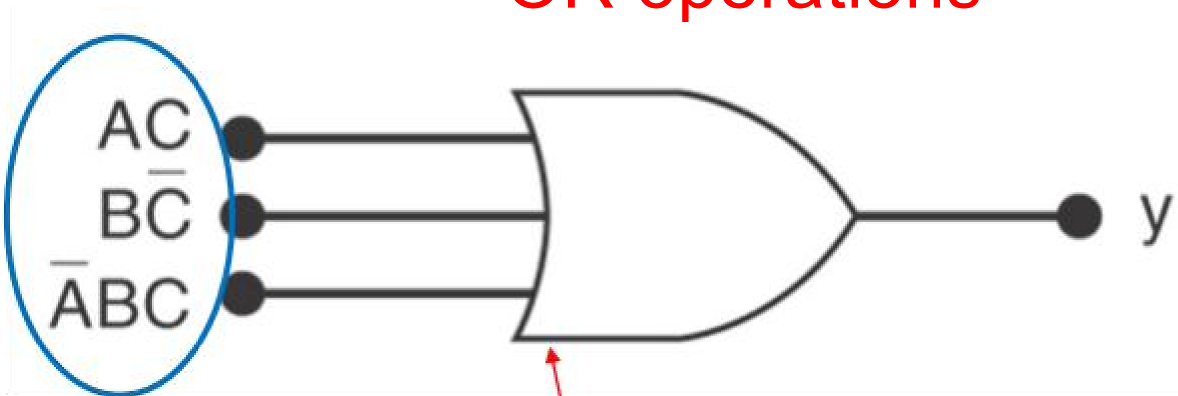
Logic circuit for the Boolean expression:

$$y = AC + B\overline{C} + \overline{A}BC$$

Logic circuit for the Boolean expression:

$$y = AC + B\overline{C} + \overline{A}BC$$

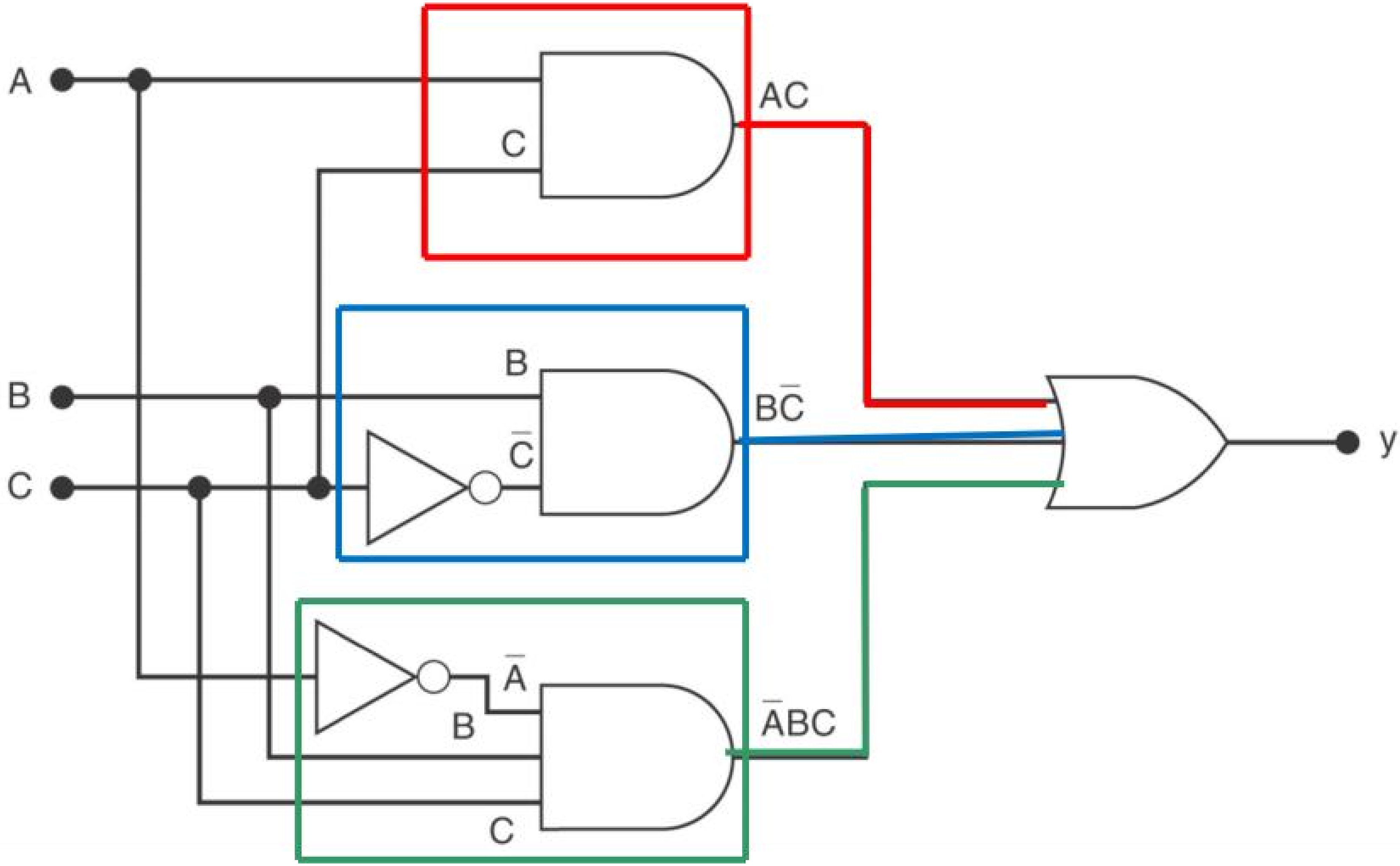
OR operations



3 inputs

OR gate

Logic circuit for the Boolean expression:



$$y = A\overline{C} + \overline{B}C + ABC$$

NOR Gate

- Combine basic AND, OR, and NOT operations.

NOR Gate

- Combine basic AND, OR, and NOT operations.

- The NOR gate is an inverted OR gate. An inversion “bubble” is placed at the output of the OR gate.

NOR Gate

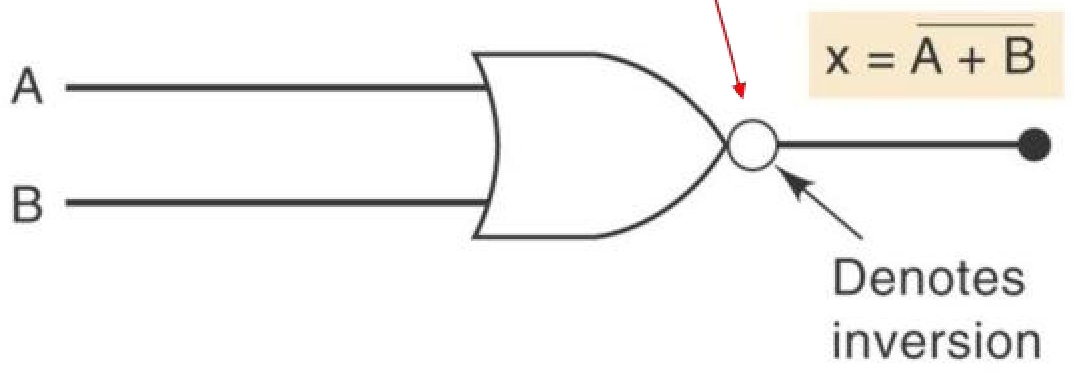
- Combine basic AND, OR, and NOT operations.

- The NOR gate is an inverted OR gate. An inversion “bubble” is placed at the output of the OR gate.

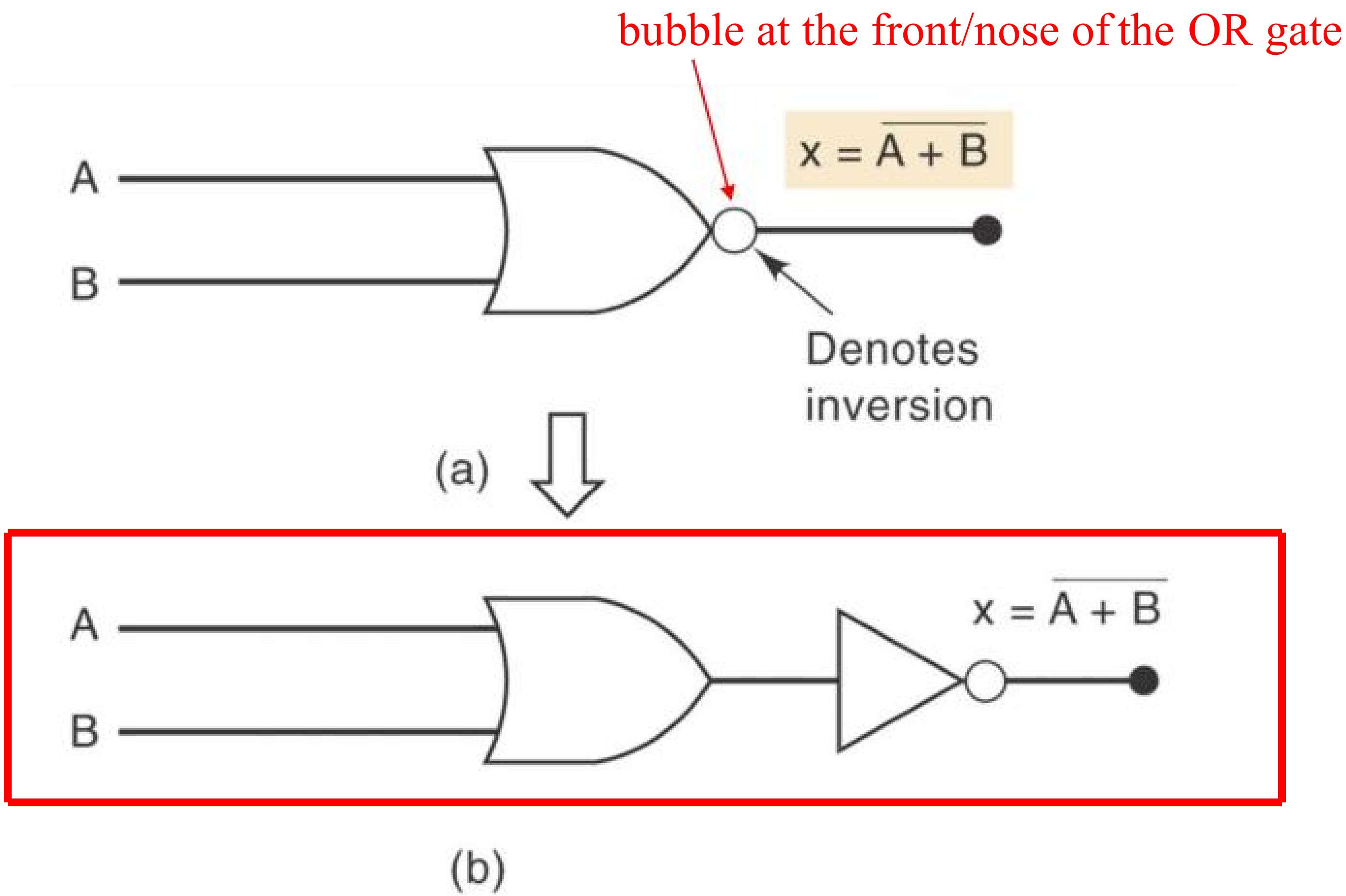
- The Boolean expression is $x = \overline{A + B}$

(a) NOR symbol

Small circle at the front/nose of the OR gate

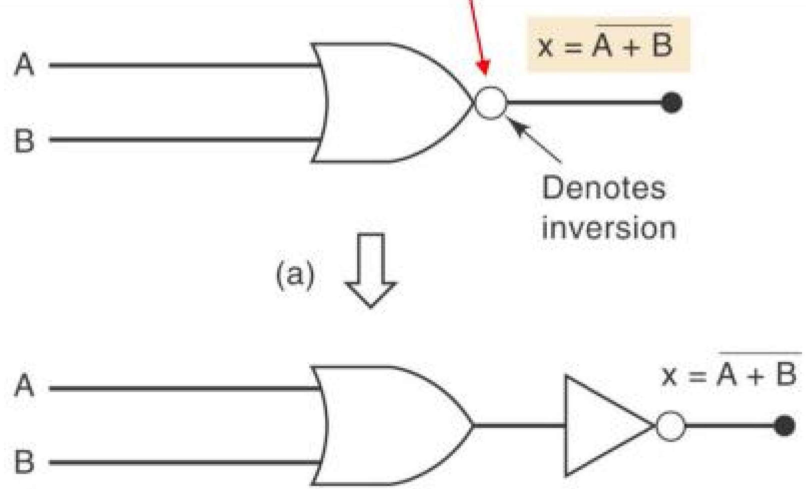


(a) NOR symbol; (b) equivalent circuit



(a) NOR symbol; (b) equivalent circuit; (c) truth table.

bubble at the front/nose of the OR gate



(b)

		OR	NOR
A	B	A + B	$\overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

(c)

Truth table

NAND Gate

- The NAND gate is an inverted AND gate.

NAND Gate

- The NAND gate is an inverted AND gate.

- An inversion “bubble” is placed at the output of an AND gate.

NAND Gate

- The NAND gate is an inverted AND gate.
- An inversion "bubble" is placed at the output of an AND gate.
- The Boolean expression is $x = \overline{AB}$

NOR Gates and NAND Gates

- The output of NAND and NOR gates may be found by simply determining the output of an AND or OR gate and inverting it.

NOR Gates and NAND Gates

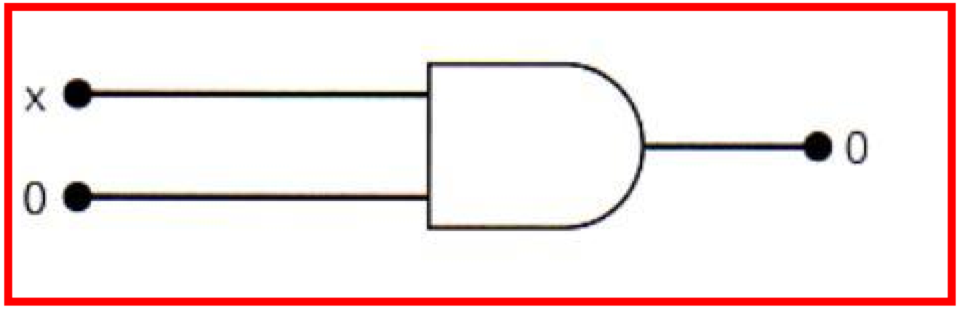
- The output of NAND and NOR gates may be found by simply determining the output of an AND or OR gate and inverting it.

- The truth tables for NOR and NAND gates show the complement of truth tables for OR and AND gates.

Boolean Theorems

Single variable theorems

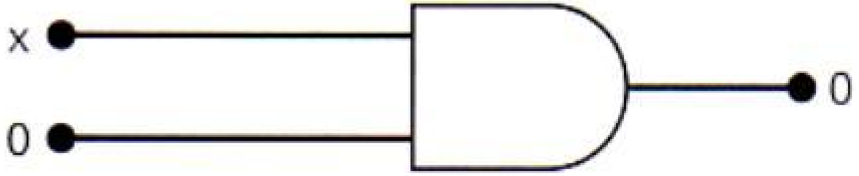
1) $x \cdot 0 = 0$



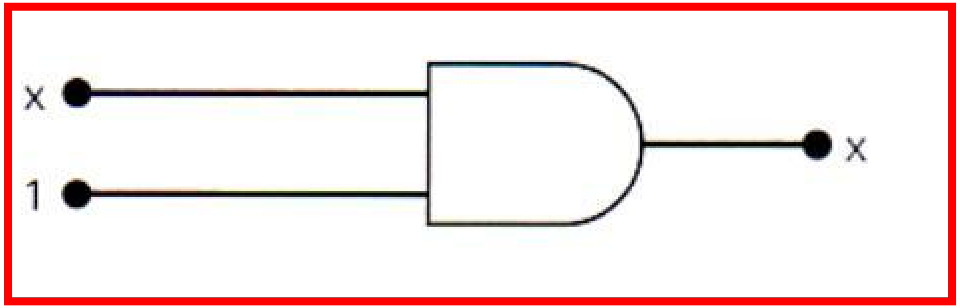
Boolean Theorems

Single variable theorems

1) $x \cdot 0 = 0$



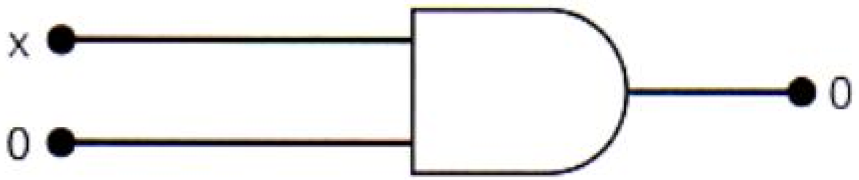
2) $x \cdot 1 = x$



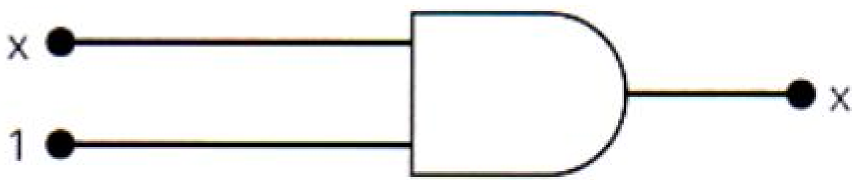
Boolean Theorems

Single variable theorems

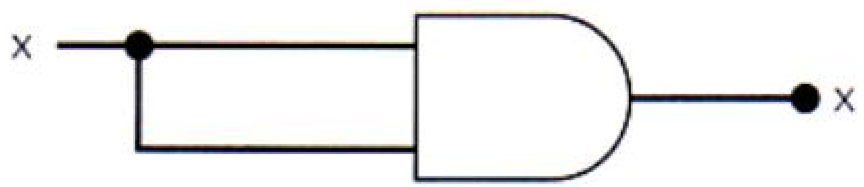
1) $x \cdot 0 = 0$



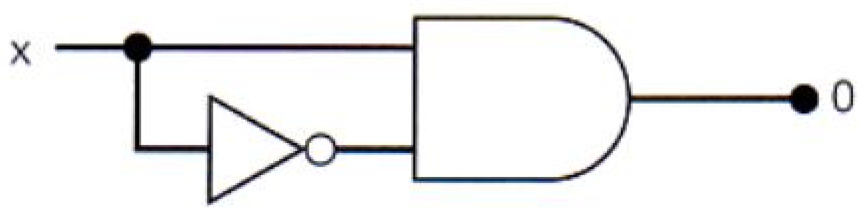
2) $x \cdot 1 = x$



3) $x \cdot x = x$



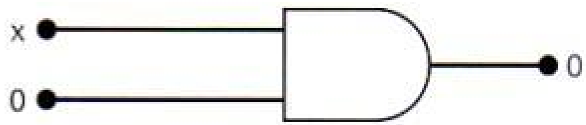
4) $x \cdot \bar{x} = 0$



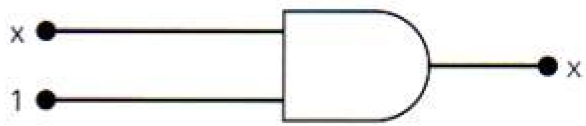
Boolean Theorems

Single variable theorems

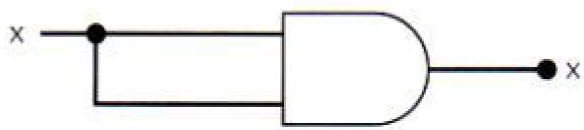
1) $x \cdot 0 = 0$



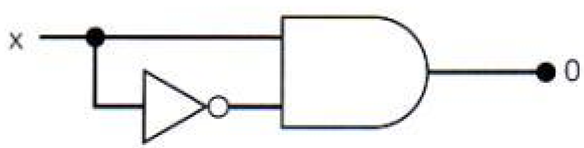
2) $x \cdot 1 = x$



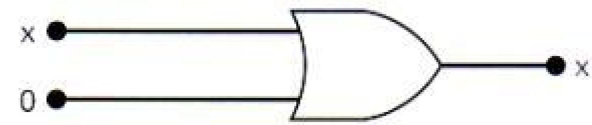
3) $x \cdot x = x$



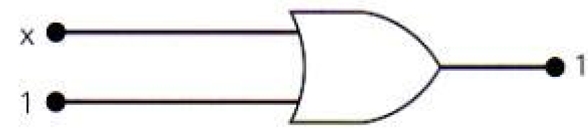
4) $x \cdot \bar{x} = 0$



5) $x + 0 = x$

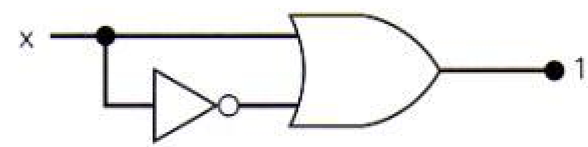


6) $x + 1 = 1$



7) $x + x = x$

8) $x + \bar{x} = 1$



Boolean Theorems

Multivariable theorems:

1) $x + y = y + x$
2) $x \,.\, y = y \,.\, x$
3) $x + (y + z) = (x + y) + z = x + y + z$
4) $x(yz) = (xy)z = xyz$
5) $x(y + z) = xy + xz$
6) $(w + x)(y + z) = wy + xy + wz + xz$
7) $x + xy = x$
8) $\underline{x} + \overline{x}y = \underline{x} + y$
9) $x + xy = x + y$

De Morgan's Theorems

- When the OR sum of two variables is inverted, it is equivalent to inverting each variable individually and ANDing them.

De Morgan’s Theorems

- When the OR sum of two variables is inverted, it is equivalent to inverting each variable individually and ANDing them.

- When the AND product of two variables is inverted, it is equivalent to inverting each variable individually and ORing them.

De Morgan's Theorems

- A NOR gate is equivalent to an AND gate with inverted inputs.

$$\overline{(x + y)} = \overline{x}.\overline{y}$$

De Morgan's Theorems

- A NOR gate is equivalent to an AND gate with inverted inputs.

$$\overline{(x + y)} = \overline{x}.\overline{y}$$

- A NAND gate is equivalent to an OR gate with inverted inputs.

$$\overline{(x.y)} = \overline{x} + \overline{y}$$

Universality of NAND and NOR gates

- NAND or NOR gates can be used to create the three basic logic operations (OR, AND, and NOT also known as the INVERTER)

Universality of NAND and NOR gates

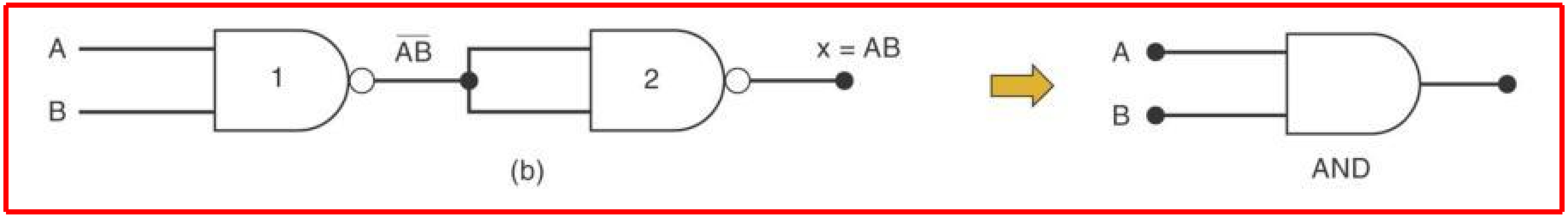
- NAND or NOR gates can be used to create the three basic logic operations (OR, AND, and INVERTER)

- How combinations of NANDs or NORs are used to create the three basic logic operations.

NAND gates can be used to implement any Boolean function.

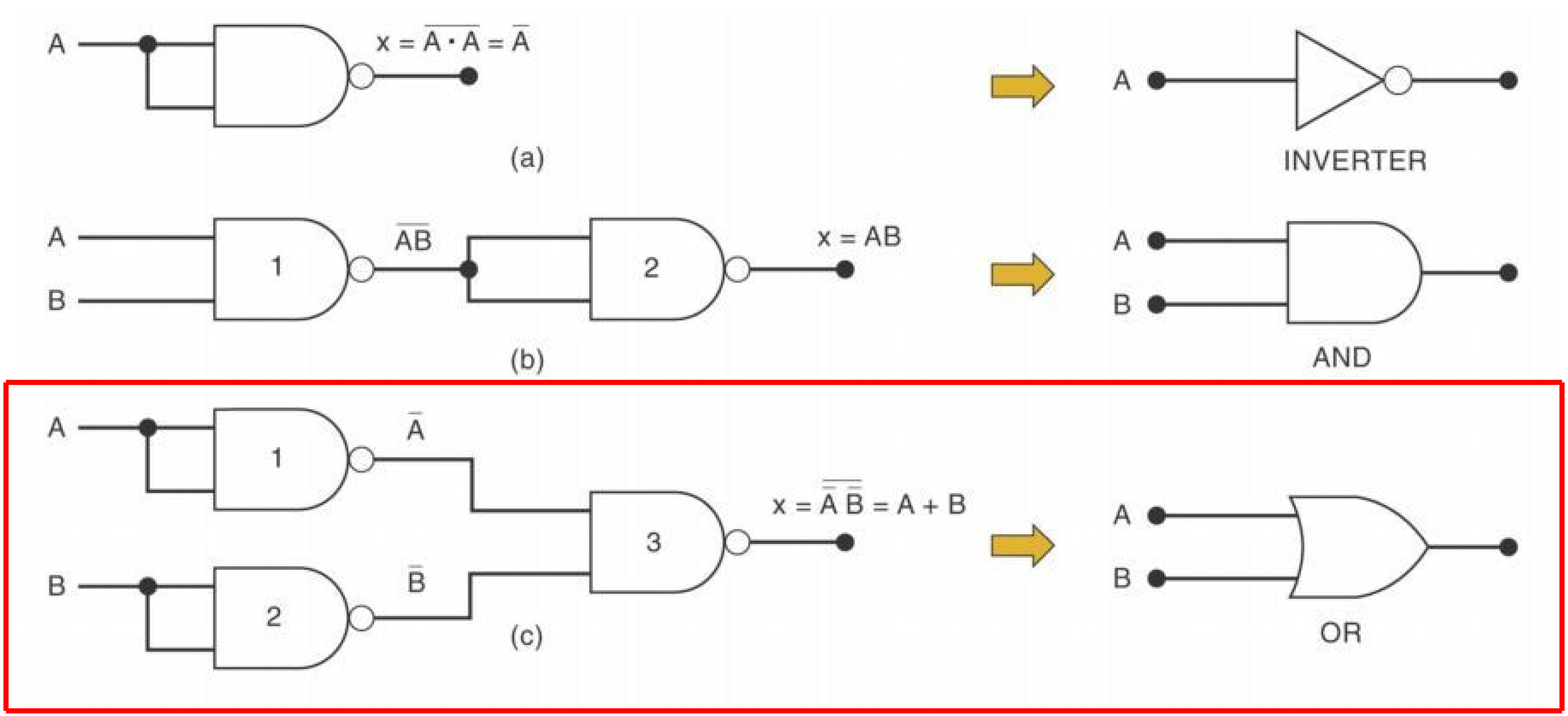
NAND gates can be used to implement any Boolean function.

(a)

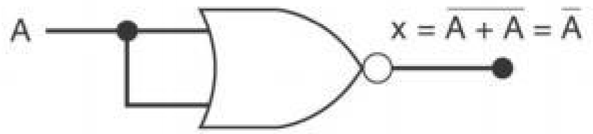


(b)

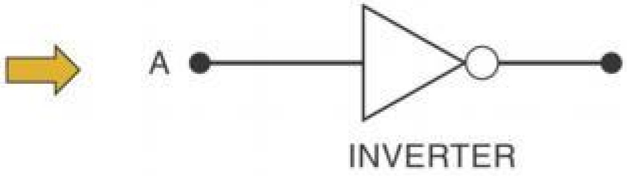
NAND gates can be used to implement any Boolean function.



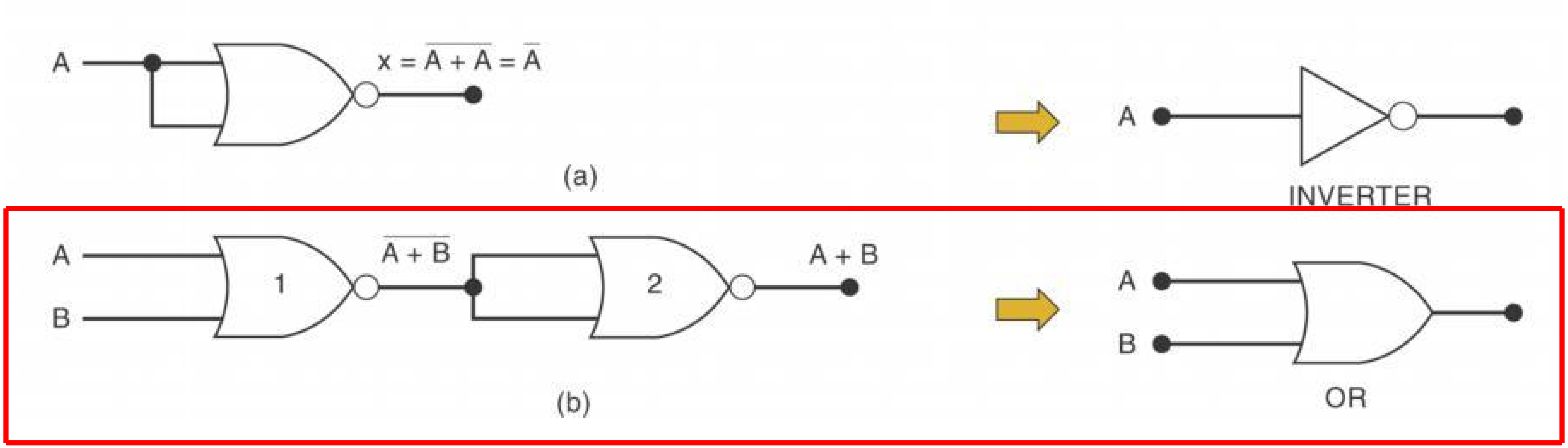
NOR gates can be used to implement any Boolean function.



(a)



NOR gates can be used to implement any Boolean function.



NOR gates can be used to implement any Boolean function.