# XMUT 202
# Digital Electronics

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# Logic Gates

- We have learned about:
  - AND, OR, NOT Boolean operations
  - Logic circuit diagrams
  - Truth tables

- Today
  - NOR and NAND gates
  - Boolean Theorems (for simplifying Boolean expressions)
  - De Morgan's Theorems (for simplifying Boolean expressions)

# Implementing Circuits From Boolean Expressions

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- A more complex example expression:

$$y = AC + B\overline{C} + \overline{A}BC$$

# Logic circuit for the Boolean expression:

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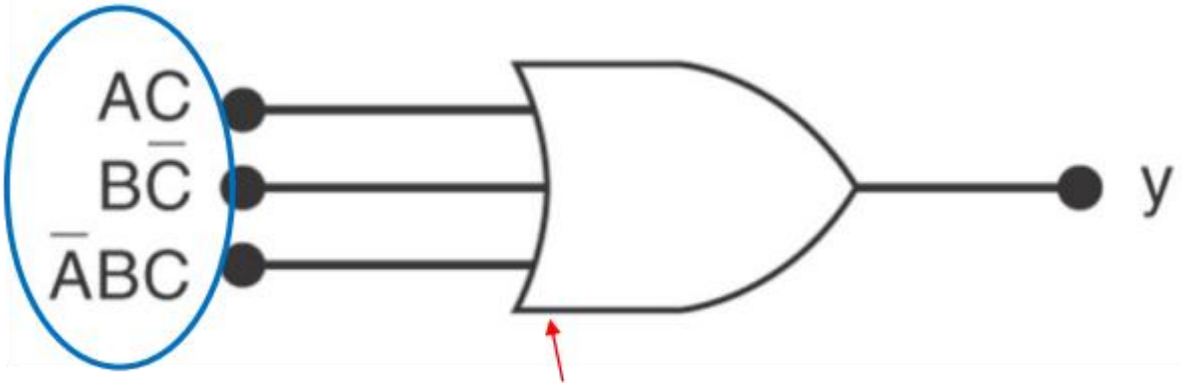
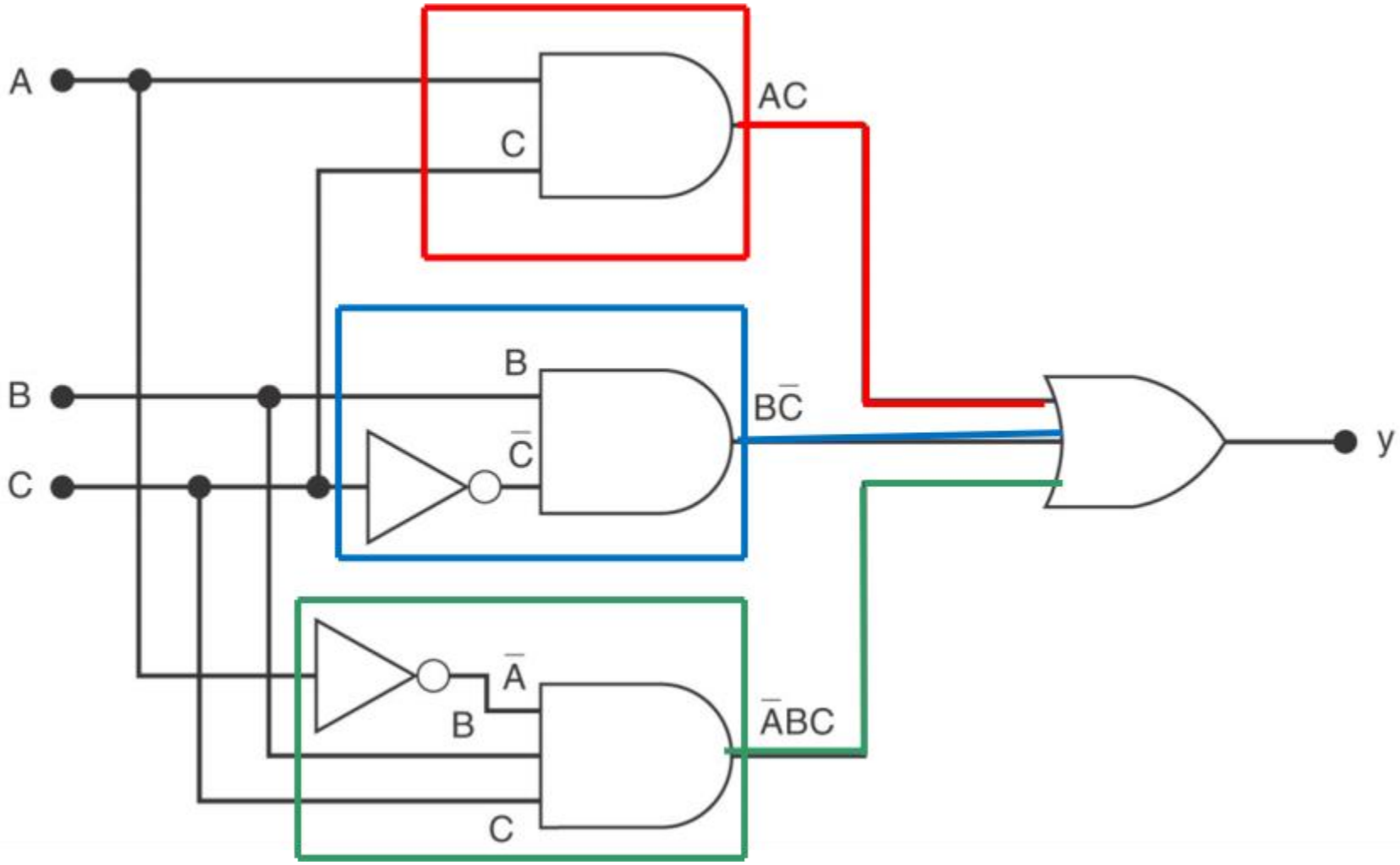
OR operations




3 inputs

OR gate

# Logic circuit for the Boolean expression:




$$y = AC + BC + \bar{A}B\bar{C}$$

# NOR Gate

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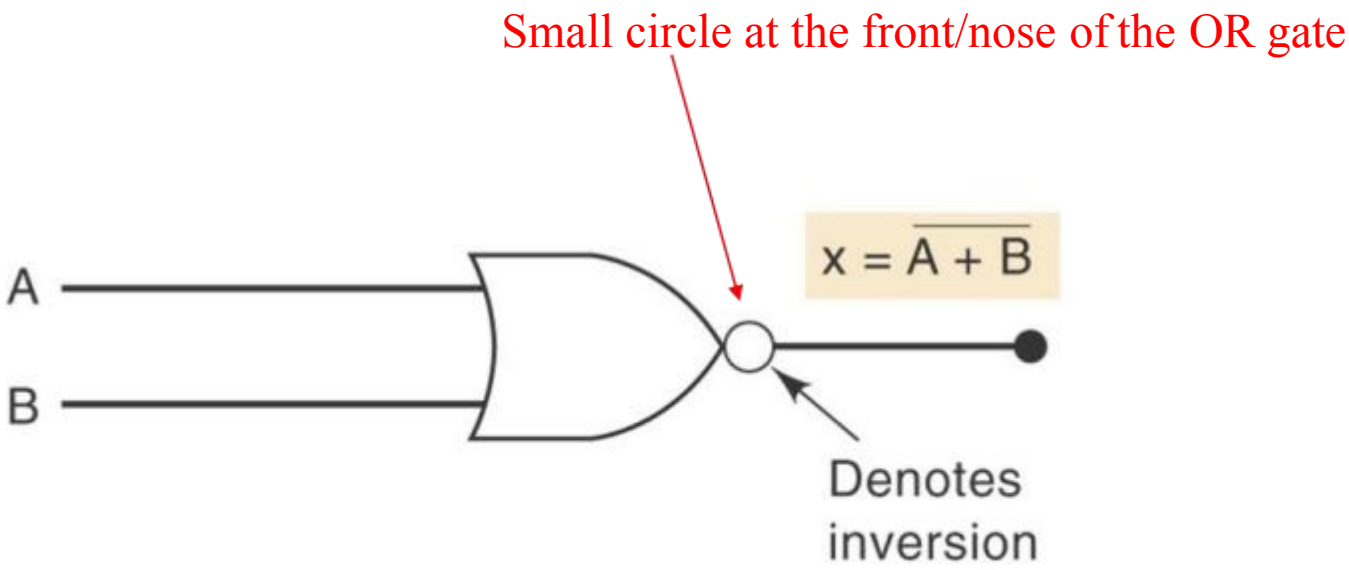
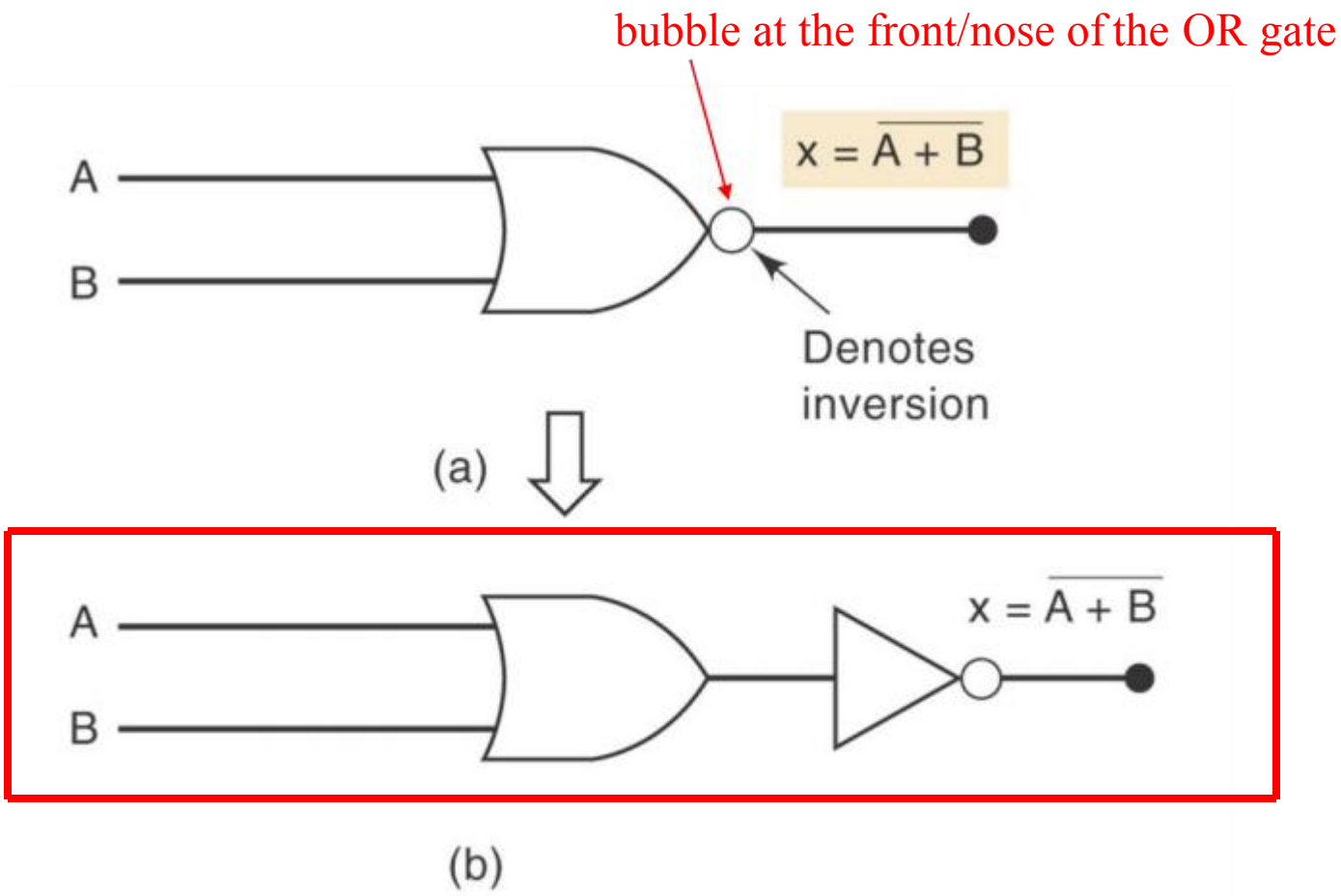
- The Boolean expression is $x = \overline{A + B}$

# (a) NOR symbol

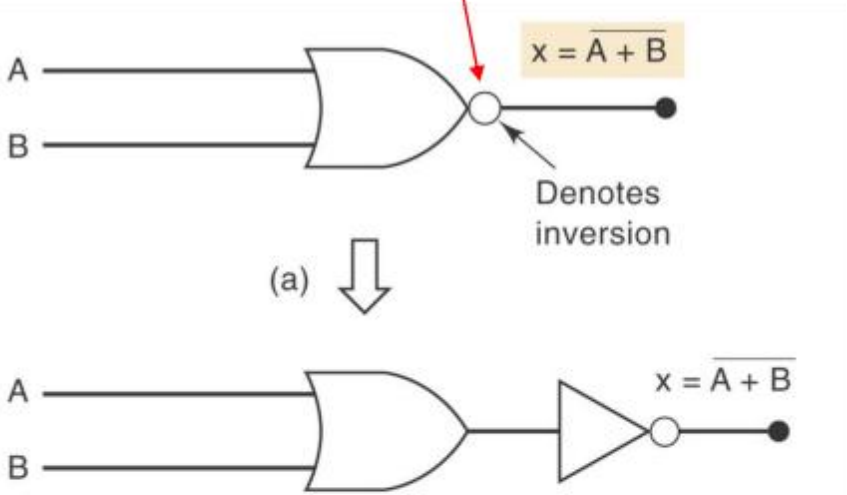

Small circle at the front/nose of the OR gate
A
B
x = A + B
Denotes
inversion

# (a) NOR symbol; (b) equivalent circuit



# (a) NOR symbol; (b) equivalent circuit; (c) truth table.

bubble at the front/nose of the OR gate




| A | B | OR: A + B | NOR: $\overline{A + B}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

(c)

Truth table

# NAND Gate

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- The Boolean expression is $x = \overline{AB}$

# NOR Gates and NAND Gates

- The output of NAND and NOR gates may be found by simply determining the output of an AND or OR gate and inverting it.

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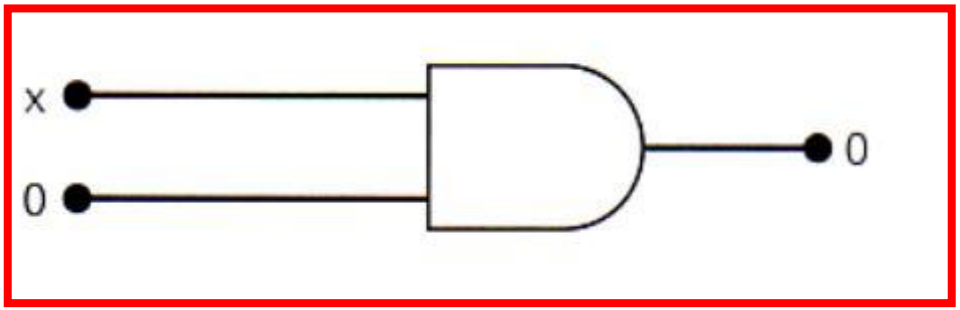
- The truth tables for NOR and NAND gates show the complement of truth tables for OR and AND gates.

# Boolean Theorems

## Single variable theorems

1) $x \cdot 0 = 0$



# Boolean Theorems

## Single variable theorems

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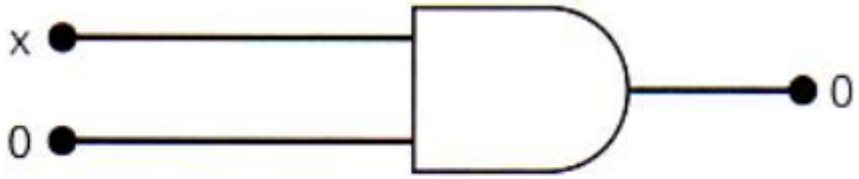
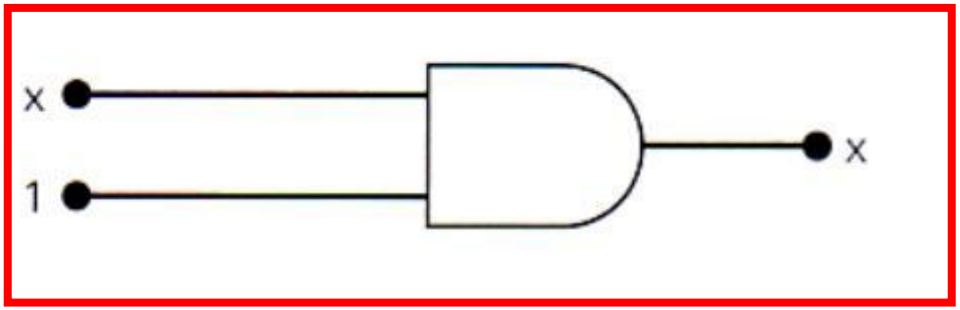



2) $x \cdot 1 = x$



# Boolean Theorems

## Single variable theorems

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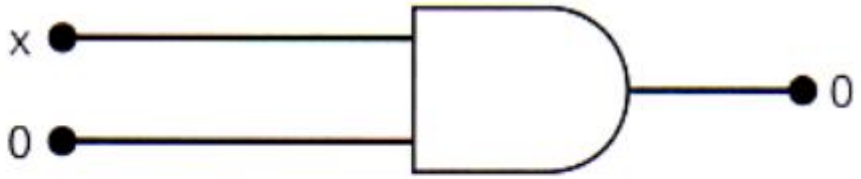


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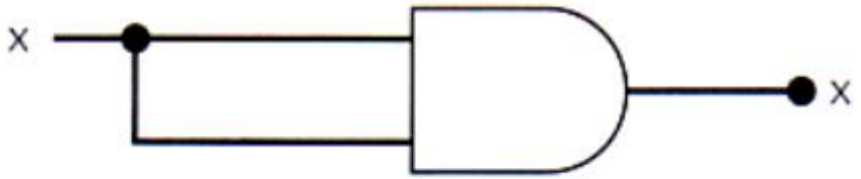
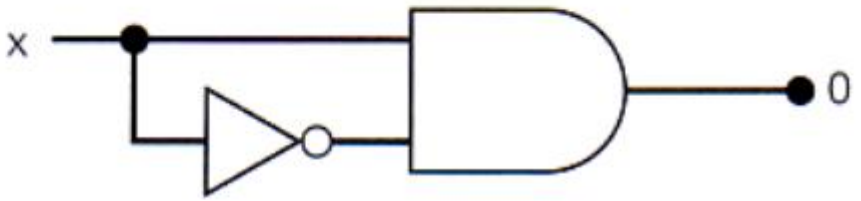
3) $x \cdot x = x$




4) $x \cdot \overline{x} = 0$



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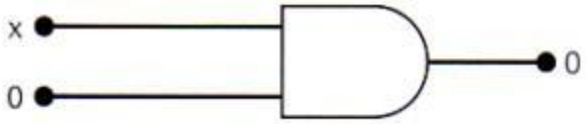


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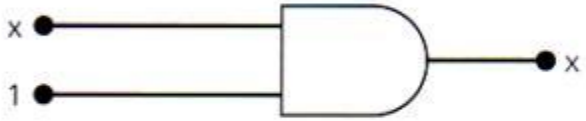


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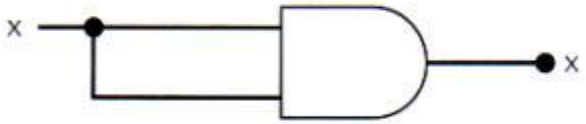
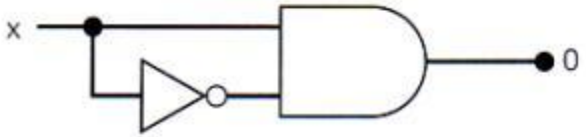
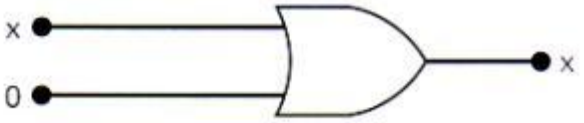


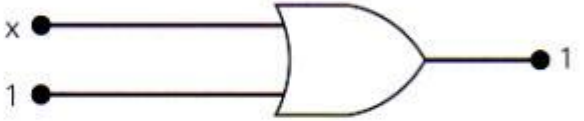

4) $x \cdot \bar{x} = 0$



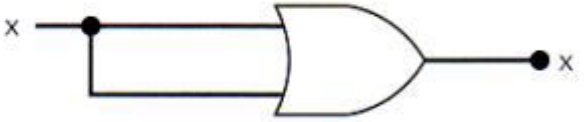

5) $x + 0 = x$



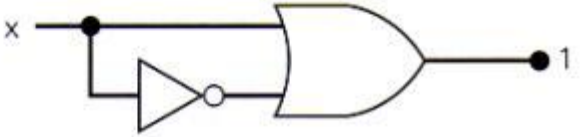

6) $x + 1 = 1$




7) $x + x = x$




8) $x + \bar{x} = 1$



# Boolean Theorems

Multivariable theorems:

1) $x + y = y + x$
2) $x \cdot y = y \cdot x$
3) $x + (y + z) = (x + y) + z = x + y + z$
4) $x(yz) = (xy)z = xyz$
5) $x(y + z) = xy + xz$
6) $(w + x)(y + z) = wy + xy + wz + xz$
7) $x + xy = x$
8) $\overline{x} + xy = \overline{x} + y$
9) $x + xy = x + y$

# De Morgan's Theorems

- When the OR sum of two variables is inverted, it is equivalent to inverting each variable individually and ANDing them.

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- When the AND product of two variables is inverted, it is equivalent to inverting each variable individually and ORing them.

# De Morgan's Theorems

- A NOR gate is equivalent to an AND gate with inverted inputs.

$$\overline{(x + y)} = \overline{x}.\overline{y}$$

# De Morgan's Theorems

- A NOR gate is equivalent to an AND gate with inverted inputs.

$$\overline{(x + y)} = \bar{x}.\bar{y}$$

- A NAND gate is equivalent to an OR gate with inverted inputs.

$$\overline{(x.y)} = \bar{x} + \bar{y}$$

# Universality of NAND and NOR gates

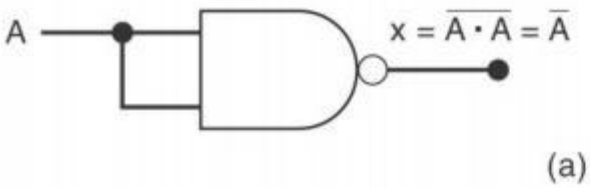
- NAND or NOR gates can be used to create the three basic logic operations (OR, AND, and NOT also known as the INVERTER)

# Universality of NAND and NOR gates

- NAND or NOR gates can be used to create the three basic logic operations (OR, AND, and INVERTER)

- How combinations of NANDs or NORs are used to create the three basic logic operations.

# NAND gates can be used to implement any Boolean function.

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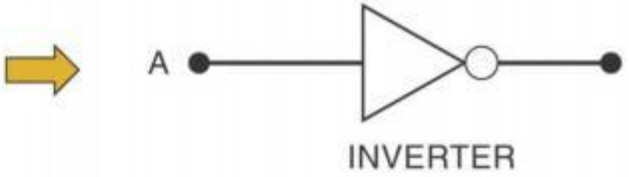

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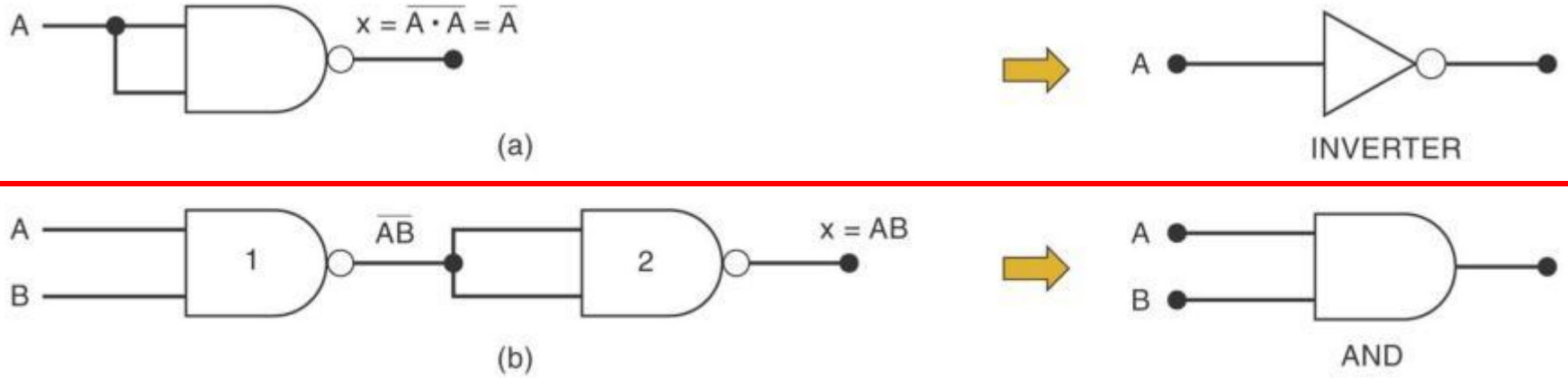

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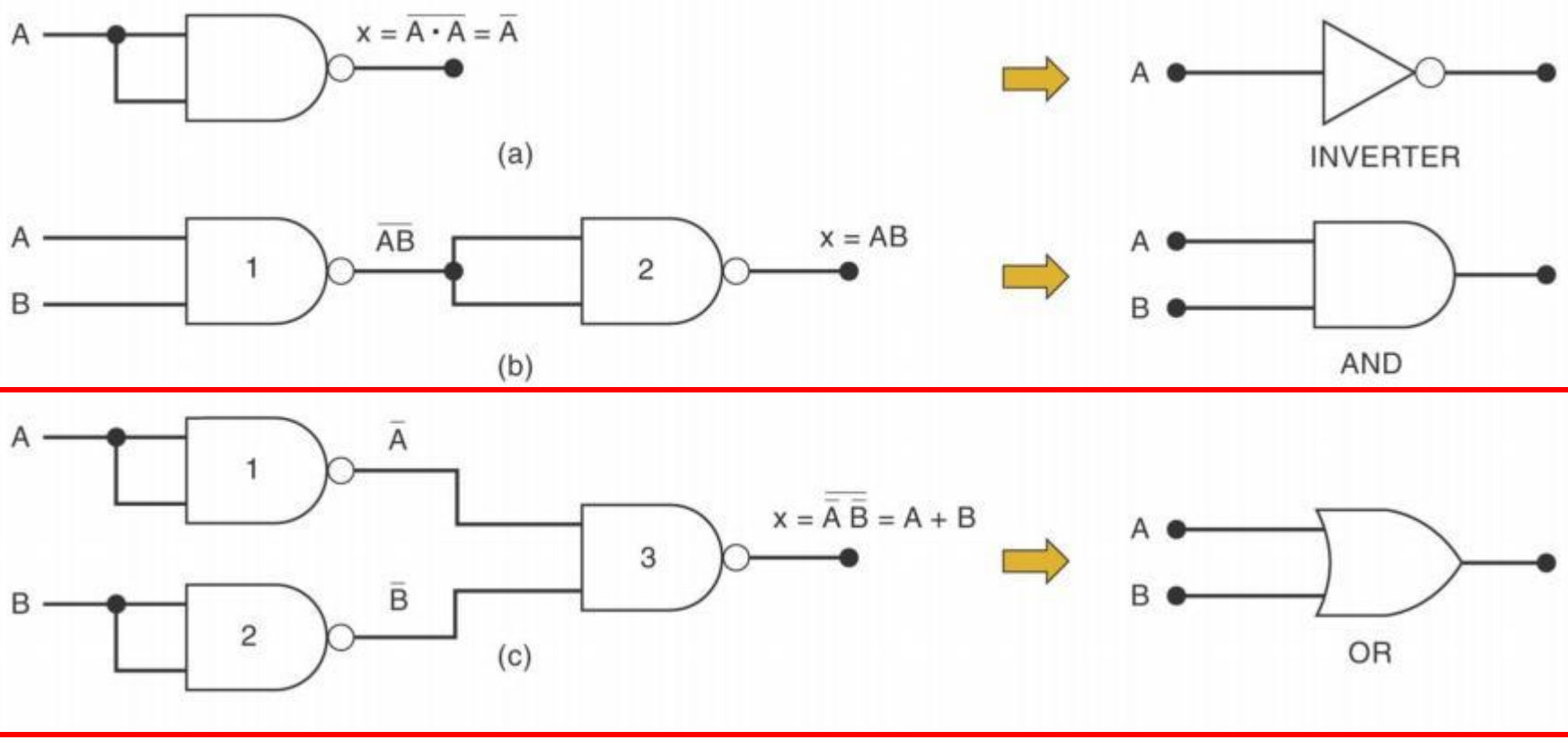

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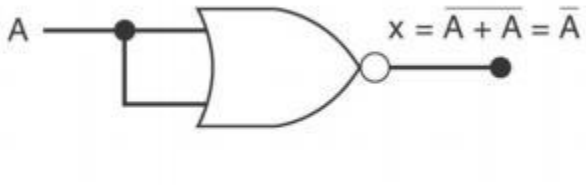


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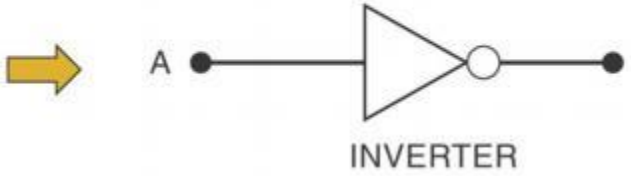

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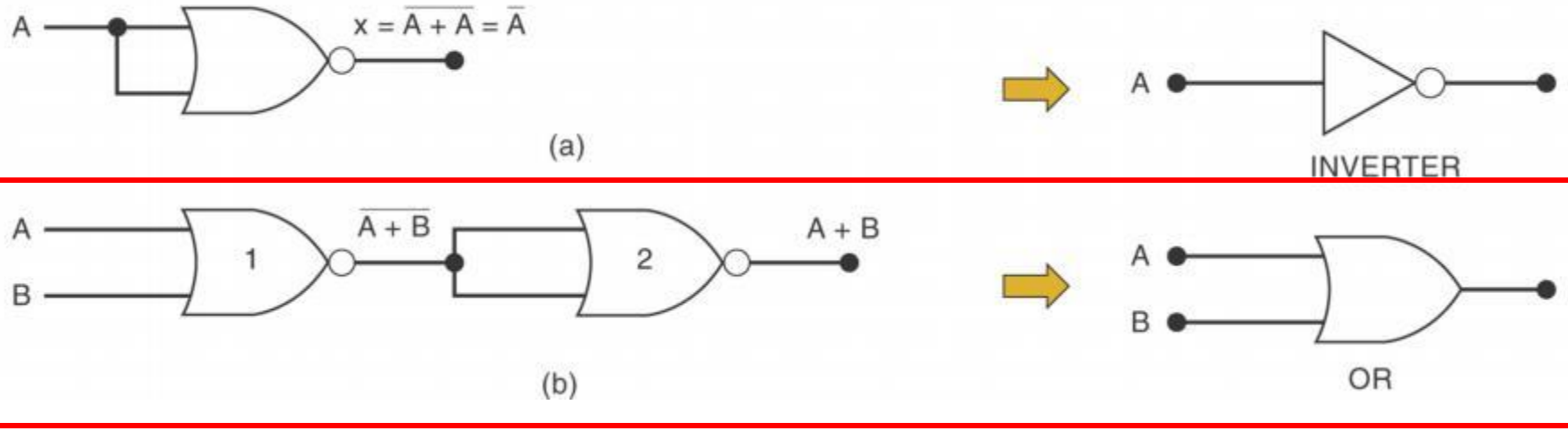

**NOR gates can be used to implement any Boolean function.**