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Lab 4 – Laplace Transforms in Circuits

XMUT203 Analogue Circuits and Systems

Topics

Laboratory 4 – Laplace Transforms in Circuits

1. Symbolic calculations.
2. Graphical manipulations.
3. First-order system circuit.
4. Second-order system circuit.

Symbolic Calculation

For solving the characteristics and behavior of a given electrical circuit in MATLAB using the transfer equation of the circuit, the following features are commonly required in the analysis process:

- `solve()` function – Solve a given transfer function equation.
- `factor()` function – Find the roots of the transfer function equation.
- `laplace()` function – Convert the equation from time-domain to s-domain.
- `ilaplace()` function – Convert the equation from s-domain to time-domain.
- `ezplot()` function – Plot the pole-zero diagram of the transfer function.

Graph Plotting

For graph plotting, these are several features in MATLAB that are typically needed for circuit simulation.

- `plot()` function – Plot graph.
- `xlim()` and `ylim()` functions – Limit range of x-axis or y-axis.
- `title()` – to print title of the graph.
- `xlabel()` and `ylabel()` functions – Label x-axis or y-axis.
- `legend()` function – Print indicators for the plots in the graph.
- `text()` function – Print a text at particular position in the graph
- `line()` function – Plot a line.
- `semilogx()` and `semilogy()` functions – Specify the graph to be log in the x-axis or y-axis.
- `loglog()` function – Specify the graph to be log on both x- and y-axis.

First Order System

Considering the transfer function of a given system is given as:

$$G(s) = \frac{Y(s)}{X(s)} = \frac{(s + z_1)(s + j\omega_2) \dots (s + z_n)}{(s + p_1)(s + p_2) \dots (s + p_n)}$$

Where:

- $z_1, z_2, \dots, z_n$ are the zeros of the system.
- $p_1, p_2, \dots, p_n$ are the poles of the system.

For a first-order system, the following equation is a typical transfer function of the system in Laplace transform with a pole at $s = -a$:

$$G(s) = \frac{a}{s + a}$$

The inverse Laplace transform of the transfer function equation given above is:

$$g(t) = e^{-at} + g(0^-)$$

First Order System

When the first-order system is given a step input ($R(s) = 1/s$), its transfer function equation is:

$$H(s) = R(s)G(s) = \left(\frac{1}{s}\right)\left(\frac{a}{s+a}\right)$$

Applying partial fraction expansion, the equation above becomes:

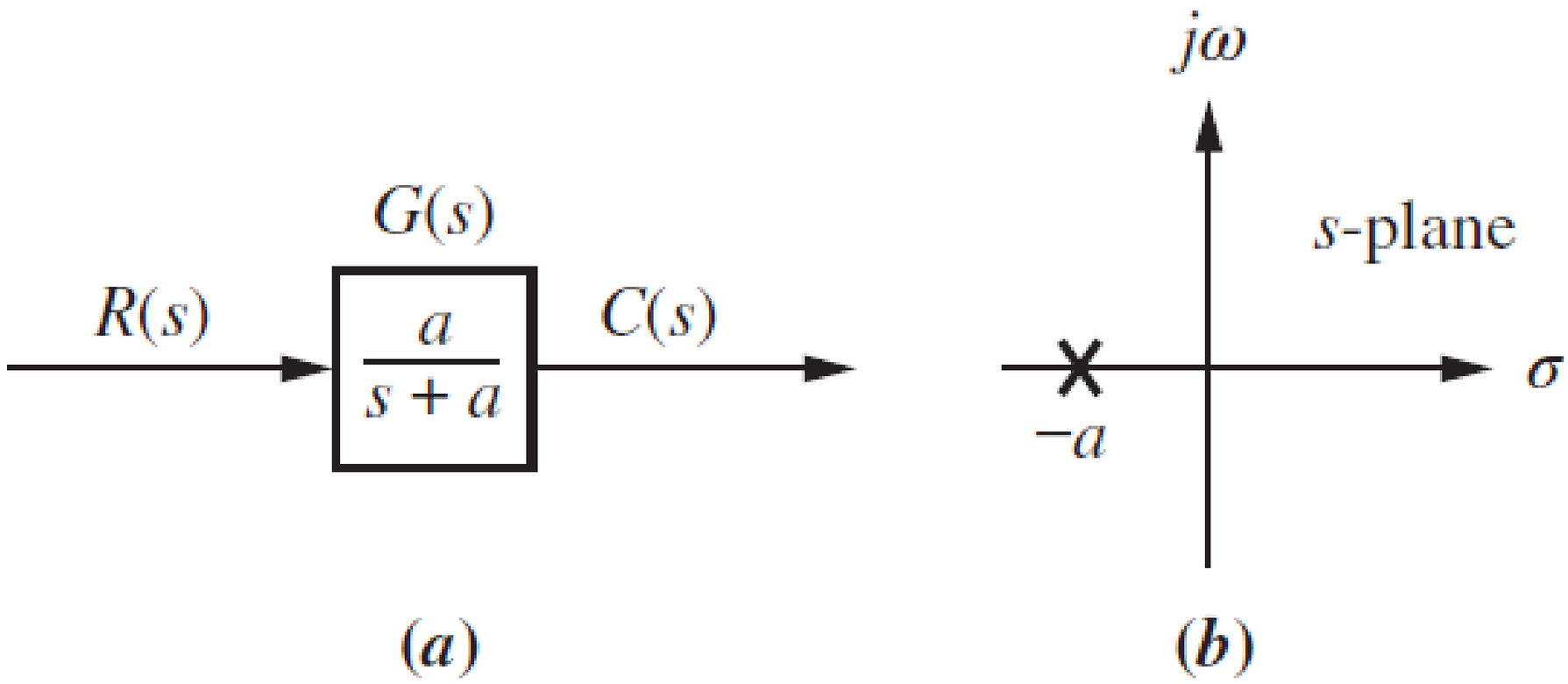
$$H(s) = \frac{1}{s} - \frac{1}{s+a}$$

Taking inverse Laplace transform, the equation for the step function of the first-order system in the time domain is:

$$h(t) = 1 - e^{-at}$$

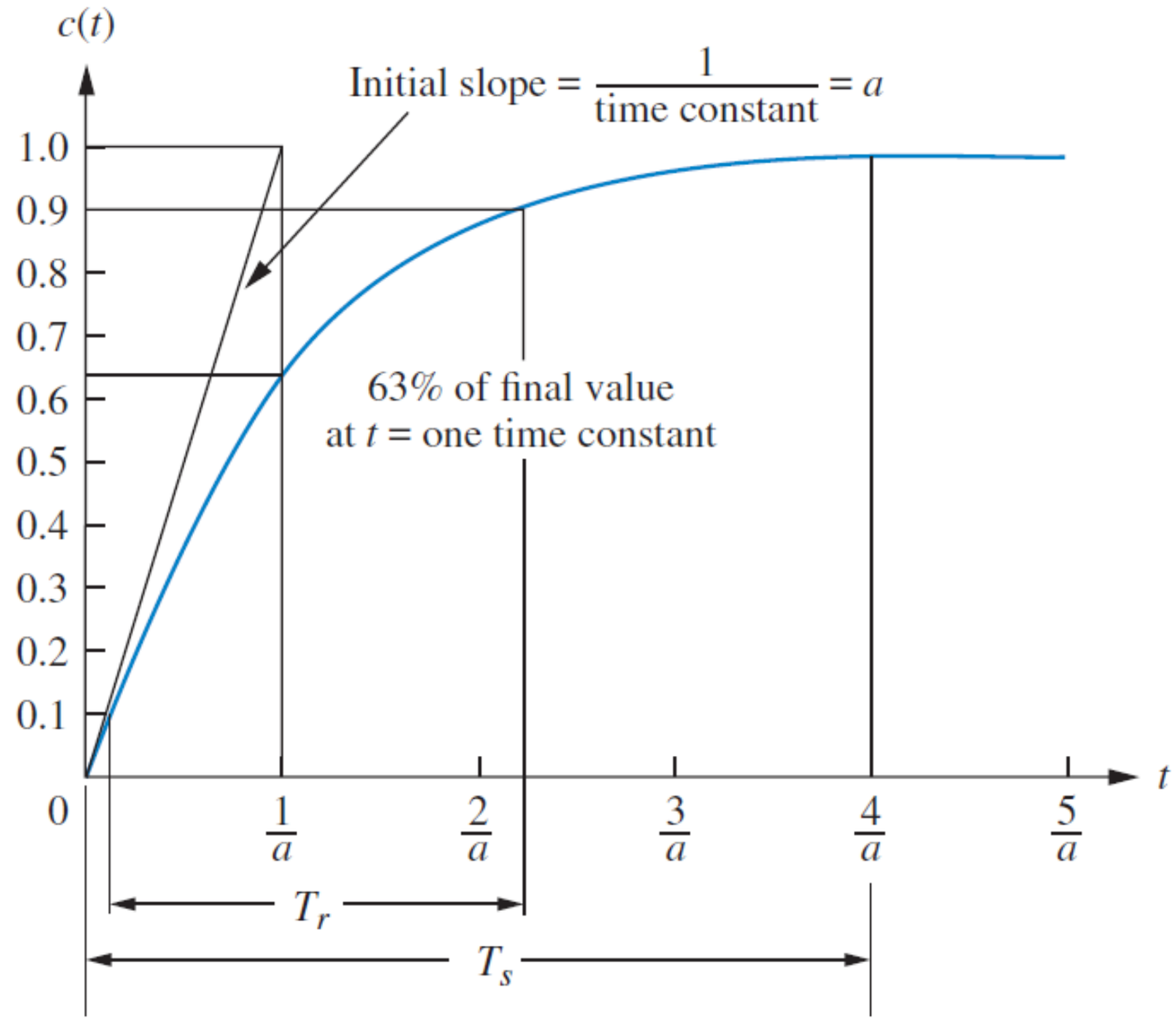
First-Order System

The pole-zero diagram of the first-order system is as shown in the figure below.



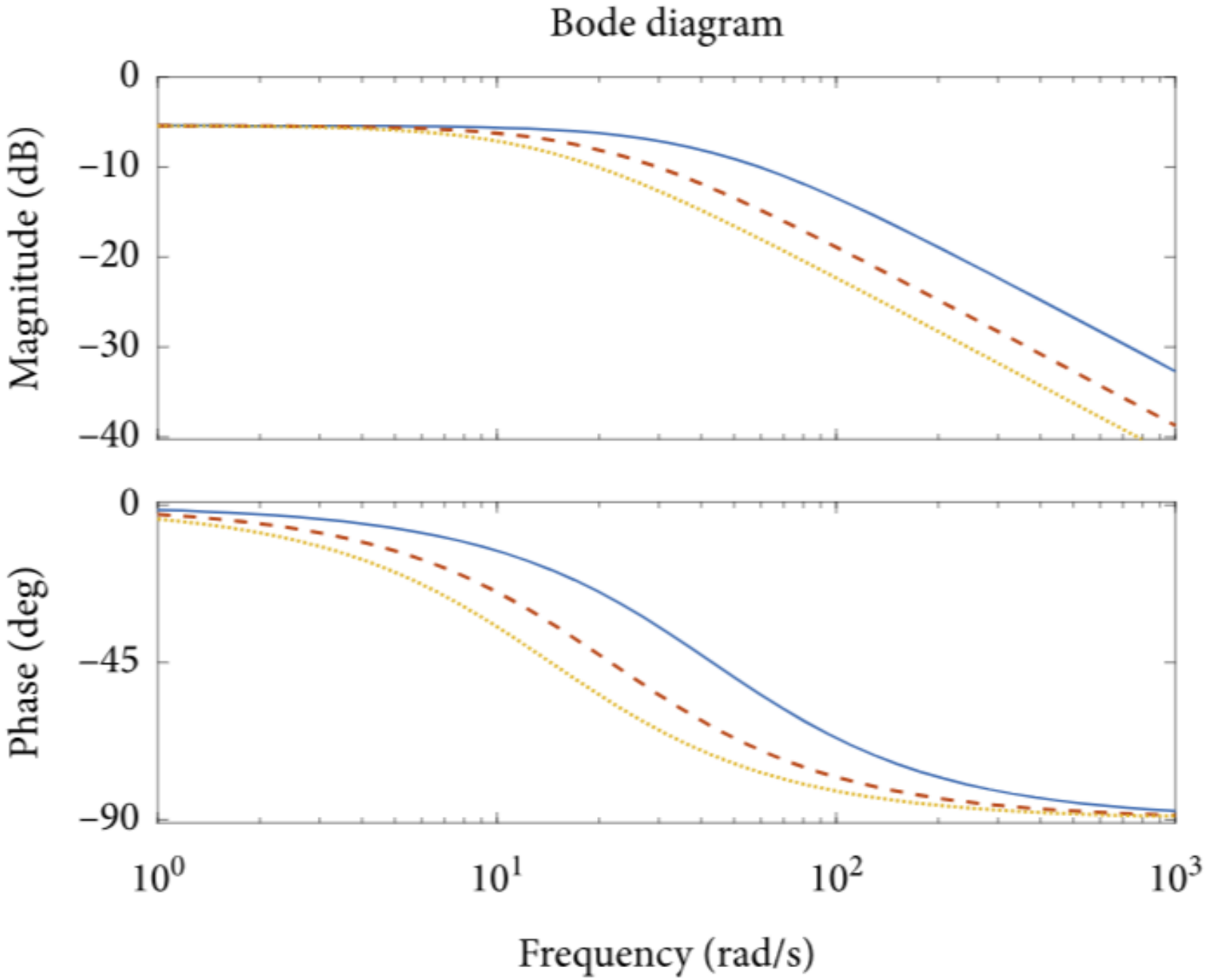
First-Order System

The figure below shows the transient response of the first-order system. Notice the parameters of the system in the time domain given in the graph.



First-Order System

The figure below shows the frequency response (Bode plot) of the first-order system with varied values of a.



Exercise 1 – First-Order Circuit

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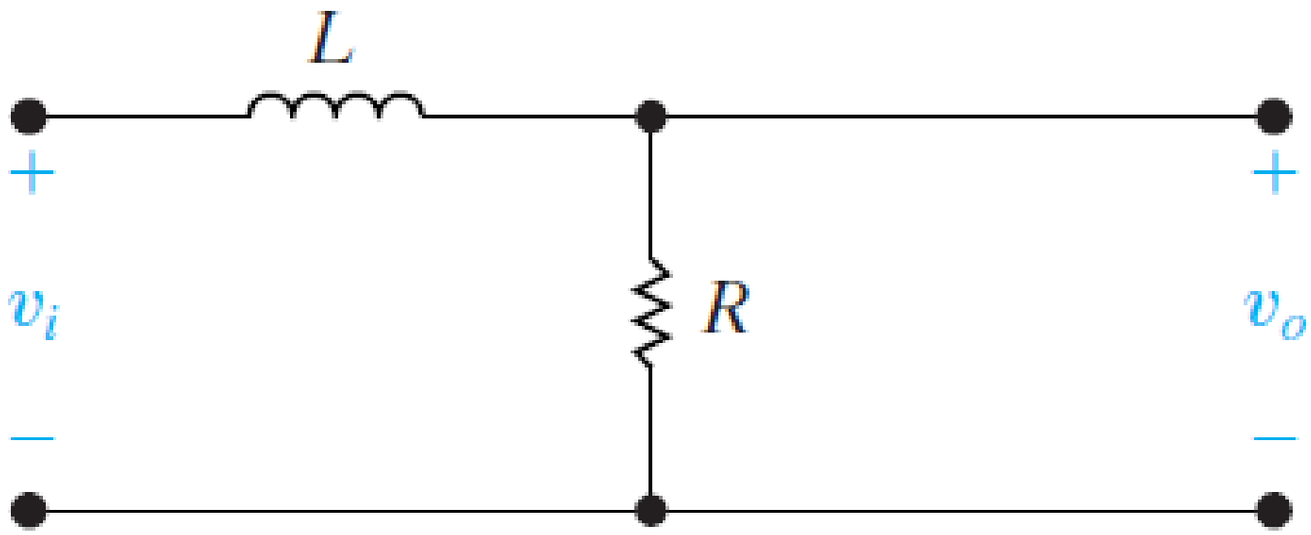
The diagram below shows a first-order RL circuit with the output voltage is taker



For the given first-order RL circuit, attempt the tasks given below and perform the simulations in MATLAB.

a. Using Laplace transform, determine the transfer function equation of the circuit. Derive expressions for the magnitude and phase of the frequency response and cut-off frequency. Determine both of the equations if R = 10 kΩ and L = 10 μH. [15 marks]

Exercise 1 – First-Order Circuit

b. Plot pole-zero diagram of the circuit. Determine the pole and zero of the circuit. [5 marks]

c. Determine expression for the time-domain response of the circuit by applying the inverse Laplace transform. [5 marks]

d. Simulate step response of the circuit. Find the rise time and settling time of the step response from the graph obtained in the simulation. [10 marks]

e. Simulate frequency response of the circuit. Determine cut-off frequency and the magnitude and phase shift of the frequency response. What type of response does the circuit produce? [15 marks]

Second Order System

For a second-order system, the following equation is a typical transfer function of the system in Laplace transform format:

$$G(s) = \frac{(\omega_n)^2}{s^2 + 2\zeta\omega_n s + (\omega_n)^2}$$

The roots of the equation given above are:

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - (\zeta)^2} = \sigma \pm s$$

Depending on the roots, the inverse Laplace transform of the transfer function equation given above is outlined in the table below.

Roots	**Time Response**	**Type of Response**
$-\sigma_1, -\sigma_2$	$g(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$	Overdamped
$-\sigma_d$	$g(t) = K_1 e^{-\sigma_d t} + K_2 t e^{-\sigma_d t}$	Critically damped
$-\sigma_d \pm j\omega_d$	$g(t) = A\, e^{-\sigma_d t}\cos(\omega_d t - \phi)$	Underdamped
$\pm j\omega_d$	$g(t) = A\cos(\omega_d t - \phi)$	Undamped

Second-Order System

For an example of the step response of a second-order system, considering a critical damped response case (e.g. $\zeta = 1$ and $\omega = \sigma_d$), the transfer function equation of the system is:

$$G(s) = \frac{(\sigma_d)^2}{s + 2\sigma_0 s + (\sigma_d)^2} = \frac{(\sigma_d)^2}{(s + \sigma_d)^2}$$

When the second-order system is given a step input ($R(s) = 1/s$), the overall equation of the system is:

$$H(s) = R(s)G(s) = \left(\frac{1}{s}\right)\frac{(\sigma_d)^2}{(s + \sigma_d)^2}$$

Taking the partial fraction expansion for repeated roots, the equation above becomes:

$$H(s) = \frac{1}{s} - \frac{K_2}{(s + \sigma_d)^2} - \frac{K_1}{s + \sigma_d}$$

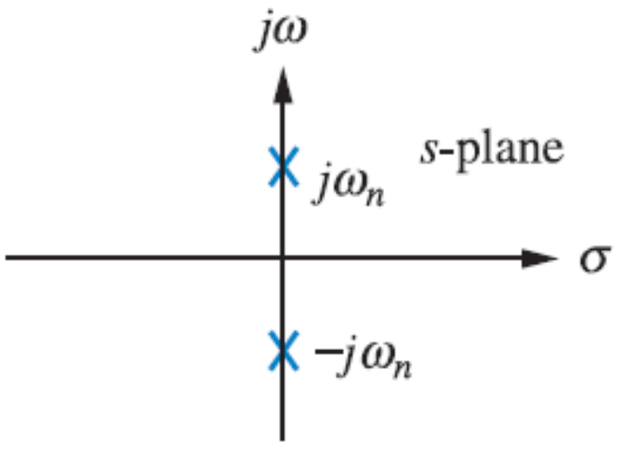
Second-Order System

The inverse Laplace transform of the equation given above is:

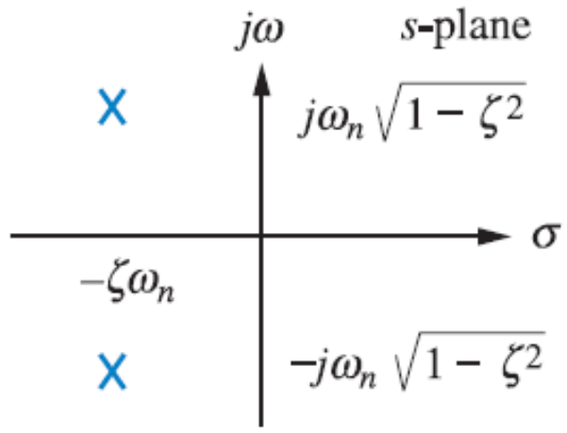
$$h(t) = 1 - K_2 t e^{-\sigma_d t} - K_1 e^{-\sigma_d t}$$

For other step response cases such as underdamped, overdamped, and undamped, please refer to the equations provided in the table above for their derivations.

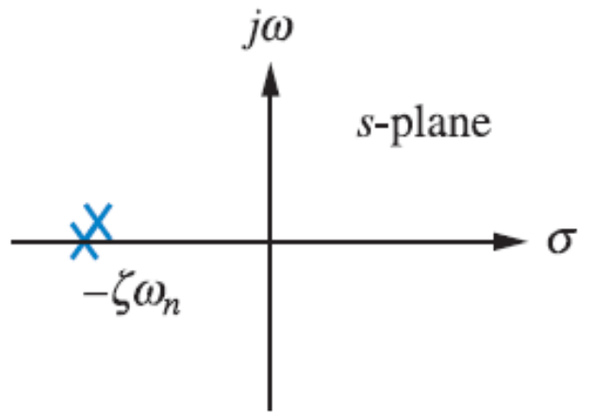
The pole-zero diagram of the second-order system is as shown in the figure below.



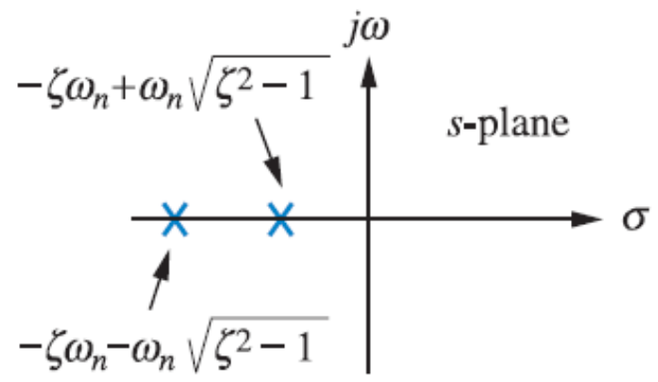
(a) Undamped system
($\zeta = 0$)



(b) Underdamped system
($0 < \zeta < 1$)



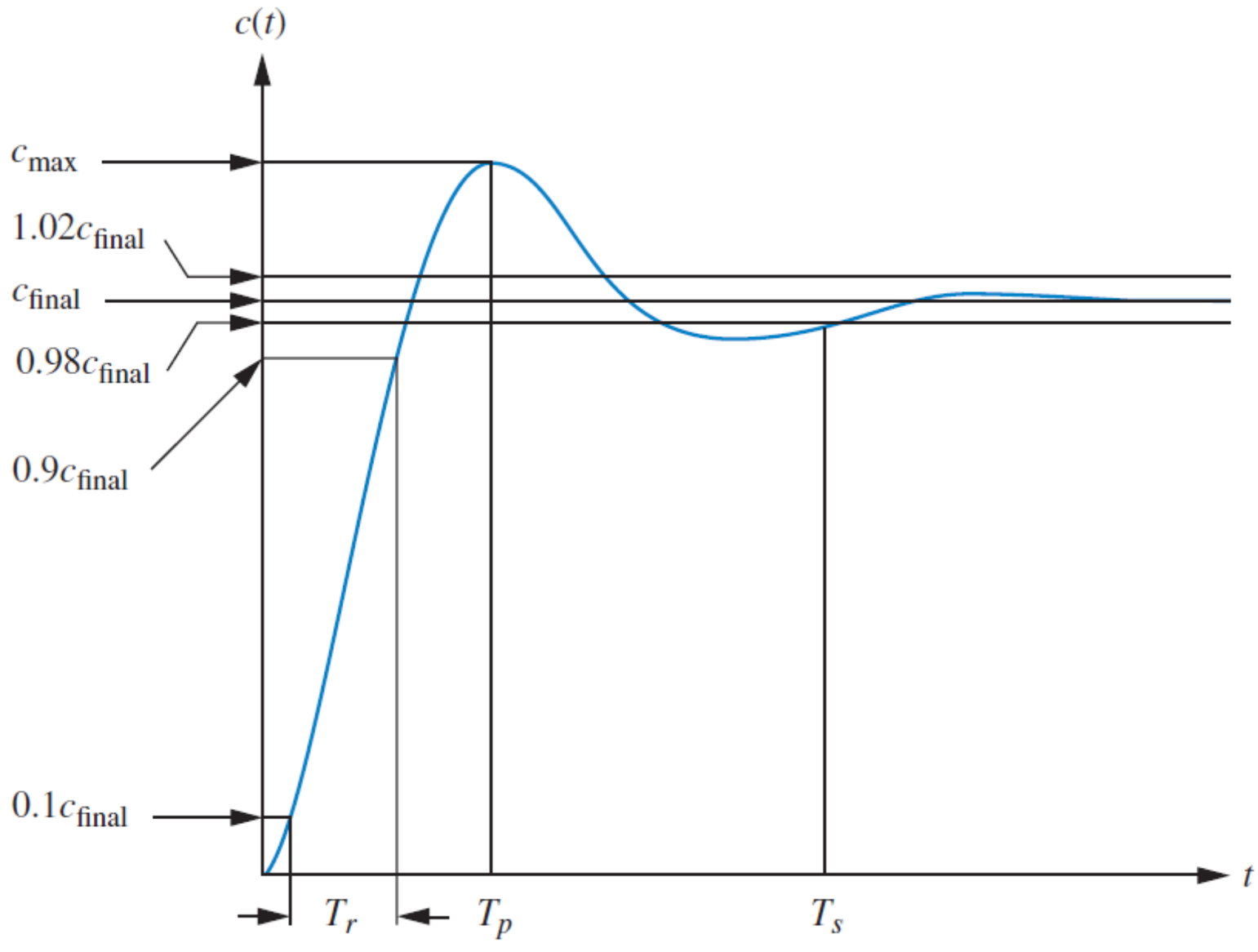
(c) Critical damped system
($\zeta = 1$)



(d) Overdamped system
($\zeta > 1$)

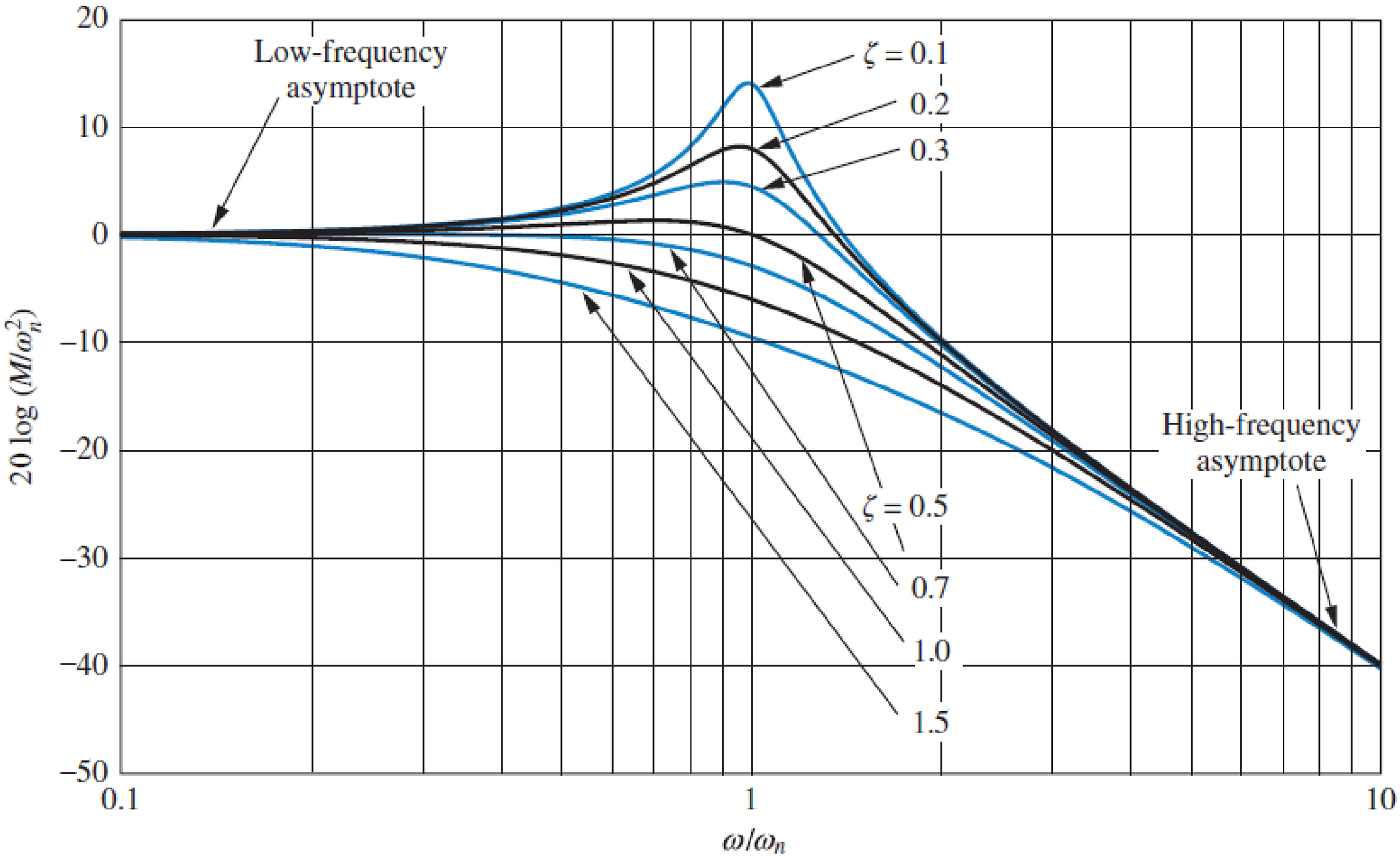
Second-Order System

The figure below shows the transient response of the second-order system. Notice the parameters of the system in the time domain given in the graph e.g. t_r = rise time, t_p = peak time, and t_s = settling time.



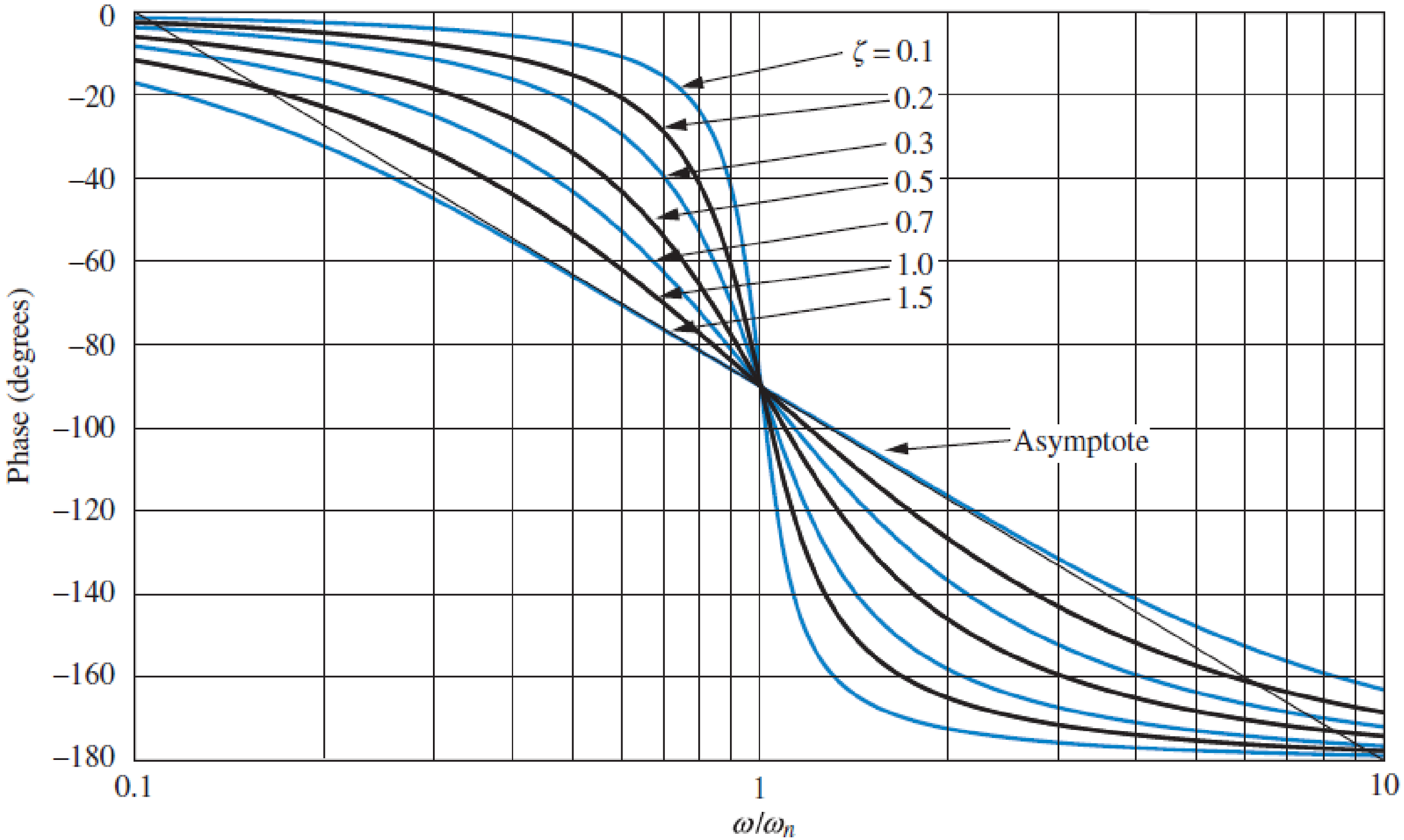
Second-Order System

The figure below shows the magnitude plot of the frequency response of the second-order system.



Second-Order System

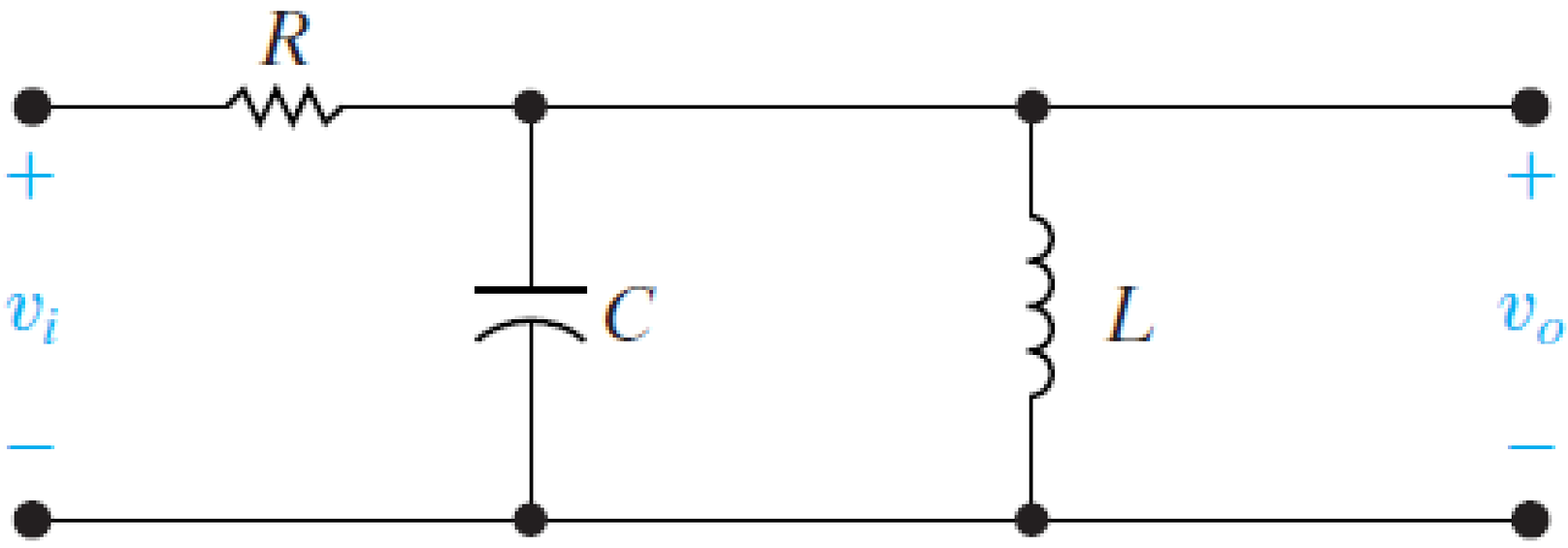
The figure below shows the phase plot of the frequency response of the second-order system.



Exercise 2 – Second-Order Circuit

Exercise 2 – Second-Order Circuit

Given in the diagram below is a second-order RLC circuit with the inductor in parallel with the capacitor and both components are in series with the resistor.



For the given second-order RLC circuit, perform the tasks and plot the following diagrams in the MATLAB.

Exercise 2 – Second-Order Circuit

a. Using Laplace transform, determine the transfer function equation of the circuit. Derive expressions for the magnitude and phase of the frequency response, center frequency, cut off frequencies, bandwidth, and Q-factor. Determine all equations if R = 1 kΩ, L = 1 μH, and C = 10 nF. [15 marks]

b. Pole-zero diagram of the circuit. Determine the pole and zero of the circuit. [5 marks]

c. Step response diagram of the circuit. Determine the rise time, percentage overshoot, and settling time of the step response. [10 marks]

d. Frequency response diagram of the circuit. Determine the center frequency, cut-off frequencies, the magnitude and phase shift of the frequency response, bandwidth, and the Q-factor of the circuit from the graphs obtained in the simulation. [10 marks]

Exercise 2 – Second-Order Circuit

e. Describe characteristics of the circuit using results in parts (b), (c), and (d). Do the calculated results match the simulation outcomes? What are two problems that exist in the system and describe a design measure to improve it? [10 marks]

NOTE: Attach snapshots of all MATLAB codes and simulation results as the appendix in your report.