XMUT203 Analogue Circuits and Systems

Laboratory 4: Laplace Transforms in Circuits

Due Date: Not for marking (no submission required).

A. Symbolic and Graph Functions in MATLAB

For solving the characteristics and behaviour of a given electrical circuit in MATLAB using the transfer equation of the circuit, the following functions are commonly used in the analysis process:

- `solve()` – Solve a given transfer function equation.
- `factor()` – Find the roots of the transfer function equation.
- `laplace()` – Convert the equation from time-domain to s-domain.
- `ilaplace()` – Convert the equation from s-domain to time-domain.
- `ezplot()` - Plot the pole-zero diagram of the transfer function.

For graph plotting, there are several functions in MATLAB that are typically needed for circuit simulation.

- `plot()` – Plot graph.
- `xlim()` and `ylim()` – Limit range of x-axis or y-axis.
- `title()` – Print title of the graph.
- `xlabel()` and `ylabel()` – Label x-axis or y-axis.
- `legend()` – Print indicators for the plots in the graph.
- `text()` – Print a text at particular position in the graph
- `line()` – Plot a line.
- `semilogx()` and `semilogy()` – Specify the graph to be log in the x-axis or y-axis.
- `loglog()` – Specify the graph to be log on both x-axis and y-axis.

B. First-Order System

Considering the transfer function of a given system is given as:

$$G(s) = \frac{Y(s)}{X(s)} = \frac{(s+z_1)(s+j\omega_2)\dots(s+z_n)}{(s+p_1)(s+p_2)\dots(s+p_n)}$$

Where:

- $z_1, z_2, \dots, z_n$ are the zeros of the system.
- $p_1, p_2, \dots, p_n$ are the poles of the system.

For a first-order system, the following equation is a typical transfer function of the system in Laplace transform with a pole at $s = -a$:

$$G(s) = \frac{a}{s+a}$$

The inverse Laplace transform of the transfer function equation given above is:

$$g(t) = e^{-at} + g(0^-)$$

When the first-order system is given a step input ($R(s) = 1/s$), its transfer function equation is:

$$H(s) = R(s)G(s) = \left(\frac{1}{s}\right)\left(\frac{a}{s+a}\right)$$

Applying partial fraction expansion, the equation above becomes:

$$H(s) = \frac{1}{s} - \frac{1}{s+a}$$

Taking inverse Laplace transform, the equation for the step function of the first-order system in the time domain is:

$$h(t) = 1 - e^{-at}$$

The pole-zero diagram of the first-order system is as shown in the figure below.

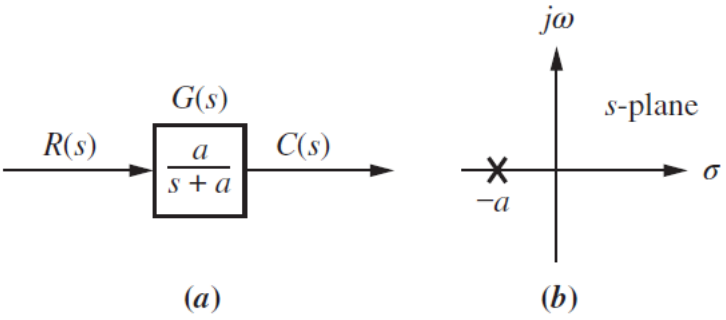


Figure 1: Pole and zero of the first-order system (a) in the s-plane diagram (b)

The figure below shows the transient response of the first-order system. Notice the parameters of the system in the time domain given in the graph.

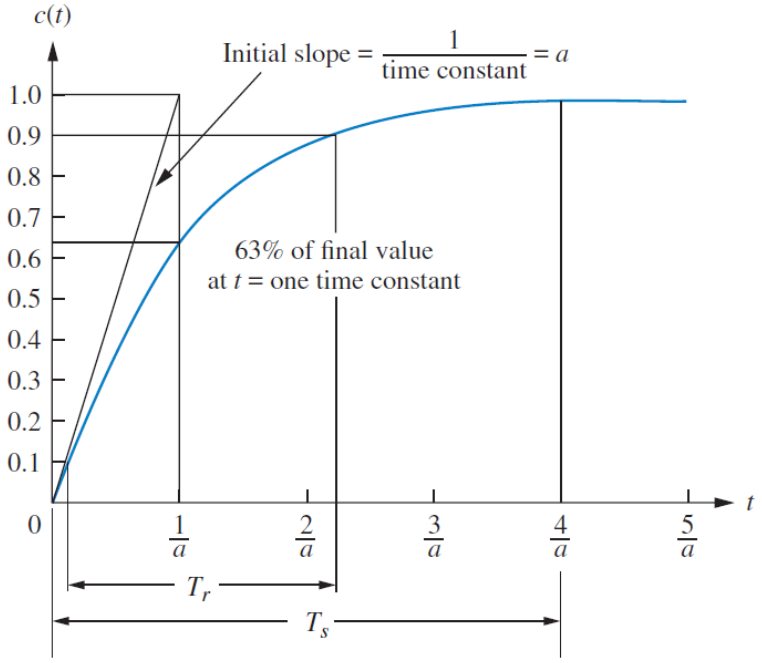


Figure 2: Transient response of the first-order system and its time-domain parameters (note: T_r = rise time, T_s = settling time

The figure below shows the frequency response (Bode plot) of the first-order system.

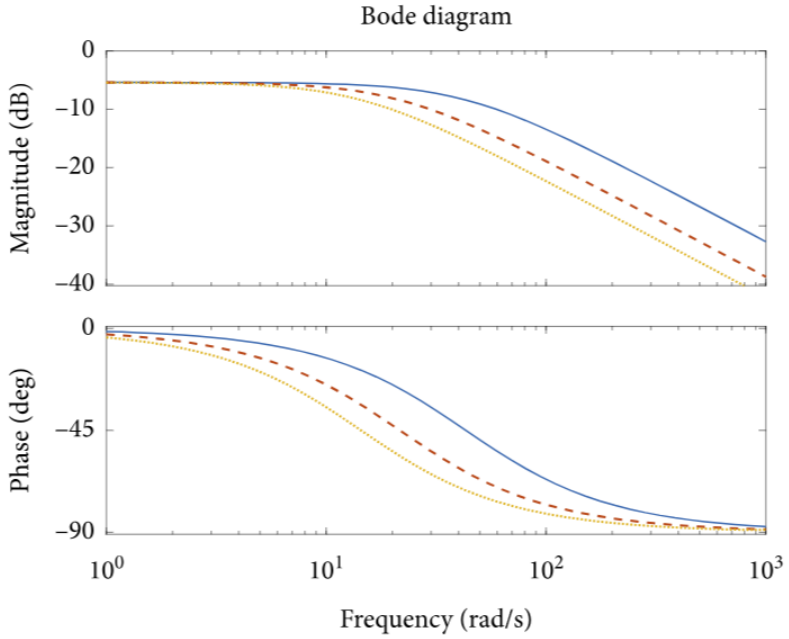


Figure 3: Frequency response of the first-order system with varied values of a

Exercise 1 – First-Order Circuit

The diagram below shows a first-order RL circuit with the output voltage taken across the resistor.

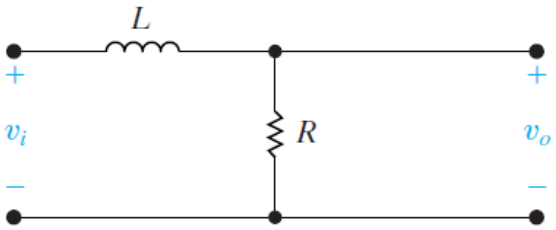


Figure 4: First-order RL circuit

For the given first-order RL circuit, attempt the tasks given below and perform the simulations in MATLAB.

a. Using Laplace transform, determine the transfer function equation of the circuit. Derive expressions for the magnitude and phase of the frequency response and cut-off frequency. Determine both equations if R = 10 kΩ and L = 10 μH. [15 marks]

b. Plot pole-zero diagram of the circuit. Determine the pole and zero of the circuit. [5 marks]

c. Determine expression for the time-domain response of the circuit by applying the inverse Laplace transform. [5 marks]

d. Simulate step response of the circuit. Find the rise time and settling time of the step response from the graph obtained in the simulation. [10 marks]

e. Simulate frequency response of the circuit. Determine cut-off frequency and the magnitude and phase shift of the frequency response. What type of response does the circuit produce? [15 marks]

C. Second-Order System

For a second-order system, the following equation is a typical transfer function of the system in Laplace transform format:

$$G(s) = \frac{(\omega_n)^2}{s^2 + 2\zeta\omega_n s + (\omega_n)^2}$$

The roots of the equation given above are:

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-(\zeta)^2} = \sigma \pm s$$

Depending on the roots, the inverse Laplace transform of the transfer function equation given above is outlined in the table below.

Roots	**Time Response**	**Type of Response**
$-\sigma_1, -\sigma_2$	$g(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$	Overdamped
$-\sigma_d$	$g(t) = K_1 e^{-\sigma_d t} + K_2 t e^{-\sigma_d t}$	Critically damped

$-\sigma_d \pm j\omega_d$	$g(t) = A\,e^{-\sigma_d t}\cos(\omega_d t - \phi)$	Underdamped
$\pm j\omega_d$	$g(t) = A\cos(\omega_d t - \phi)$	Undamped

Note: σ_d is real (damped) part of the roots and $\omega_d = \omega_n\sqrt{1-\zeta^2}$.

Table 1: Roots of transfer function equation and transient response of the system

For an example of the step response of a second-order system, considering a critical damped response case (e.g. $\zeta = 1$ and $\omega = \sigma_d$), the transfer function equation of the system is:

$$G(s) = \frac{(\sigma_d)^2}{s + 2\sigma_0 s + (\sigma_d)^2} = \frac{(\sigma_d)^2}{(s+\sigma_d)^2}$$

When the second-order system is given a step input ($R(s) = 1/s$), the overall equation of the system is:

$$H(s) = R(s)G(s) = \left(\frac{1}{s}\right)\frac{(\sigma_d)^2}{(s+\sigma_d)^2}$$

Taking the partial fraction expansion for repeated roots, the equation above becomes:

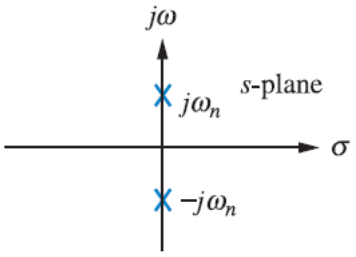
$$H(s) = \frac{1}{s} - \frac{K_2}{(s+\sigma_d)^2} - \frac{K_1}{s+\sigma_d}$$

The inverse Laplace transform of the equation given above is:

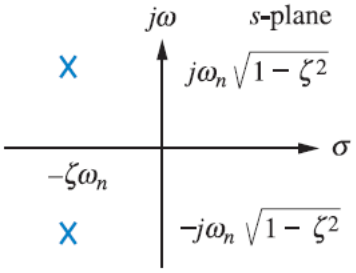
$$h(t) = 1 - K_2 t e^{-\sigma_d t} - K_1 e^{-\sigma_d t}$$

For other step response cases such as underdamped, overdamped, and undamped, please refer to the equations provided in the table above for their derivations.

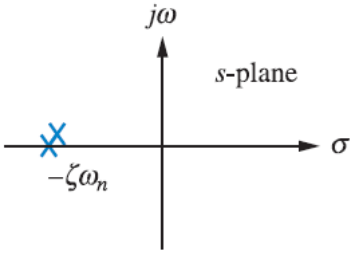
The pole-zero diagram of the second-order system is as shown in the figure below.



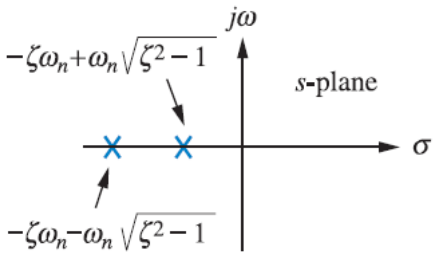
(a) Undamped system ($\zeta = 0$)



(b) Underdamped system ($0 < \zeta < 1$)



(c) Critical damped system ($\zeta = 1$)



(d) Overdamped system ($\zeta > 1$)

Figure 5: Pole-zero of the second-order system in the s-plane diagram at various damping ratio (ζ)

The figure below shows the transient response of the underdamped second-order system. Notice the parameters of the system in the time domain given in the graph.

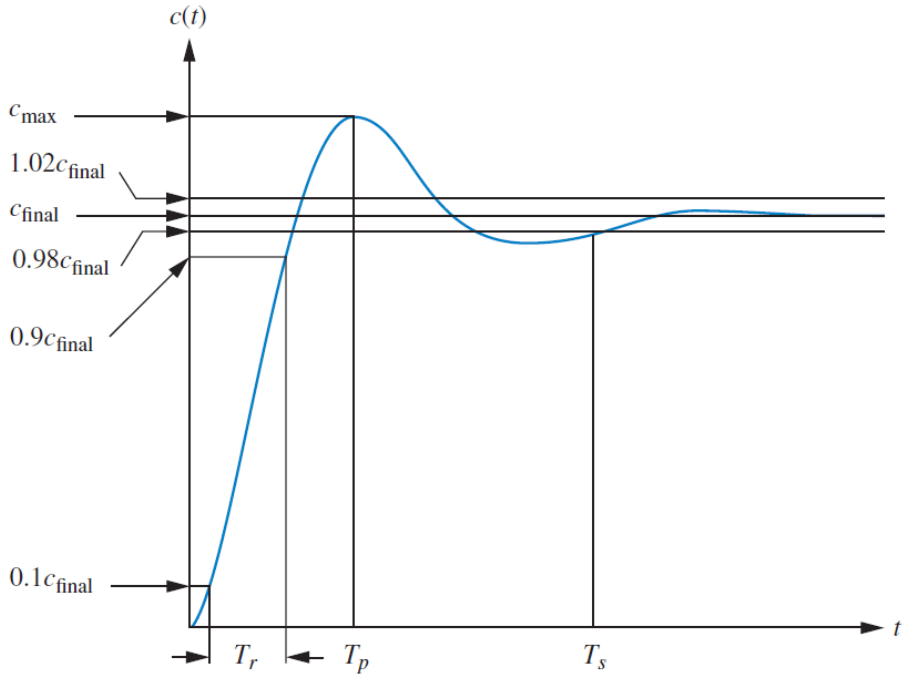
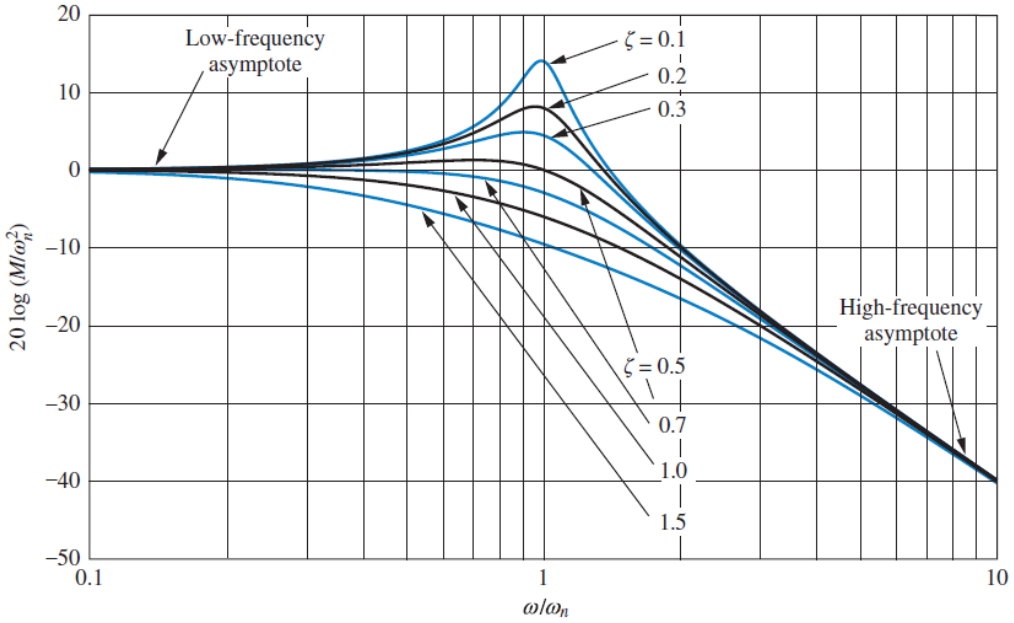


Figure 6: Transient response of the underdamped second-order system and its time-domain parameters (note: t_r = rise time, t_p = peak time, and t_s = settling time).

The figure below shows the frequency response of the underdamped second-order system.



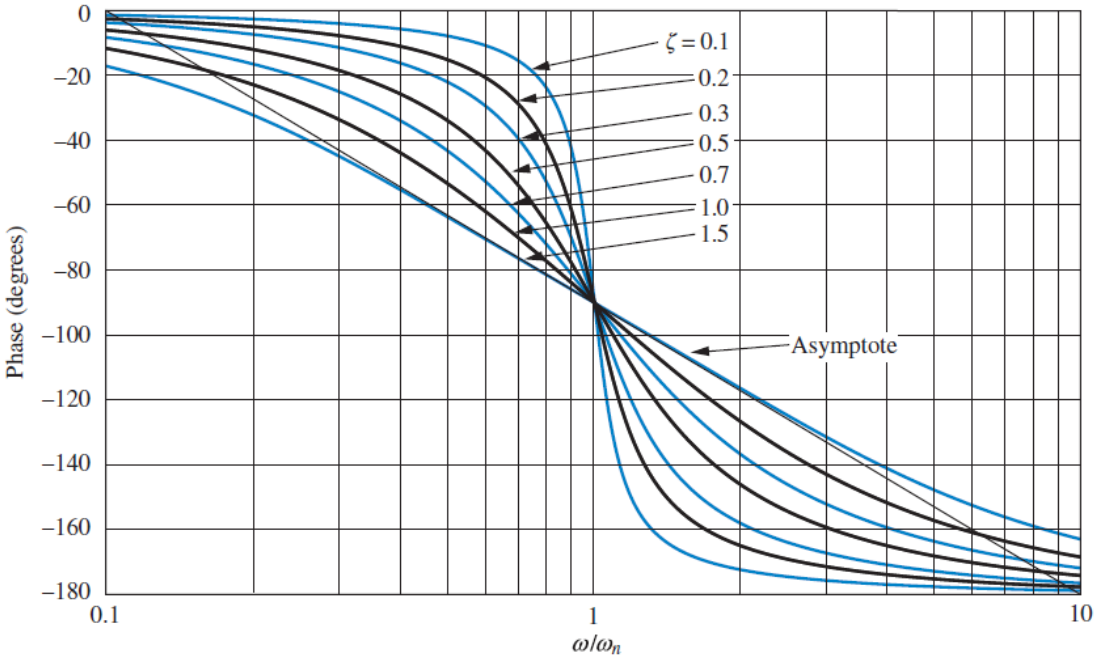


Figure 7: Magnitude plot (top) and phase plot (bottom) of the frequency response of the underdamped second-order system at various damping ratio (ζ)

Exercise 2 – Second-Order Circuit

Given in the diagram below is a second-order RLC circuit with the inductor in parallel with the capacitor and both components are in series with the resistor.

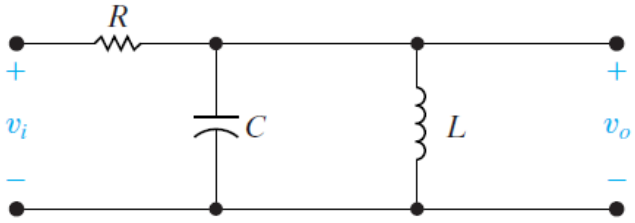


Figure 8: Second-order RLC circuit

For the given second-order RLC circuit, perform the following tasks and plot the relevant diagrams in the MATLAB.

a. Using Laplace transform, determine the transfer function equation of the circuit. Derive expressions for the magnitude and phase of the frequency response, center frequency, cut-off frequencies, bandwidth, and Q-factor. Determine all equations if R = 1 kΩ, L = 1 μH, and C = 10 nF. [15 marks]

b. Pole-zero diagram of the circuit. Determine the pole and zero of the circuit. [5 marks]

c. Step response diagram of the circuit. Determine the rise time, percentage overshoot, and settling time of the step response. [10 marks]

d. Frequency response diagram of the circuit. Determine the center frequency, cut-off frequencies, the magnitude and phase shift of the frequency response, bandwidth, and the Q-factor of the circuit from the graphs obtained in the simulation. [10 marks]

e. Describe characteristics of the circuit using results in parts (b), (c), and (d). Do the calculated results match the simulation outcomes? What are two problems that exist in the system and describe a design measure to improve it? [10 marks]

NOTE: Attach snapshots of all MATLAB codes and simulation results as the appendix in your report.

Marking Schedule

Student Name : _____

Student ID : _____

No	Description	Mark	Your Mark	Remark
	Exercise 1 – First-Order RL Circuit			
1a	Transfer function equation of the circuit, expressions for magnitude and phase of frequency response and cut off frequency at R = 10 kΩ and L = 10 μH.	15		
1b	Pole-zero diagram of the circuit. Pole and zero of the circuit.	5		
1c	Expression for the time-domain response of the circuit.	5		
1d	Step response diagram of the circuit. The rise time and settling time of the step response.	10		
1e	Frequency response diagram of the circuit. The cut-off frequency and magnitude and phase shift of the frequency response. Type of response of the circuit.	15		
	Exercise 2 – Second-Order RLC Circuit			
2a	Transfer function equation of the circuit. Expressions for the magnitude and phase of the frequency response, cut off frequencies, bandwidth, and Q-factor and at R = 1 kΩ, L = 1 μH, and C = 10 nF.	15		
2b	Pole-zero diagram of the circuit. The pole and zero of the circuit.	5		
2c	Step response diagram of the circuit. The rise time, percentage	10		

	overshoot, and settling time of the step response.			
2d	Frequency response diagram of the circuit. The cut-off frequency, the magnitude and phase shift of the frequency response, bandwidth, and the Q-factor of the circuit.	10		
2e	Characteristics of the circuit and comparison between calculated and simulation results. Two problems of the circuit and a design measure to fix it.	10		
	Total	100		

Comment: