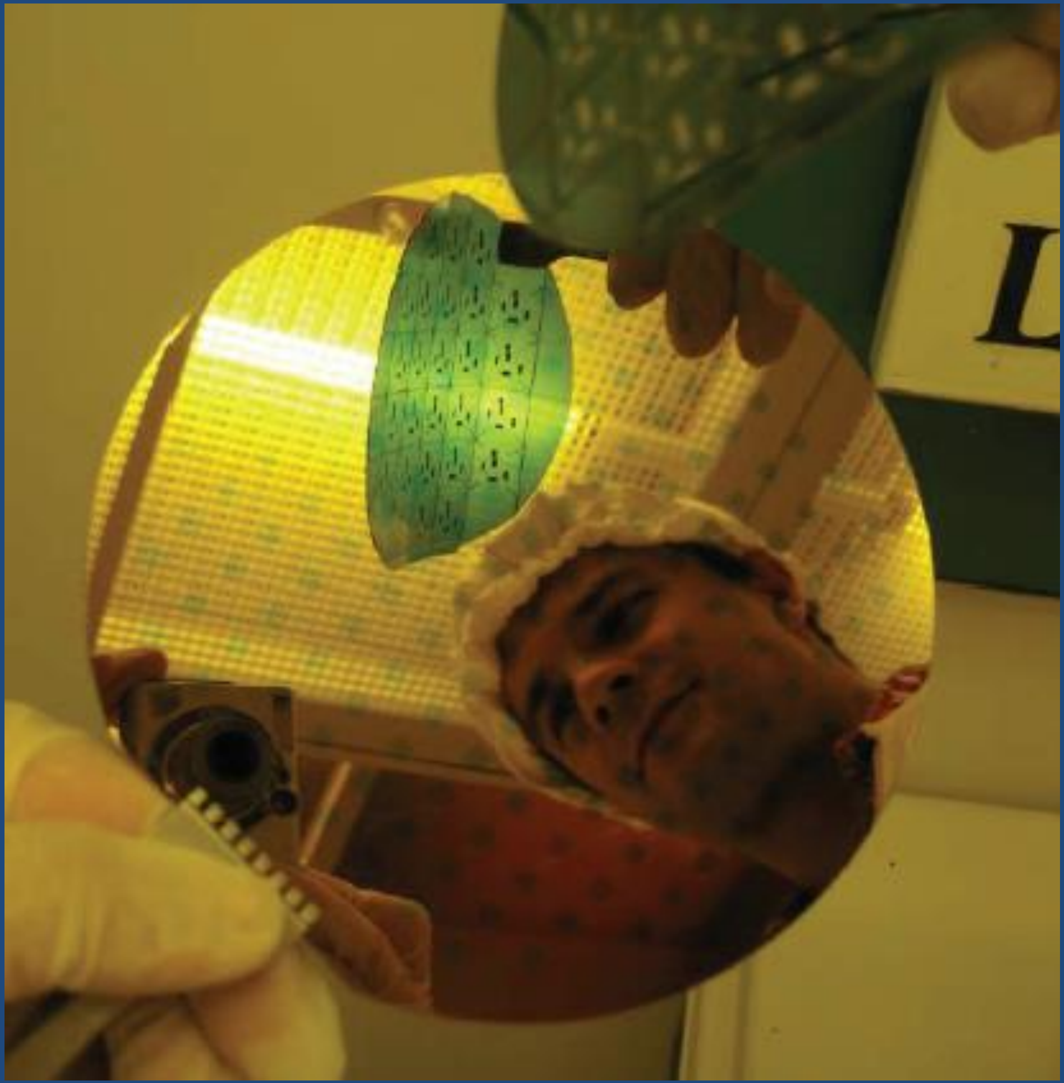


3. ANALYSIS TECHNIQUES

ECEN203

Analogue Circuits and Systems

Analysis Techniques

- Apply node-voltage and mesh-current methods to analyse an electric circuit of any configuration, as long as it is linear and planar.
- Apply the by-inspection methods to circuits that satisfy certain conditions.
- Use the source-superposition method to evaluate the sensitivity of a circuit to the various sources in the circuit.
- Determine the Thevenin and Norton equivalent circuits

Revision: Linear circuits

- A linear circuit s a circuit of independent sources and linear element.
- A linear element: it is passive and exhibits a linear i-v relationship.
- Passive: it does not generate energy of its own, eg a resistor.
- Use Ohm's law: v= i x R to show a few properties of a linear circuit.

Properties of a linear circuit

- Homogeneity: if i through R is increased by a factor of K, so will v. Also known as the scaling property.
- Superposition principle:

If current i_1 can give rise to voltage $v_1 = Ri_1$, and another current i_2 can give rise to voltage $v_2 = Ri_2$, then the *simultaneous* presence of both currents gives rise to

$$v = R(i_1 + i_2) = Ri_1 + Ri_2 = v_1 + v_2.$$

so the output v due to two inputs (i1 and i2) is equal to the sum of the two outputs (v1 and v2) when each input is introduced separately.

Linear elements

- Resistors, R
- Capacitors, C
- Inductors, L

A quick overview on C where $i = C\,\dfrac{d\upsilon}{dt}$

If we multiply both sides by a factor K, we get

$$Ki = KC\,\frac{d\upsilon}{dt} = C\,\frac{d}{dt}\,(K\upsilon).$$

Hence, increasing υ by a factor K leads to an increase in i by the same factor, which implies that the d/dt differentiation operator has no bearing on the homogeneity property linking i to υ. The time derivative does not impact the additivity property either.

Non-linear elements

- Diodes,
- Transistors,
- Integrated circuits (IC)

The analysis techniques learnt here cannot be applied directly to them;

But, it is often possible to convert into equivalent circuits with linear elements and dependant sources.

(not covered in this course)

Node-Voltage Method

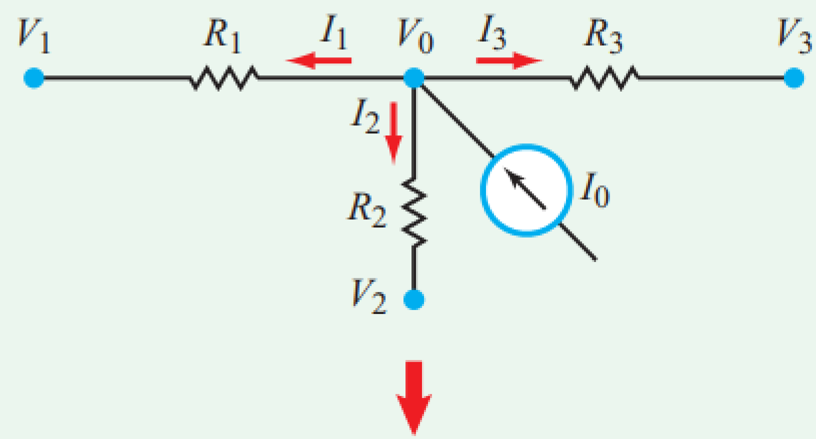
Solution Procedure: Node Voltage

Step 1: Identify all extraordinary nodes, select one of them as a reference node (ground), and then assign node voltages to the remaining $(n_{ex} - 1)$ extraordinary nodes.

Step 2: At each of the $(n_{ex} - 1)$ extraordinary nodes, apply the form of KCL requiring the sum of all currents *leaving* a node to be zero.

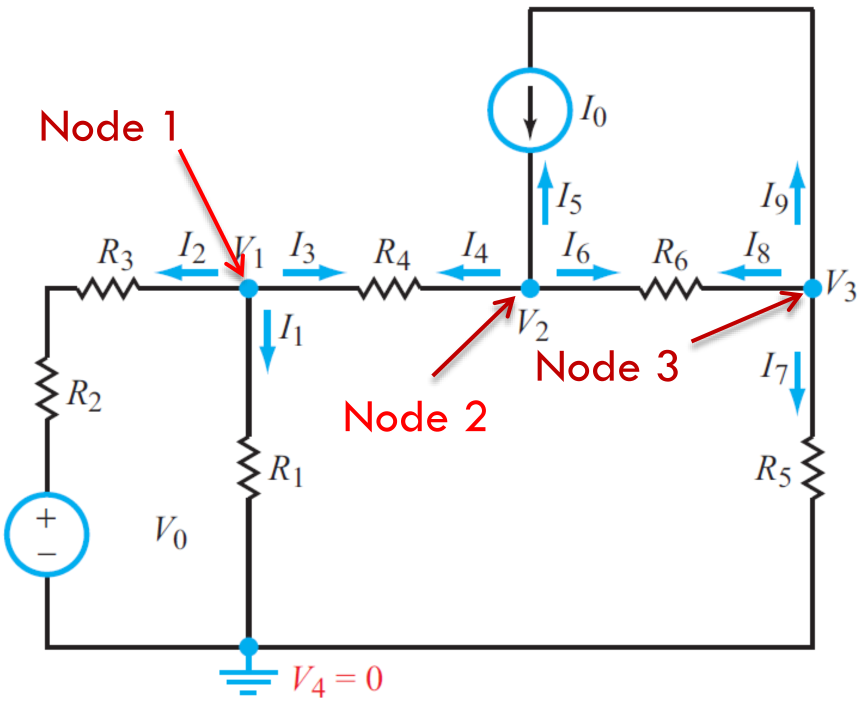
Step 3: Solve the $(n_{ex} - 1)$ independent simultaneous equations to determine the unknown node voltages.

KCL Template



$$\frac{V_0 - V_1}{R_1} + \frac{V_0 - V_2}{R_2} + \frac{V_0 - V_3}{R_3} - I_0 = 0$$

Example: Node-Voltage Method



Solution Procedure: Node Voltage

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Step 3: Solve the $(n_{ex} - 1)$ independent simultaneous equations to determine the unknown node voltages.

Node 1:

$$I_1 + I_2 + I_3 = 0.$$

$$\frac{V_1}{R_1} + \frac{V_1 - V_0}{R_2 + R_3} + \frac{V_1 - V_2}{R_4} = 0 \qquad \text{(node 1)}.$$

Node 2

$$\frac{V_2 - V_1}{R_4} - I_0 + \frac{V_2 - V_3}{R_6} = 0 \qquad \text{(node 2)},$$

Node 3

$$\frac{V_3}{R_5} + \frac{V_3 - V_2}{R_6} + I_0 = 0 \qquad \text{(node 3)}.$$

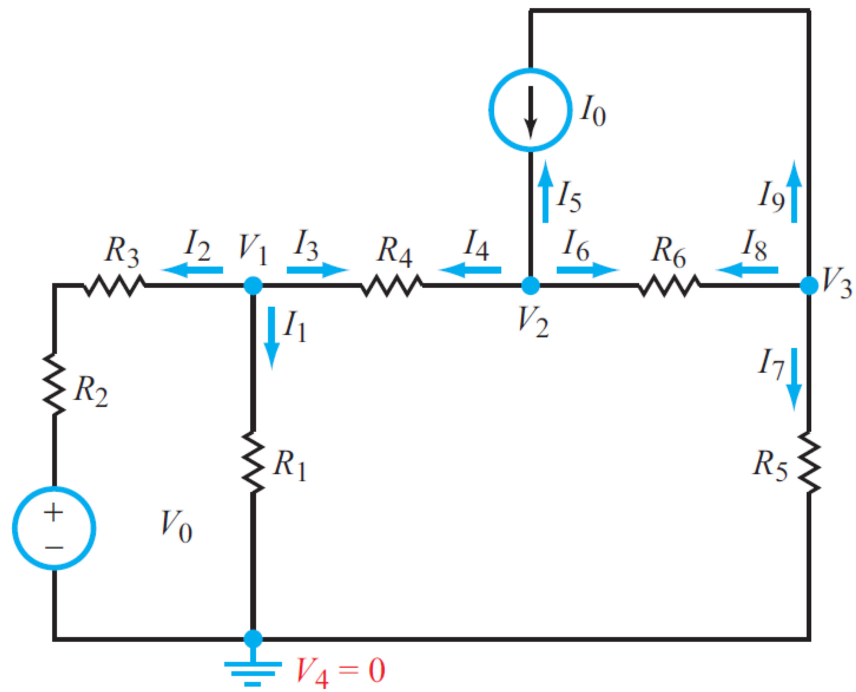
Example: Node-Voltage Method

$$\left(\frac{1}{R_1}+\frac{1}{R_2+R_3}+\frac{1}{R_4}\right)V_1-\left(\frac{1}{R_4}\right)V_2=\frac{V_0}{R_2+R_3}, \tag{3.8a}$$

$$-\left(\frac{1}{R_4}\right)V_1+\left(\frac{1}{R_4}+\frac{1}{R_6}\right)V_2-\frac{V_3}{R_6}=I_0, \tag{3.8b}$$

and

$$-\left(\frac{1}{R_6}\right)V_2+\left(\frac{1}{R_5}+\frac{1}{R_6}\right)V_3=-I_0. \tag{3.8c}$$



Three equations in 3 unknowns:
Solve using Cramer's rule, matrix
inversion, or MATLAB

$$a_{11}V_1+a_{12}V_2+a_{13}V_3=b_1$$

$$a_{21}V_1+a_{22}V_2+a_{23}V_3=b_2$$

$$a_{31}V_1+a_{32}V_2+a_{33}V_3=b_3$$

Example: Node-Voltage method

- Try it out yourself

Problem 3.3 Use nodal analysis to determine the current I_x and amount of power supplied by the voltage source in the circuit of Fig. P3.3.

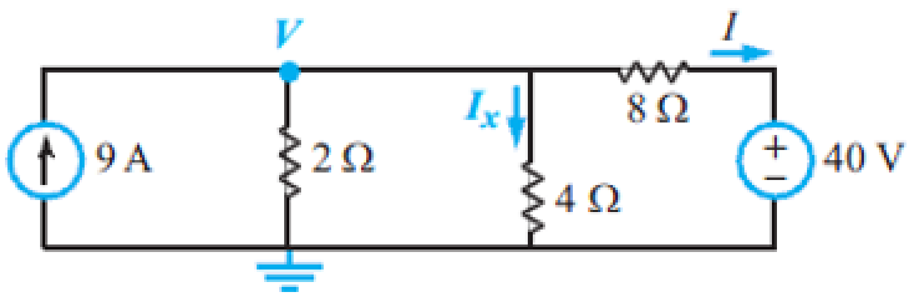


Figure P3.3: Circuit for Problem 3.3.

Solution: At node V, application of KCL gives

$$-9+\frac{V}{2}+\frac{V}{4}+\frac{V-40}{8}=0$$

$$V\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)=9+\frac{40}{8}$$

$$\frac{7V}{8}=9+5$$

$$V=16\text{ V.}$$

The current I_x is then given by

$$I_x=\frac{V}{4}=\frac{16}{4}=4\text{ A.}$$

To find the power supplied by the 40-V source, we need to first find the current I flowing into its positive terminal,

$$I=\frac{V-40}{8}=\frac{16-40}{8}=-3\text{ A.}$$

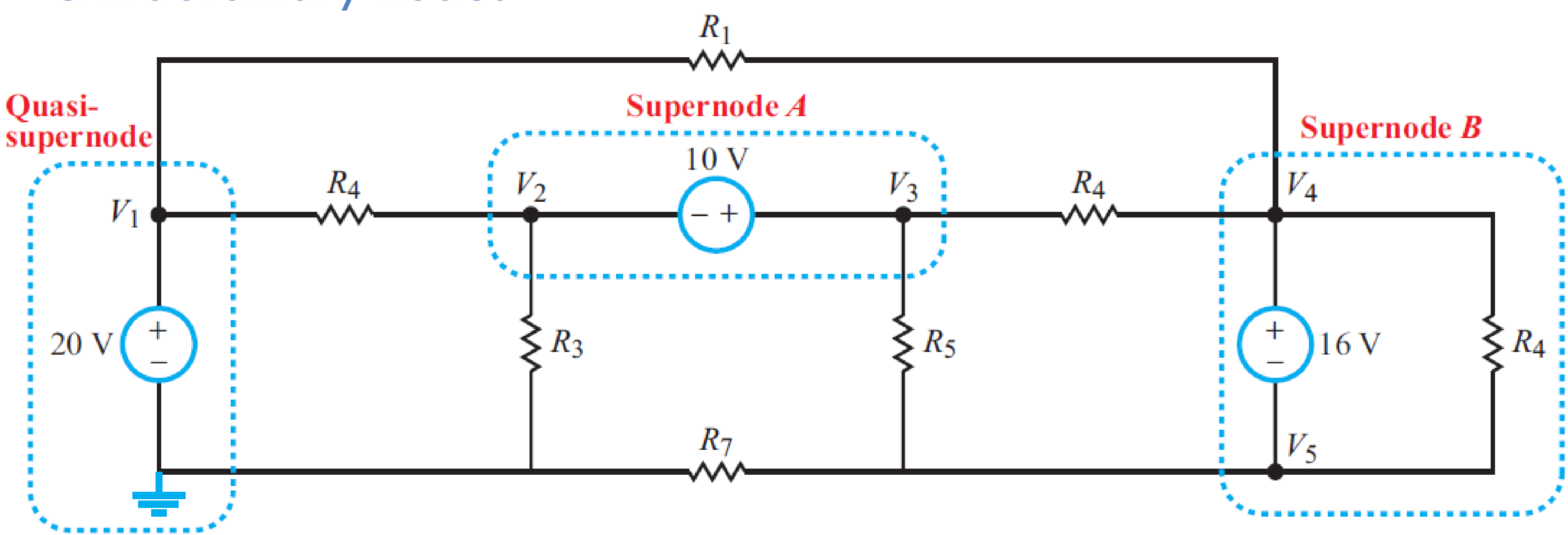
Hence,

$$P=VI=40\times(-3)=-120\text{ W}$$

(The minus sign confirms that the voltage source is a supplier of power.)

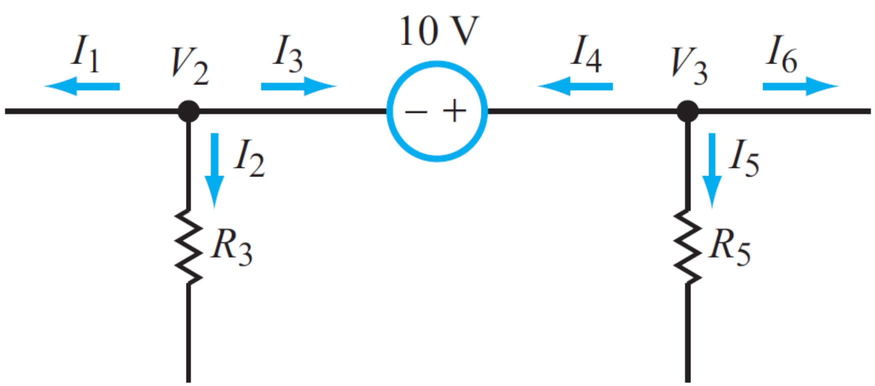
Supernode

A supernode is formed when a voltage source connects two extraordinary nodes



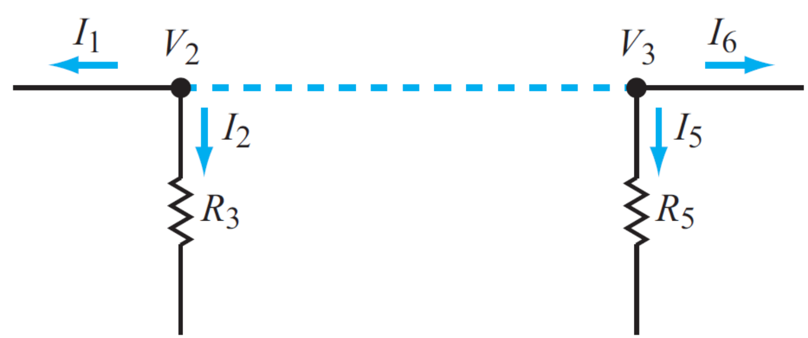
- Less nodes to worry about, less work!
- Write KVL equation for supernode
- Write KCL equation for closed surface around supernode
- V1 = 20 V directly. Why?

KCL at Supernode: special note



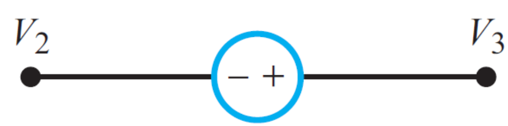
Supernode A

=



$$I_1 + I_2 + I_5 + I_6 = 0 \quad \text{(KCL)}$$

+



$$V_3 - V_2 = 10\ \text{V}, \qquad \text{(Auxiliary Eq.)}$$

- Note that “internal” current (I3 and I4, because I3 = -I4) in supernode cancels, simplifying KCL expressions
- Takes care of unknown current in a voltage source

Supernode Attributes

(1) At a supernode, Kirchhoff's current law (KCL) can be applied to the combination of the two nodes as if they are a single node, but the two nodes retain their own identities.

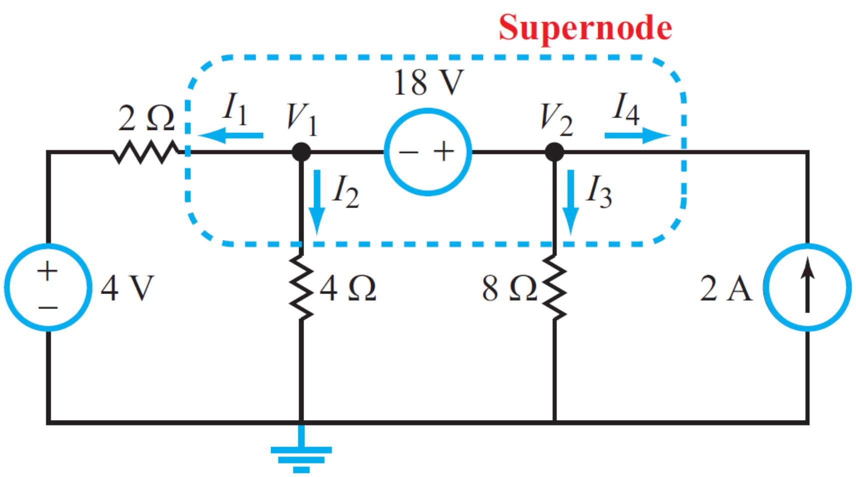
(2) Kirchhoff's voltage law (KVL) is used to express the voltage difference between the two nodes in terms of the voltage of the source between them. This provides the ***supernode auxiliary equation***.

(3) If a supernode contains a resistor in parallel with the voltage source, the resistor exercises no influence on the currents and voltages in the other parts of the circuit, and therefore, it may be ignored altogether.

(4) For a quasi-supernode, the node-voltage of the non-reference node is equal to the voltage magnitude of the source.

=> This means, voltage between nodes 4 and 5 is specified by the 16 V sources, regardless of the value of R6 (as long as R6 is not a short circuit).

Example 3-3: Supernode



Determine: V1 and V2

Solution:

$$I_1 + I_2 + I_3 + I_4 = 0$$ Supernode

$$\frac{V_1 - 4}{2} + \frac{V_1}{4} + \frac{V_2}{8} - 2 = 0,$$ equivalently

which may be simplified to

$$6V_1 + V_2 = 32.$$

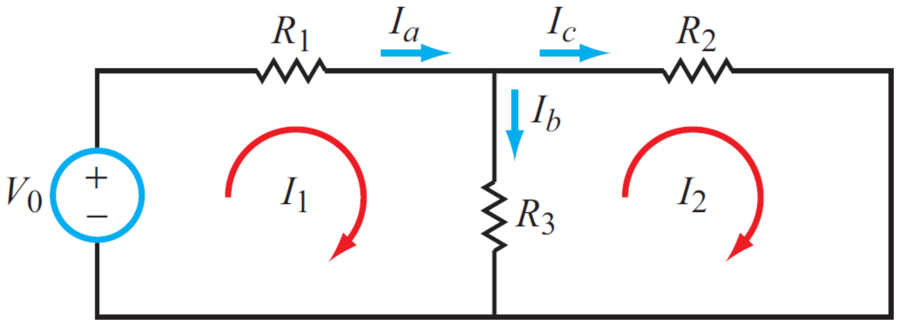
Additionally, the supernode KVL equation is

$$V_2 - V_1 = 18.$$

Simultaneous solution of the two equations yields

$$V_1 = 2 \text{ V}, \qquad V_2 = 20 \text{ V}.$$

Mesh-Current Method



Solution Procedure: Mesh Current

Step 1: Identify all meshes and assign each of them an unknown mesh current. For convenience, define the mesh currents to be clockwise in direction.

Step 2: Apply kirchhoff's voltage law (KVL) to each mesh.

Step 3: Solve the resultant simultaneous equations to determine the mesh currents.

$$-V_0 + I_1 R_1 + (I_1 - I_2) R_3 = 0 \qquad \text{(mesh 1)}$$

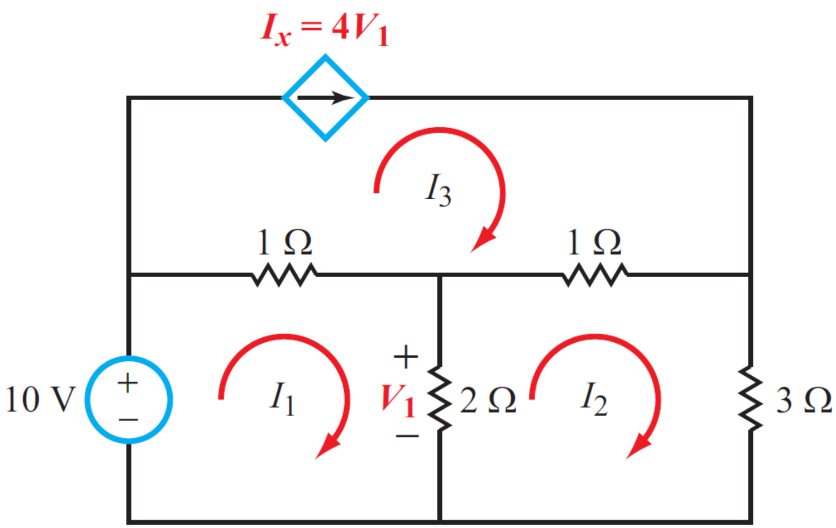
$$(I_2 - I_1) R_3 + I_2 R_2 = 0 \qquad \text{(mesh 2)}$$

Two equations in 2 unknowns:
Solve using Cramer's rule, matrix inversion, or MATLAB

$$(R_1 + R_3) I_1 - I_2 R_3 = V_0 \qquad \text{(mesh 1)}$$

$$-R_3 I_1 + (R_2 + R_3) I_2 = 0 \qquad \text{(mesh 2)}$$

Example 3-5: Mesh Analysis



Solution Procedure: Mesh Current

Step 1: Identify all meshes and assign each of them an unknown mesh current. For convenience, define the mesh currents to be clockwise in direction.

Step 2: Apply kirchhoff's voltage law (KVL) to each mesh.

Step 3: Solve the resultant simultaneous equations to determine the mesh currents.

$$(1+2)I_1 - 2I_2 - I_3 = 10,$$ Mesh 1

$$-2I_1 + (2+1+3)I_2 - I_3 = 0.$$ Mesh 2

$$I_3 = I_x = 4V_1.$$ Mesh 3

But $V_1 = 2(I_1 - I_2).$

Hence

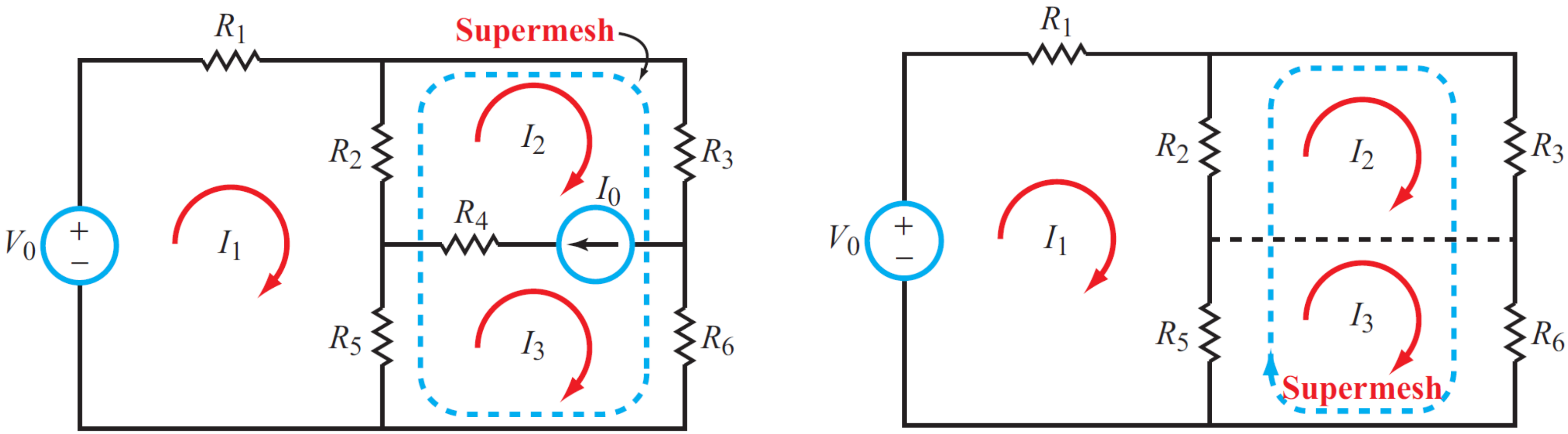
$$-5I_1 + 6I_2 = 10,$$
$$-10I_1 + 14I_2 = 0.$$

$$I_1 = -14 \text{ A}, \qquad I_2 = -10 \text{ A}.$$

$$\begin{aligned} I_x &= 8(I_1 - I_2) \\ &= 8(-14+10) \\ &= -32 \text{ A}. \end{aligned}$$

Supermesh

A supermesh results when two meshes have a current source(with or w/o a series resistor) in common



- I0 removed and replaced by dash line;

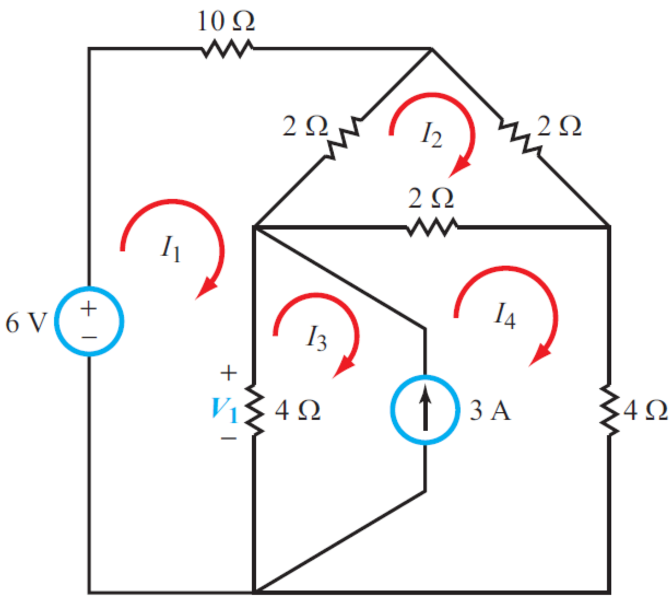
remember that $I_0 = I_2 - I_3$ **(auxiliary eq.)**.

$$(R_1 + R_2 + R_5)I_1 - R_2 I_2 - R_5 I_3 = V_0 \qquad \text{Mesh 1}$$

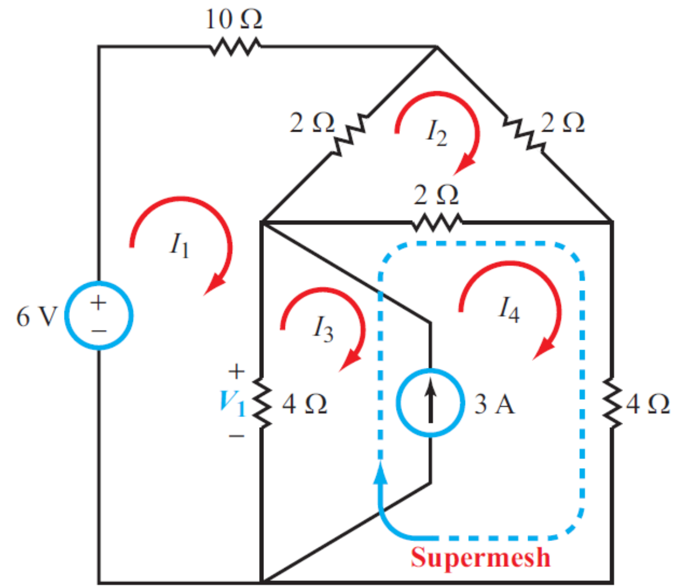
$$-(R_2 + R_5)I_1 + (R_2 + R_3)I_2 + (R_5 + R_6)I_3 = 0 \qquad \text{supermesh}$$

These 3 equations are enough to solve for three mesh currents.

Example 3-6: Supermesh



(a) Original circuit



(b) Meshes 3 and 4 constitute a supermesh

$$(10 + 2 + 4)I_1 - 2I_2 - 4I_3 = 6,$$ Mesh 1

$$-2I_1 + (2 + 2 + 2)I_2 - 2I_4 = 0,$$ Mesh 2

$$-4I_1 - 2I_2 + 4I_3 + (2 + 4)I_4 = 0.$$ SuperMesh 3/4

$$I_4 - I_3 = 3.$$ Supermesh Auxiliary Equation

Solution gives:

$$I_1 = 0,$$

$$I_2 = \frac{3}{7}\ \text{A},$$

$$I_3 = -\frac{12}{7}\ \text{A},$$

$$I_4 = \frac{9}{7}\ \text{A}.$$

Nodal versus Mesh

When do you use one vs. the other?

- Nodal Analysis
  - Node Voltages (voltage difference between each node and ground reference) are *UNKNOWNS*
  - KCL Equations at Each *UNKNOWN* Node Constrain Solutions (*N* KCL equations for *N* Node Voltages)
- Mesh Analysis
  - "Mesh Currents" Flowing in Each Mesh Loop are *UNKNOWNS*
  - KVL Equations for Each Mesh Loop Constrain Solutions (*M* KVL equations for *M* Mesh Loops)

Count nodes, meshes, look for supernode/supermesh

Linearity

A circuit is linear if output is proportional to input

- A function $f(x)$ is linear if $f(ax) = af(x)$
- All circuit elements will be assumed to be linear or can be modeled by linear equivalent circuits
  - Resistors $V = IR$
  - Linearly Dependent Sources
  - Capacitors
  - Inductors

We will examine theorems and principles that apply to linear circuits to simplify analysis

Superposition

> If a circuit contains more than one independent source, the voltage (or current) response of any element in the circuit is equal to the algebraic sum of the individual responses associated with the individual independent sources, as if each had been acting alone.

Superposition breaks apart the examination of one complex circuit in place of several simpler circuits

Superposition: solution procedure

Step 1: Set all independent sources equal to zero (by replacing voltage sources with short circuits and current sources with open circuits), except for source 1.

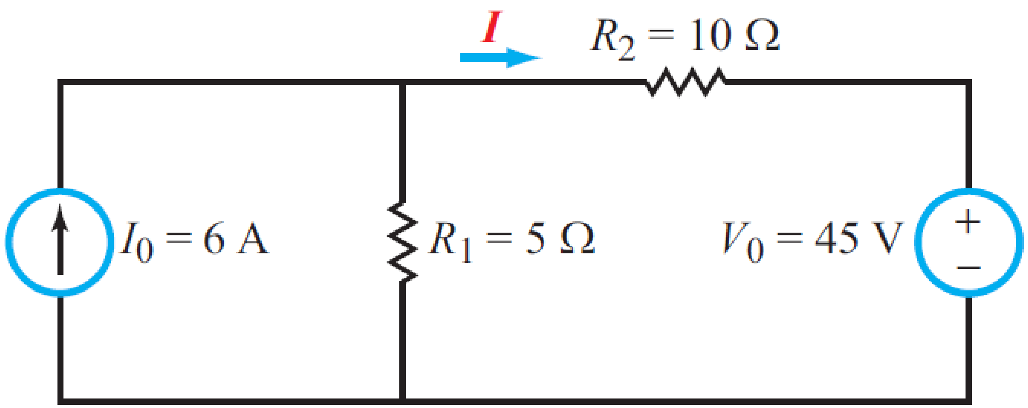
Step 2: Apply node-voltage, mesh-current, or any other convenient analysis techniques to solve for the response v_1 due to source 1.

Step 3: Repeat the process for sources 2 through n, calculating in each case the response due to that one source acting alone.

Step 4: Add up all the voltages, v_1 to v_n.

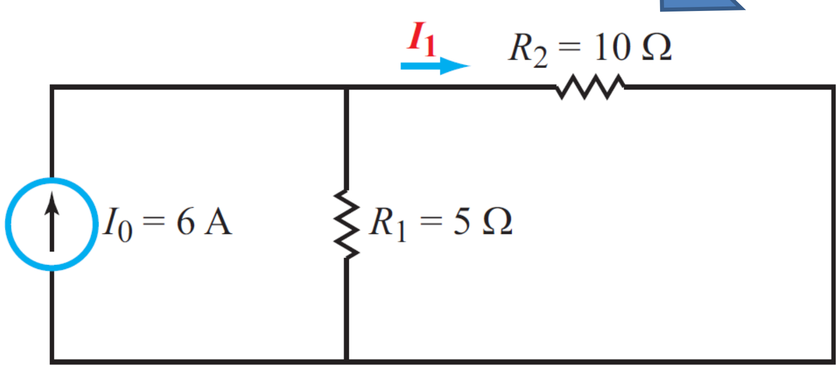
Alternatively, the procedure can be used to find currents i_1 to i_n and then add them up to find the total current.

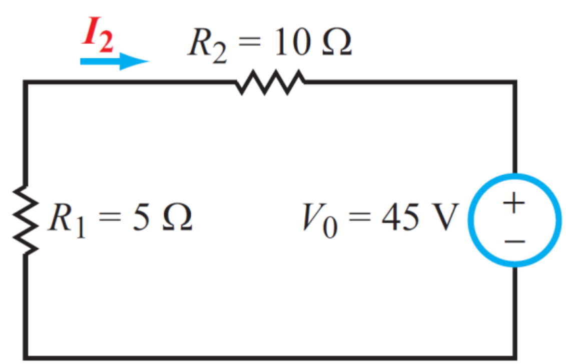
Example 3-9: Superposition



Contribution from I_0 alone

Contribution from V_0 alone



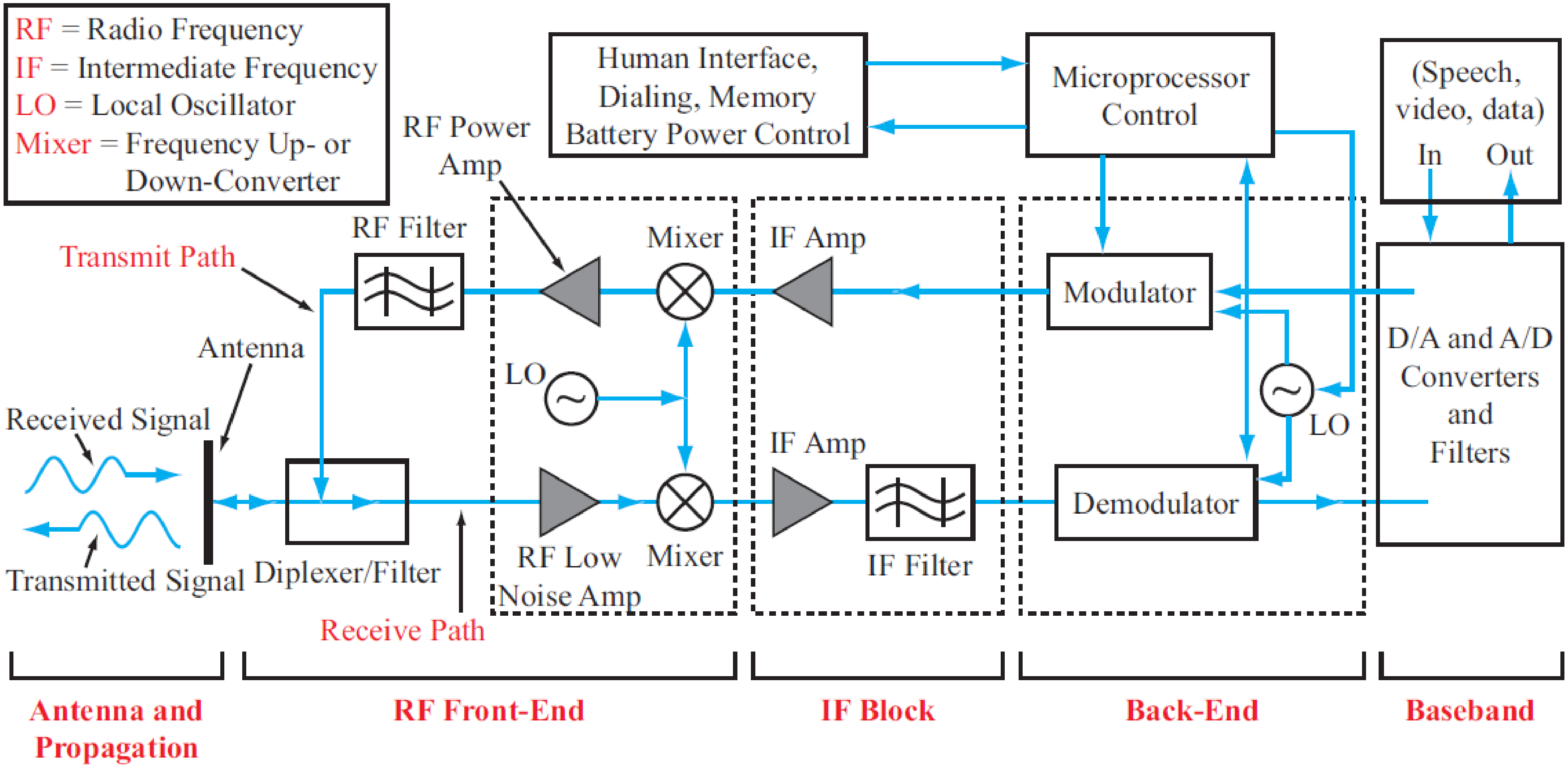


$I_1 = 2$ A

$$\boxed{I = I_1 + I_2 = 2 - 3 = -1 \text{ A}}$$

$I_2 = -3$ A

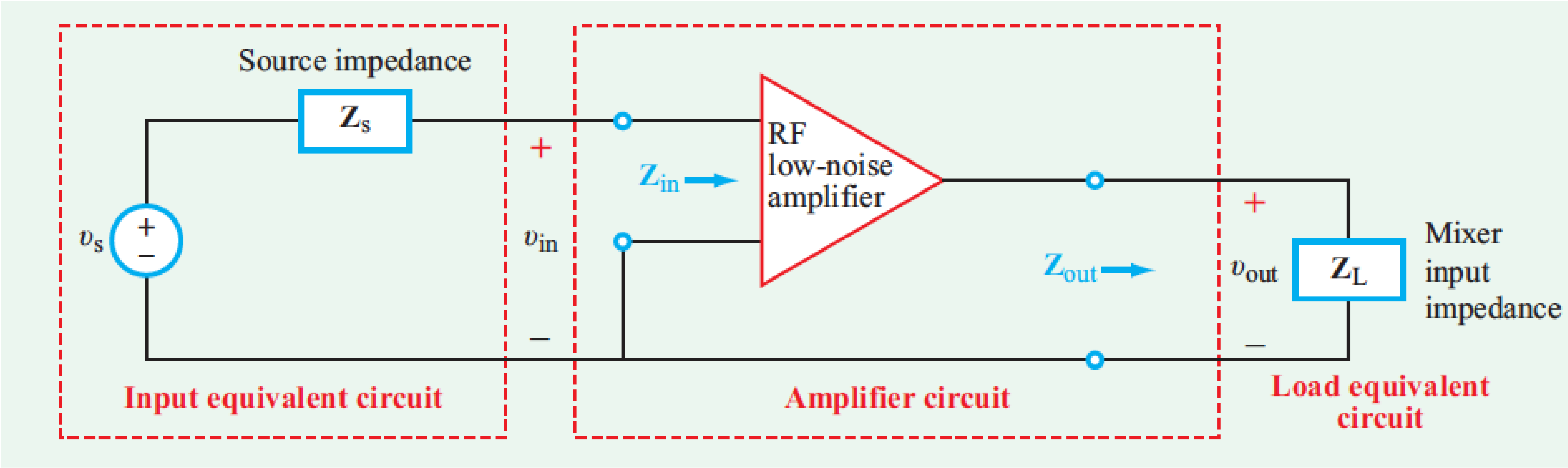
Cell Phone



Today's systems are complex. We use a block diagram approach to represent circuit sections.

Equivalent Circuit Representation

- Fortunately, many circuits are linear
- Simple equivalent circuits may be used to represent complex circuits



Thévenin's Theorem

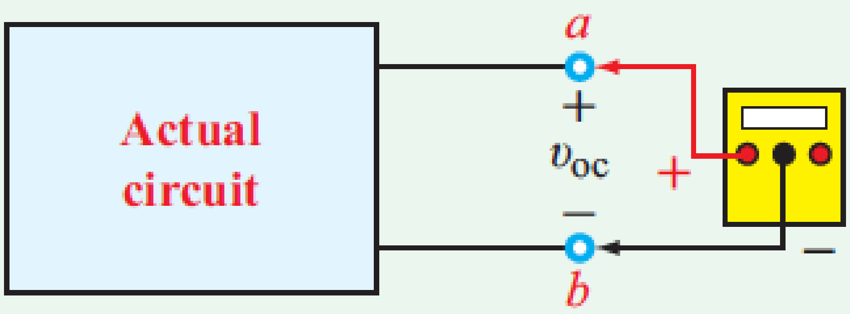
Linear two-terminal circuit can be replaced by an equivalent circuit composed of a voltage source and a series resistor

$$v_{\mathrm{Th}} = v_{\mathrm{oc}}$$

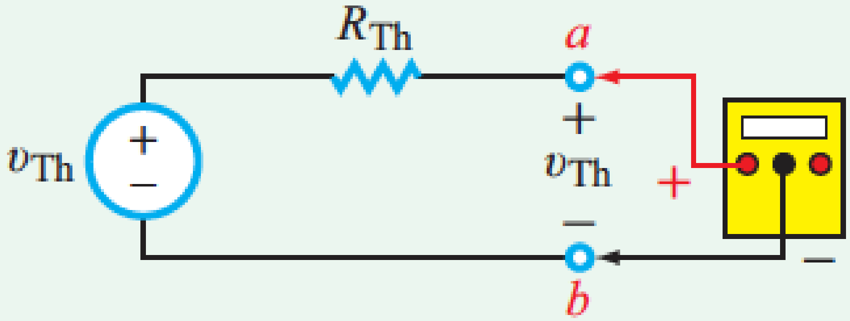
voltage across output with no load (open circuit)

$$R_{\mathrm{Th}} = R_{\mathrm{in}}$$

Resistance at terminals with all independent circuit sources set to zero



(a) Measuring v_{oc} on actual circuit



(b) Measuring v_{Th} of equivalent circuit

Norton's Theorem

Linear two-terminal circuit can be replaced by an equivalent circuit composed of a current source and parallel resistor

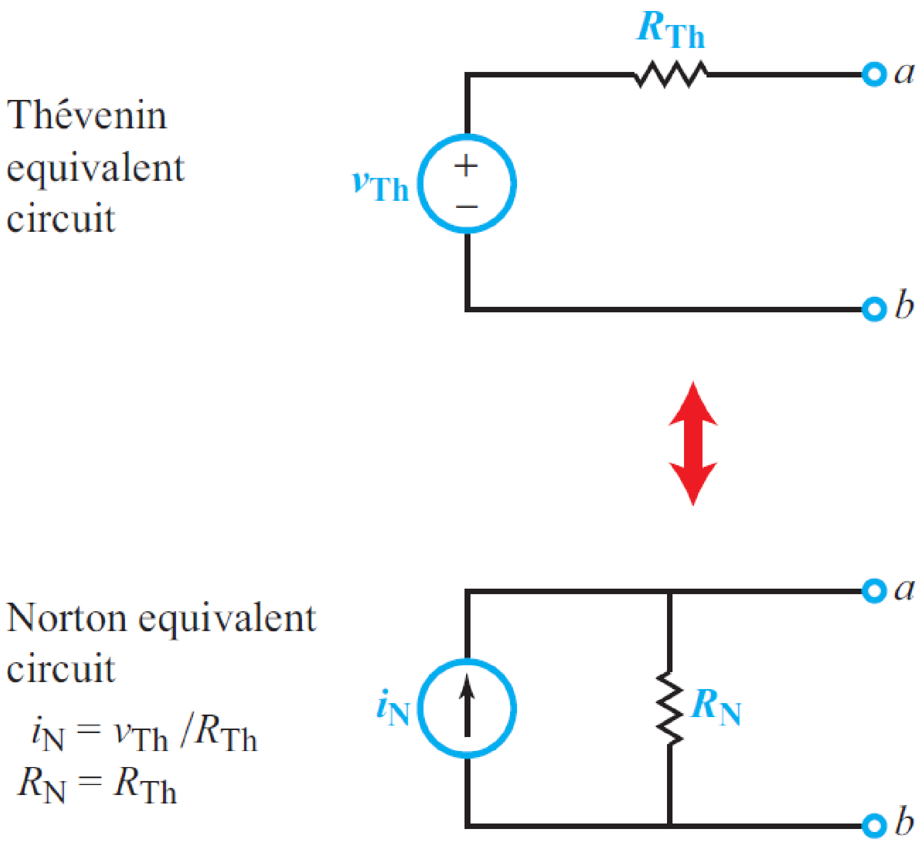
$$i_N = \frac{v_{Th}}{R_{Th}}$$

Current through output with short circuit

$$R_N = R_{Th}.$$

Resistance at terminals with all circuit sources set to zero

Thévenin and Norton Equivalency



How Do We Find Thévenin/Norton Equivalent Circuits ?

- Method 1: Open circuit/Short circuit

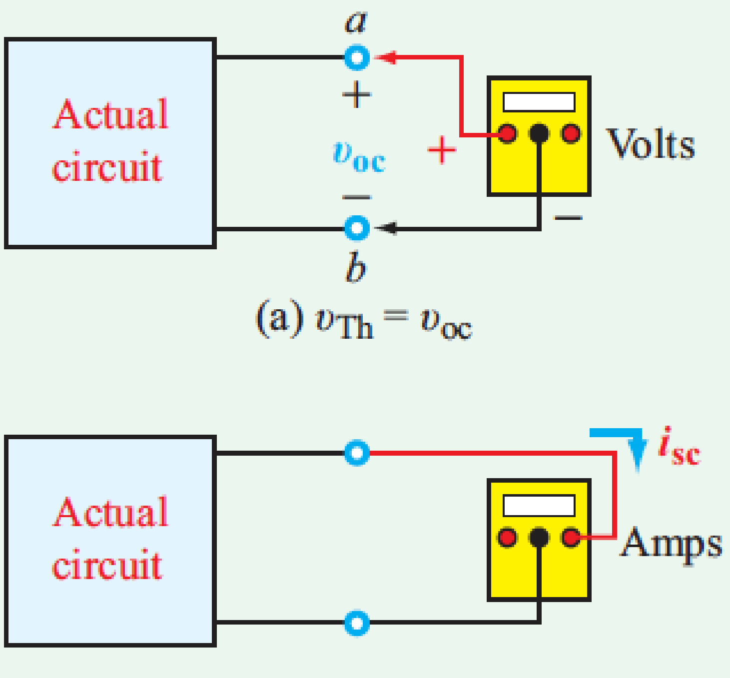
1. Analyze circuit to find v_{oc}
2. Analyze circuit to find i_{sc}

$$v_{Th} = v_{oc}$$

$$R_{Th} = \frac{v_{Th}}{i_{sc}}$$

Note: This method is applicable to any circuit, whether or not it contains dependent sources.

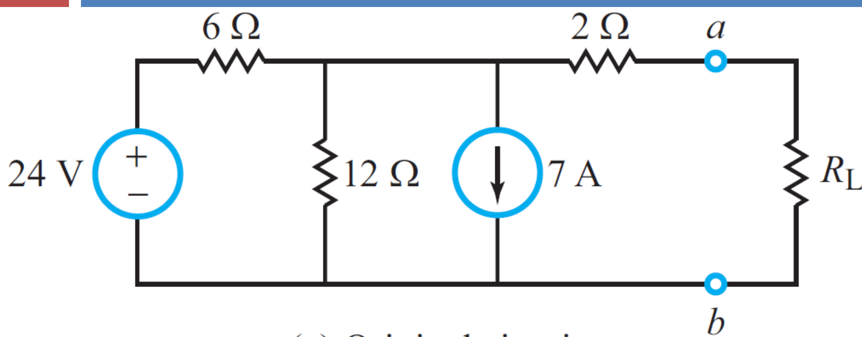
Open-Circuit / Short-Circuit Method



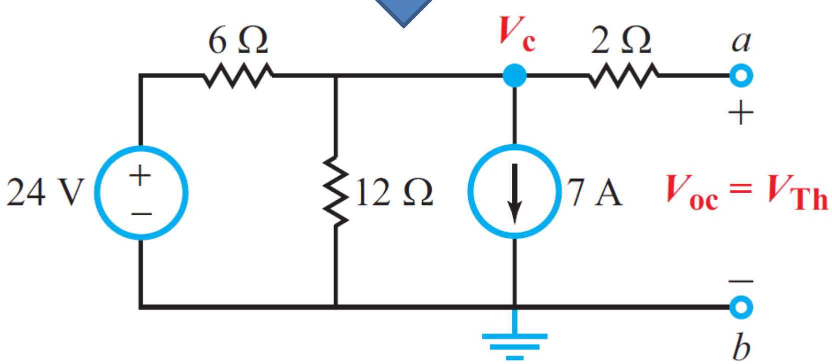
(a) $v_{Th} = v_{oc}$

(b) $R_{Th} = v_{oc}/i_{sc}$

Example 3-11: Thévenin Equivalent



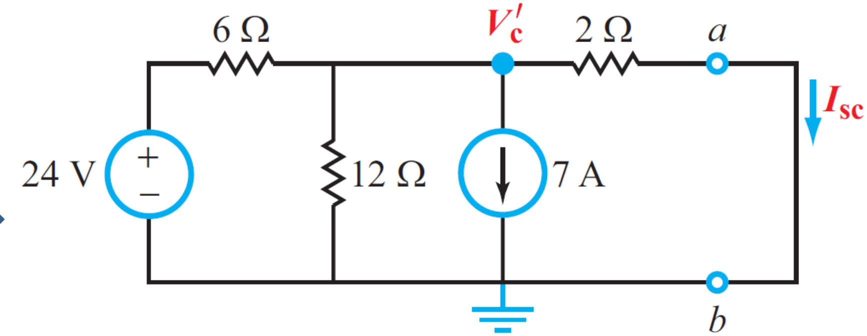
(a) Original circuit



(b) Replacing R_L with open circuit

$$\frac{V_c - 24}{6} + \frac{V_c}{12} + 7 = 0$$

$$V_{Th} = -12 \text{ V}$$

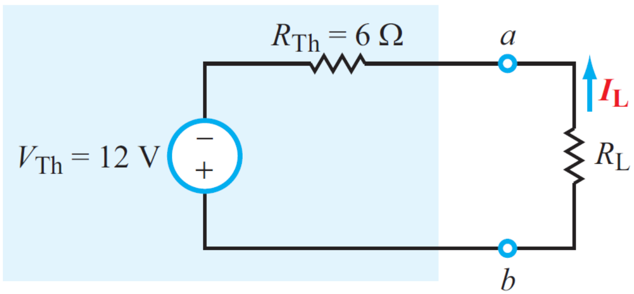


(c) Replacing R_L with short circuit

$$\frac{V_c' - 24}{6} + \frac{V_c'}{12} + 7 + \frac{V_c'}{2} = 0 \rightarrow V_c' = -4 \text{ V},$$

$$I_{sc} = \frac{V_c'}{2} = -\frac{4}{2} = -2 \text{ A}$$

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{-12}{-2} = 6 \ \Omega$$



(d) Thévenin equivalent circuit

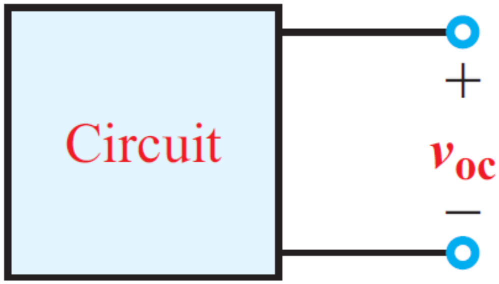
How Do We Find Thévenin/Norton Equivalent Circuits?

Method 2: Equivalent Resistance

1. Analyze circuit to find either

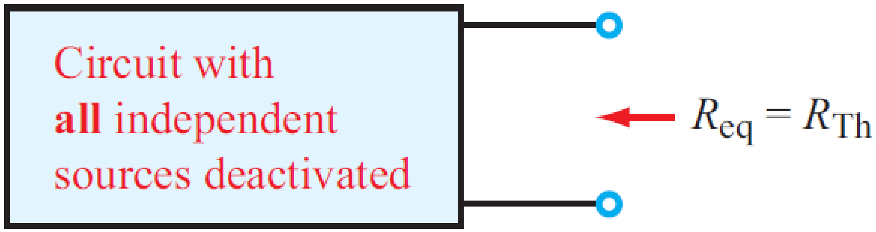
v_{oc} or i_{sc}

2. Deactivate all independent sources by replacing voltage sources with short circuits and current sources with open circuits.
3. Simplify circuit to find equivalent resistance



(a) $v_{Th} = v_{oc}$

Equivalent-Resistance Method



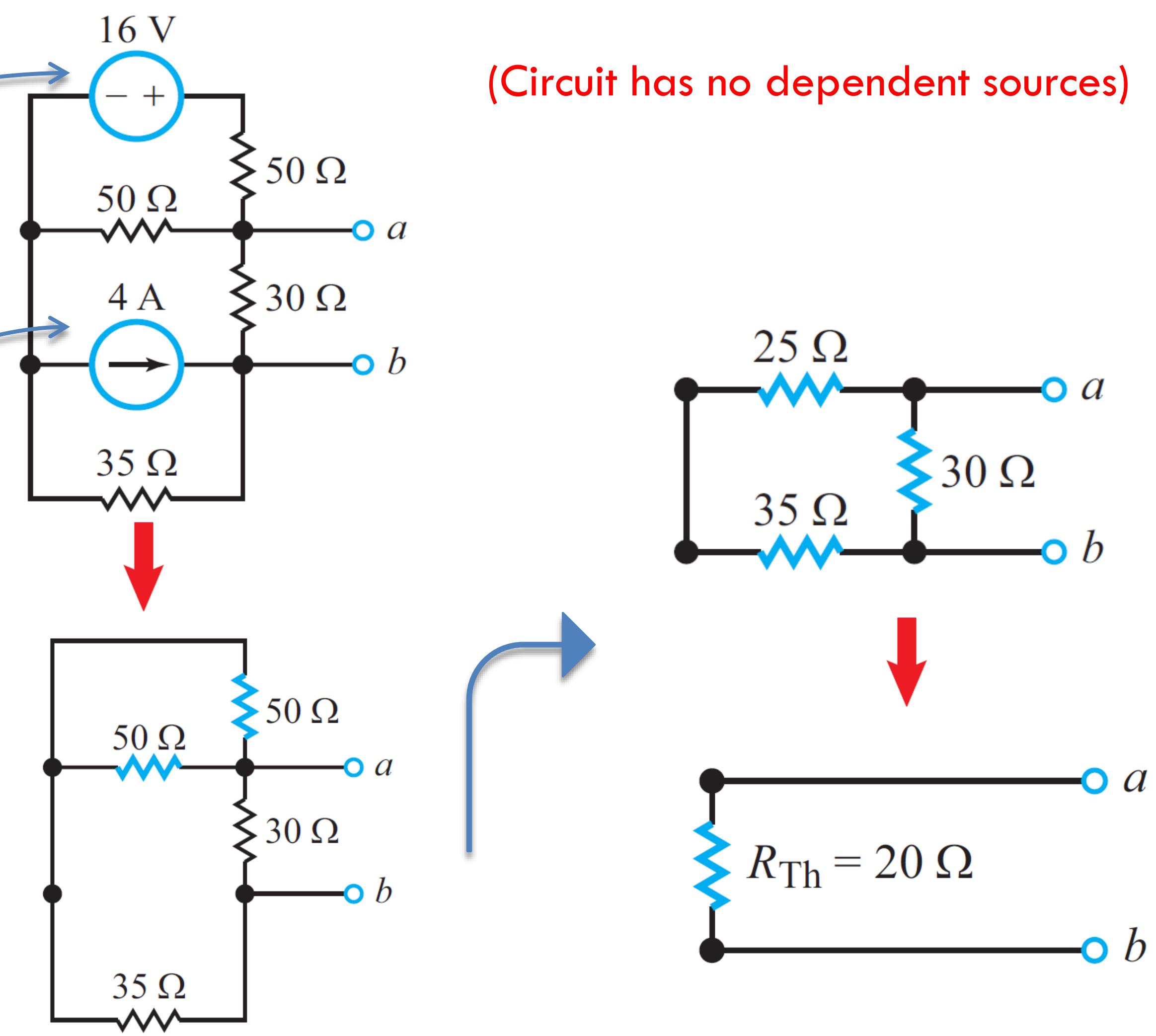
Note: This method does not apply to circuits that contain dependent sources.

Example 3-12: R_{Th}

Replace with SC

Replace with OC

(Circuit has no dependent sources)



How Do We Find Thévenin/Norton Equivalent Circuits?

Method 3:

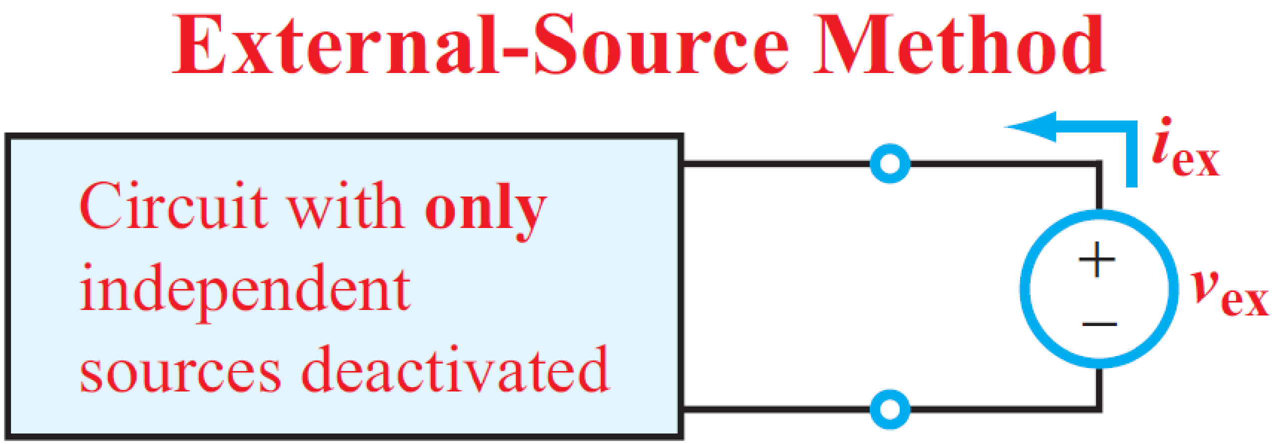


Figure 3-22: If a circuit contains both dependent and independent sources, R_{Th} can be determined by (a) deactivating independent sources (only), (b) adding an external source v_{ex}, and then (c) solving the circuit to determine i_{ex}. The solution is $R_{Th} = v_{ex}/i_{ex}$.

Example 3-13

Find the Thévenin equivalent circuit at terminals (a, b) for the circuit

Solution:

The equations for mesh currents I_1 and I_2 in Fig. 3-23(a) are given by

$$-68 + 6I_1 + 2(I_1 - I_2) + 4I_x = 0$$

and

$$-4I_x + 2(I_2 - I_1) + 6I_2 + 4I_2 = 0.$$

Recognizing that $I_x = I_2$, solution of these two simultaneous equations leads to

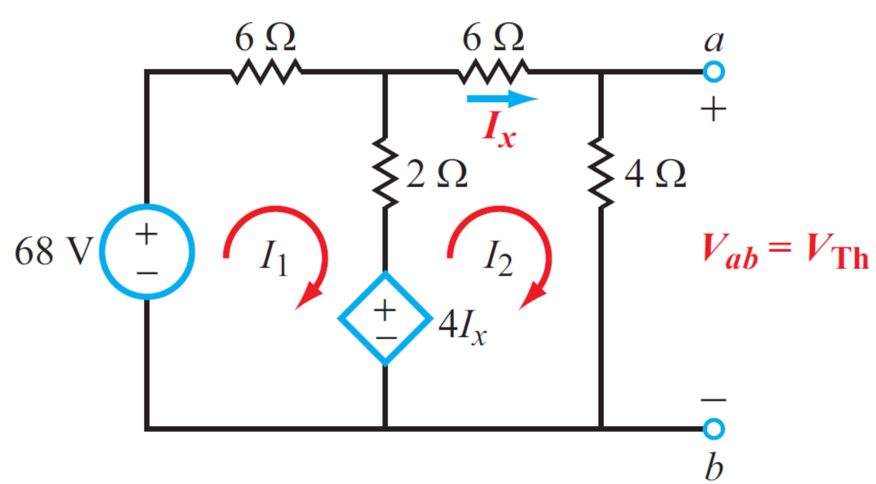
$$I_1 = 8 \text{ A},$$

and

$$I_2 = 2 \text{ A}.$$

The Thévenin voltage is V_{ab}. Hence,

$$V_{Th} = V_{ab} = 4I_2 = 8 \text{ V}.$$



(a) Solving for V_{Th}

Example 3-13 (cont.)

To find R_{Th}

$$6I_1' + 2(I_1' - I_2') + 4I_x = 0,$$
$$-4I_x + 2(I_2' - I_1') + 6I_2' + 4(I_2' - I_3') = 0,$$

$$4(I_3' - I_2') + V_{ex} = 0.$$

After replacing I_x with I_2' and solving the three simultaneous equations, we obtain

$$I_1' = \frac{1}{18} V_{ex},$$
$$I_2' = -\frac{2}{9} V_{ex},$$

and

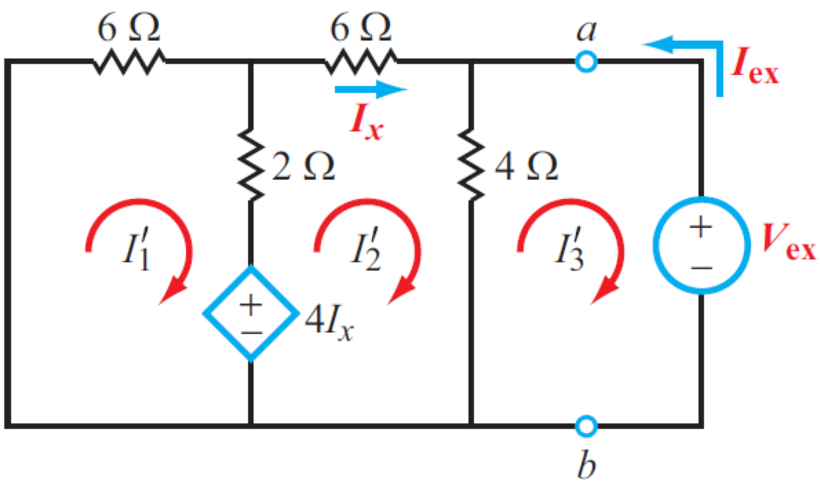
$$I_3' = -\frac{17}{36} V_{ex}.$$

For the equivalent circuit shown in Fig. 3-23(c),

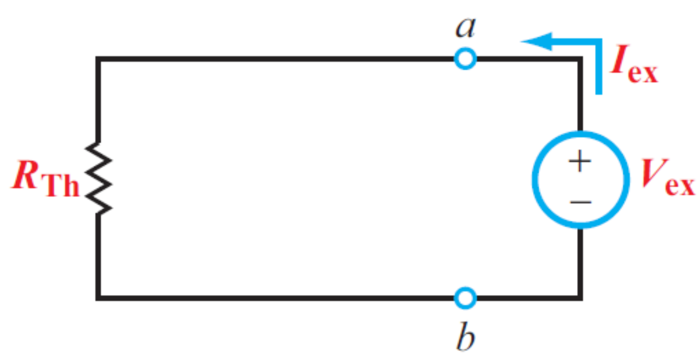
$$R_{Th} = \frac{V_{ex}}{I_{ex}}.$$

In terms of our solution, $I_{ex} = -I_3'$. Hence,

$$R_{Th} = -\frac{V_{ex}}{I_3'} = \frac{36}{17}\ \Omega.$$



(b) Solving for I_{ex}



(c) Equivalent circuit for calculating R_{Th}

Cascaded system:
Repeated application of Thevenin equivalence

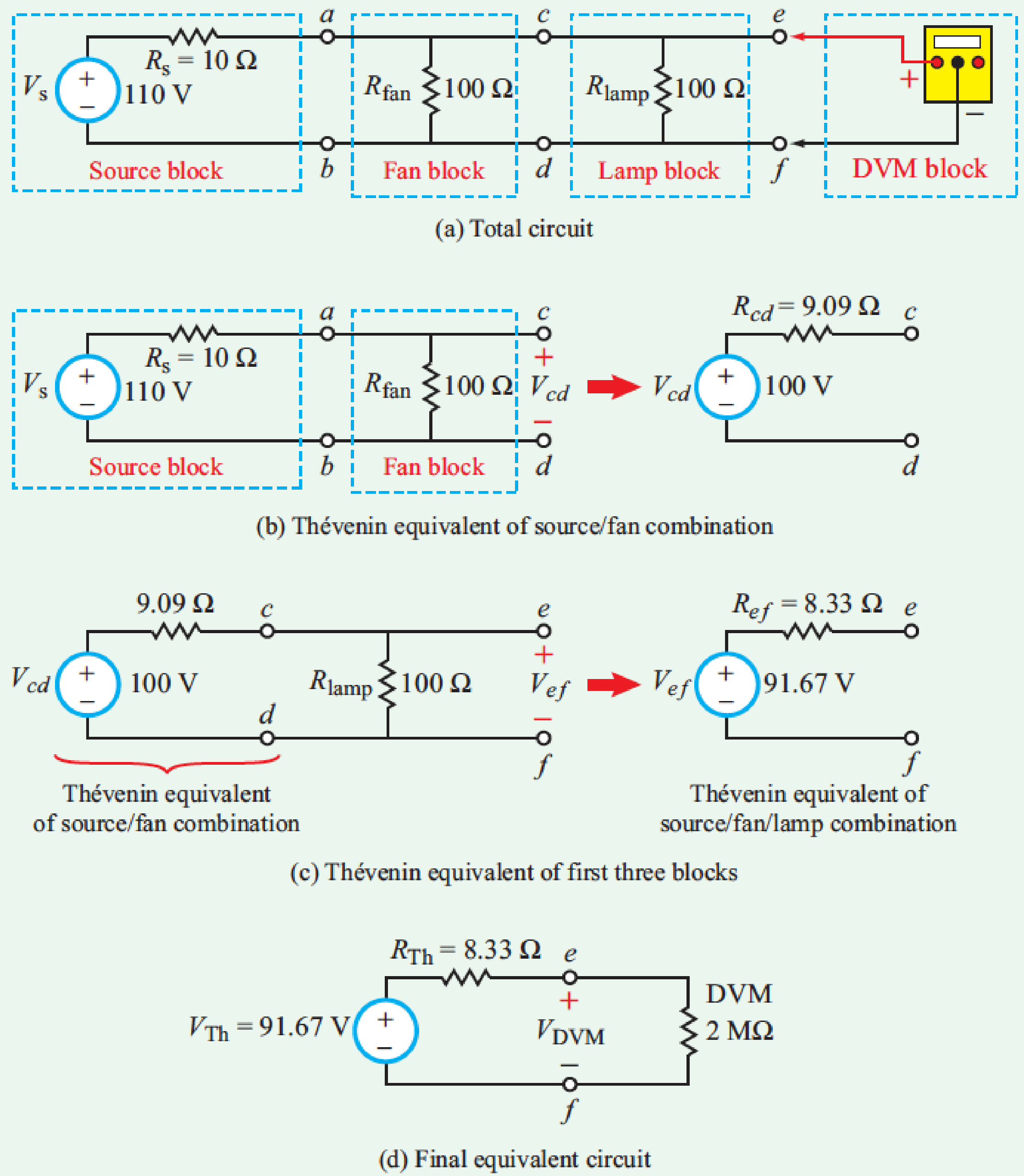
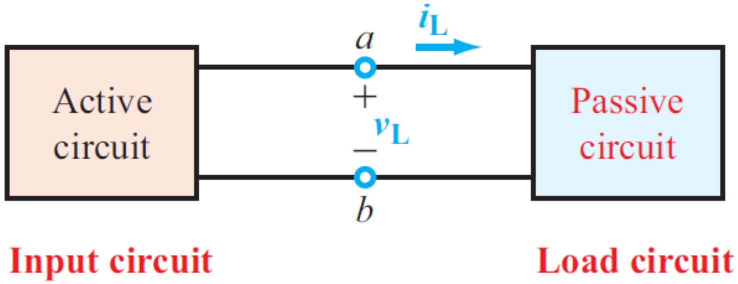


Figure 3-29: Repeated application of Thévenin-equivalent circuit technique.

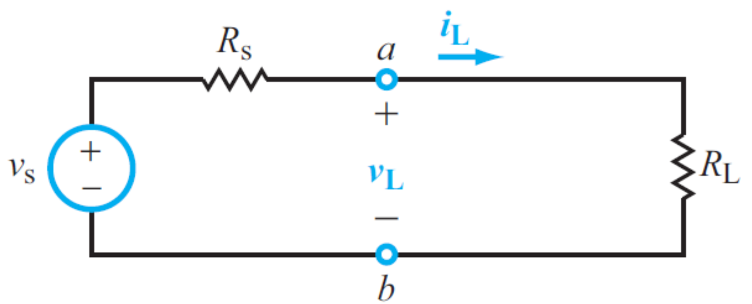
Power Transfer

In many situations, we want to maximize power transfer to the load

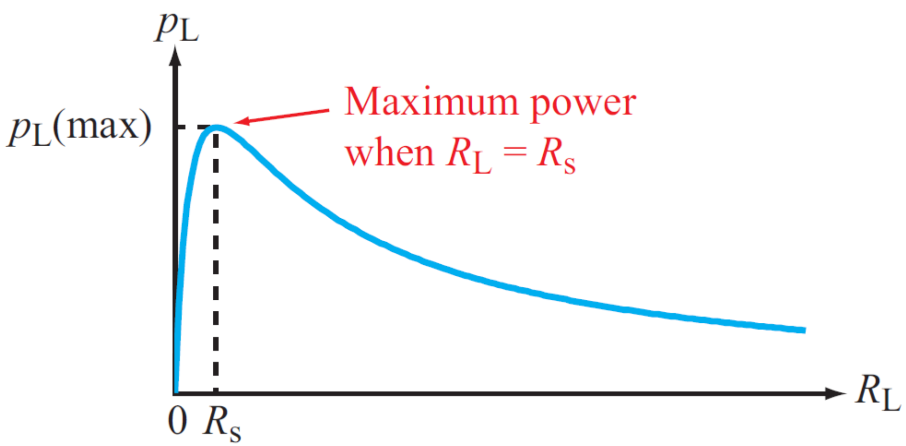
$$p_L = i_L v_L = \frac{v_s^2 R_L}{(R_s + R_L)^2}.$$



(a) Source and load circuits

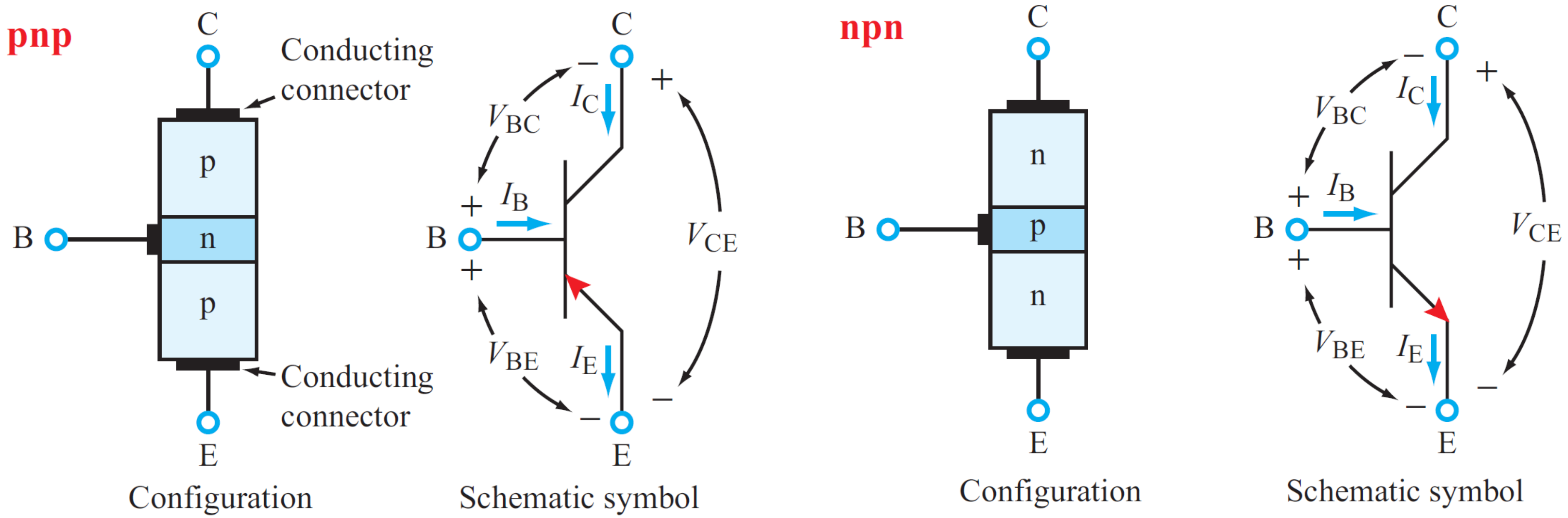


(b) Replacing source and load circuits with their Thévenin equivalents



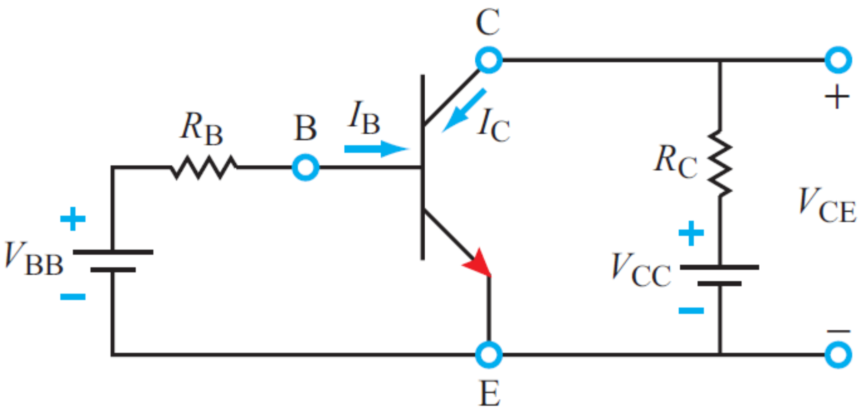
$$p_L(\max) = \frac{v_s^2 R_L}{(R_L + R_L)^2} = \frac{v_s^2}{4R_L},$$

BJT: Our First 3 Terminal Device!

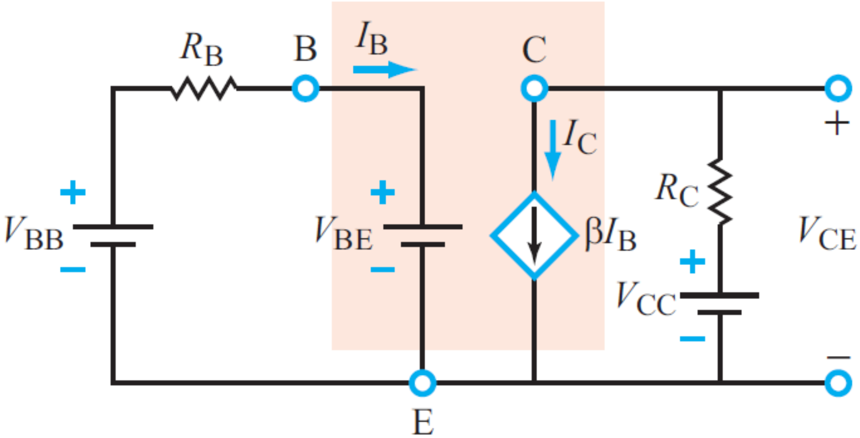


- Active device with dc sources
- Allows for input/output, gain/amplification, etc

BJT Equivalent Circuit



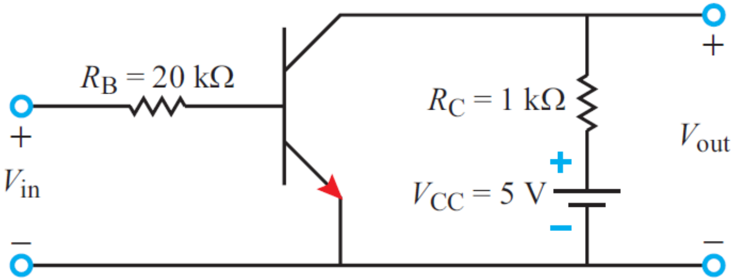
(a) Transistor circuit



(b) Equivalent circuit

Looks like a current amplifier with gain β

Digital Inverter With BJTs



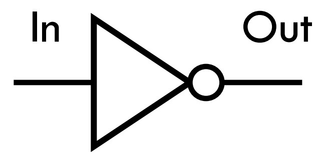
(a) Inverter circuit

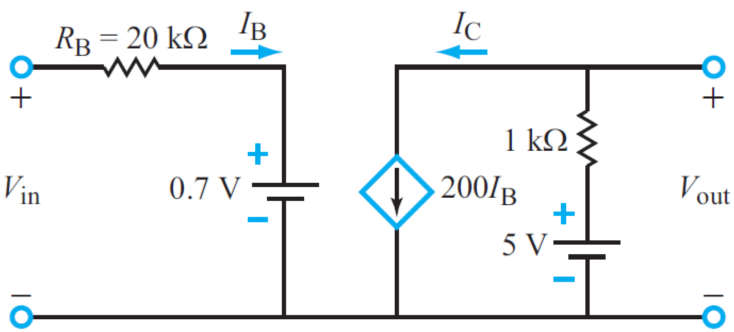
BJT Rules:

Vout cannot exceed Vcc=5V

Vin cannot be negative

In	Out
0	1
1	0





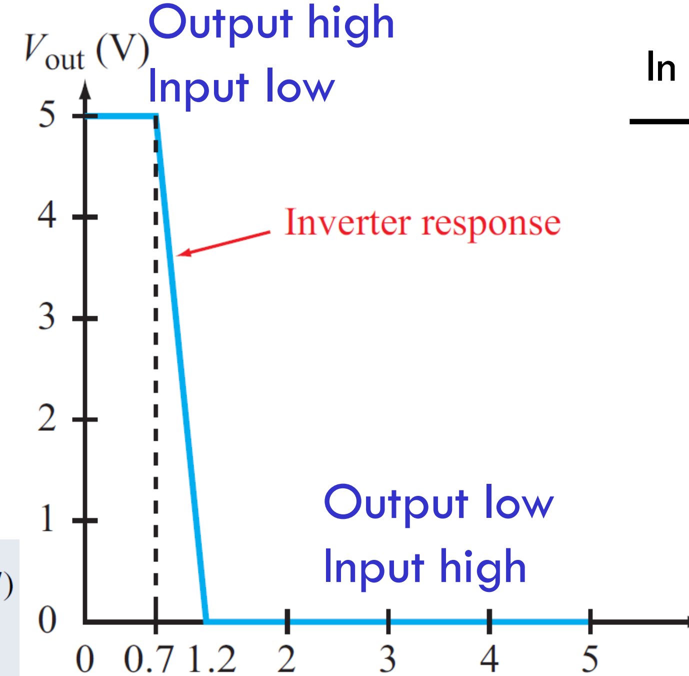
(b) Equivalent circuit

$$I_B = \frac{V_{in} - 0.7}{20k},$$

$$I_C = \beta I_B = 200 I_B,$$

$$V_{out} = V_{CC} - I_C R_C.$$

$$V_{out} = V_{CC} - \frac{\beta R_C}{R_B}(V_{in} - 0.7)$$
$$= 12 - 10 V_{in} \qquad (V).$$



(c) V_{out} versus V_{in}

Summary

Chapter 3 Relationships

Node-voltage method	
$\sum$ of all current leaving a node $= 0$ [current entering a node is $(-)$]	
Mesh-current method	
$\sum$ of all voltages around a loop $= 0$ [passive sign convention applied to mesh currents in clockwise direction]	
Nodal analysis by inspection	$\mathbf{GV} = \mathbf{I}_t$
Mesh analysis by inspection	$\mathbf{RI} = \mathbf{V}_t$
Thévenin equivalent circuit	$\upsilon_{Th} = \upsilon_{oc}$ $R_{Th} = \upsilon_{oc}/i_{sc}$
Norton equivalent circuit	$i_N = i_{sc}$ $R_N = R_{Th}$
Maximum power transfer	$R_L = R_s$ $P_L(\max) = \dfrac{\upsilon_s^2}{4R_L}$