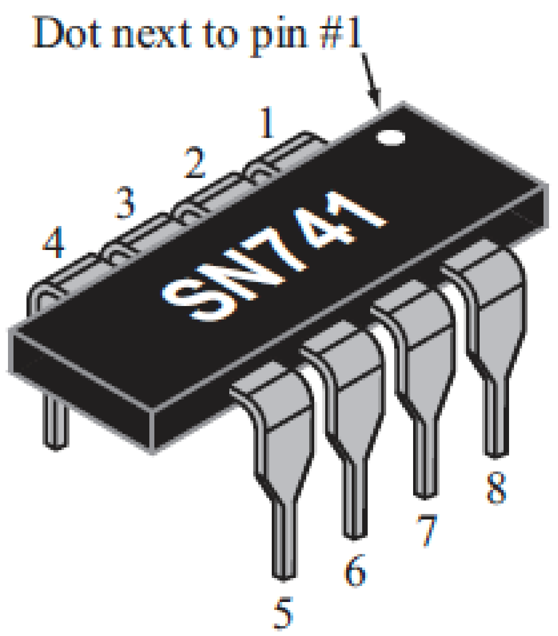
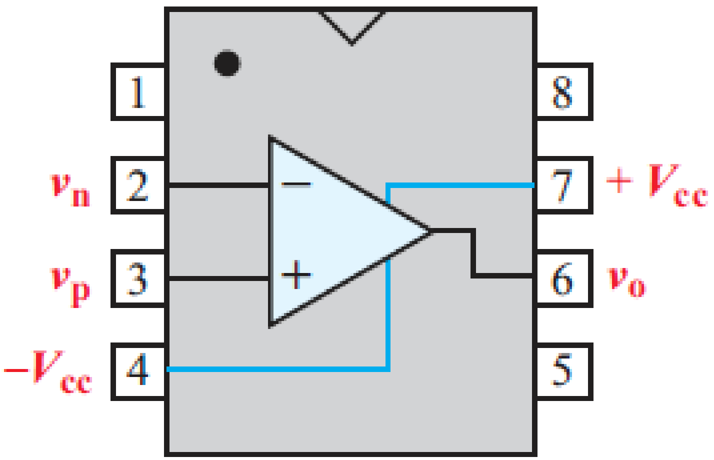




LEC 4. OPERATIONAL AMPLIFIERS

ECEN203

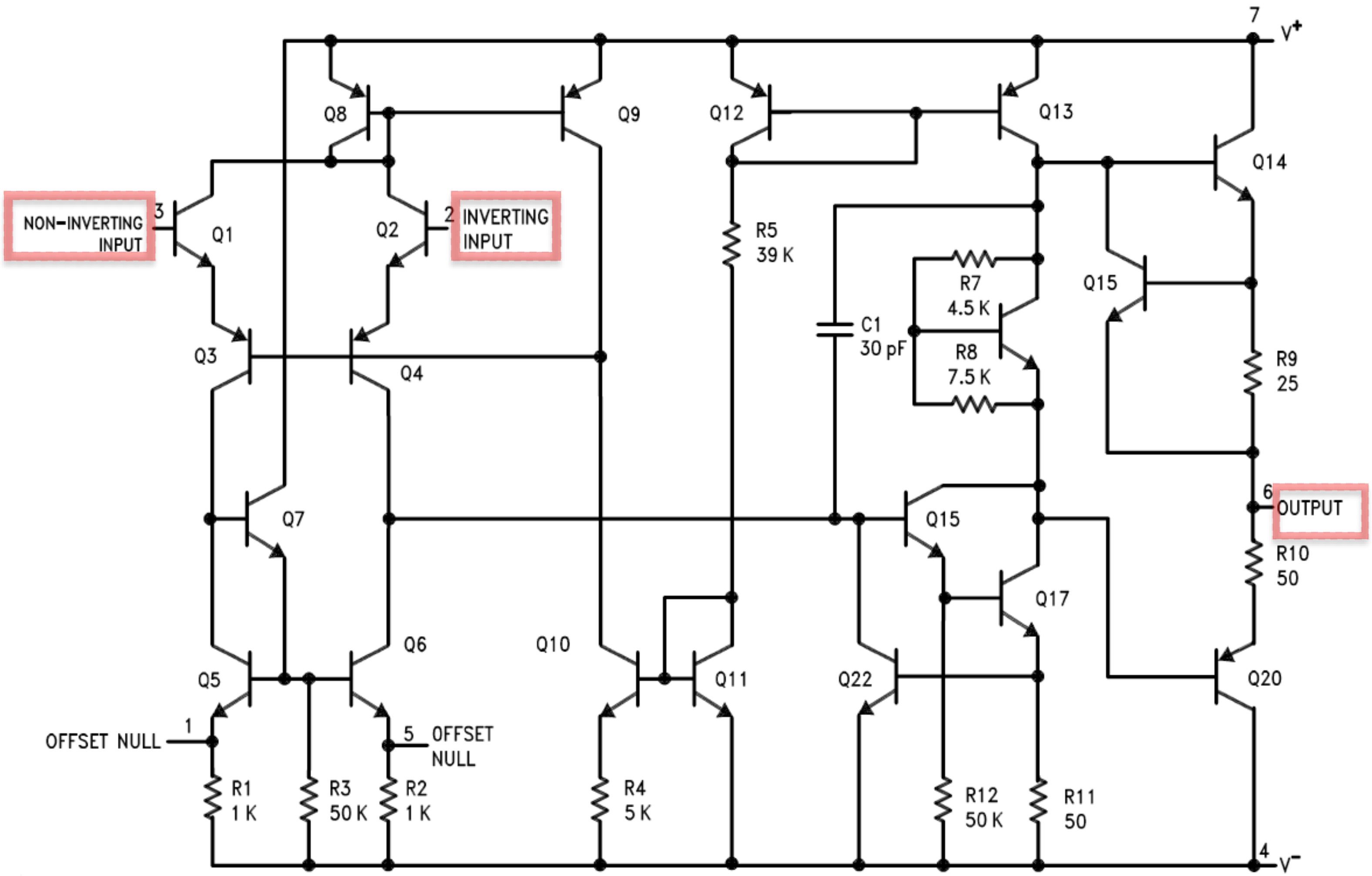
Analogue Electronics and Systems

Operational Amplifiers (Op-Amps)

- Describe the basic properties of an op-amp and state the constraints of the ideal op-amp model.
- Explain the role of negative feedback and the trade-off between circuit gain and dynamic range.
- Analyse and design inverting amplifiers, summing amplifiers, difference amplifiers, and voltage followers.

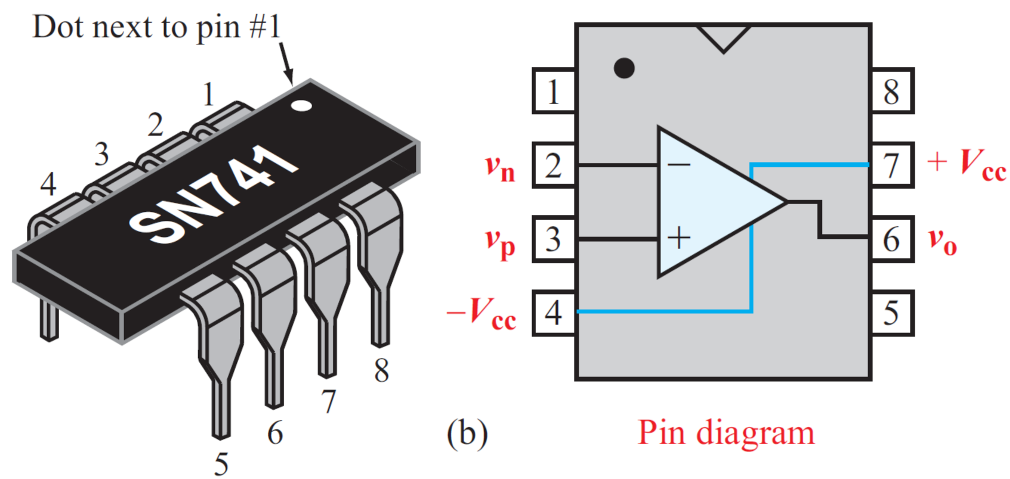
Inside The Op-Amp (741)

Schematic Diagram



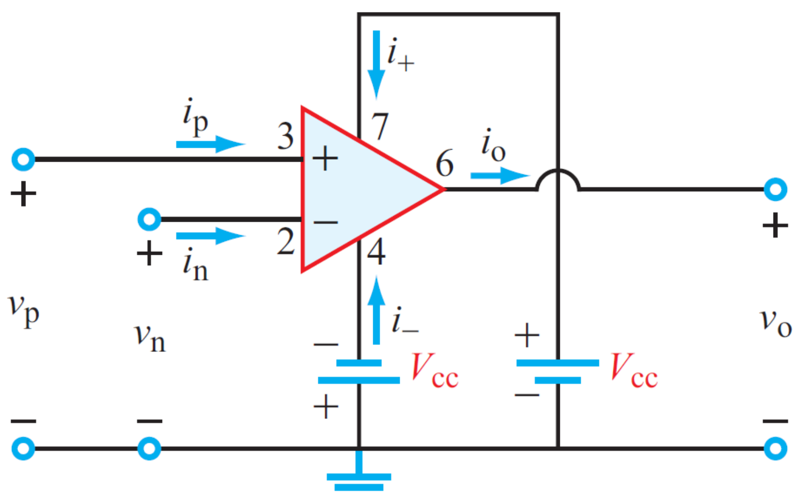
Operational Amplifier “Op Amp”

- Two input terminals, positive (non- inverting) and negative (inverting)
- One output
- Power supply $+V_{cc}$ and $-V_{cc}$
- *Active* device!

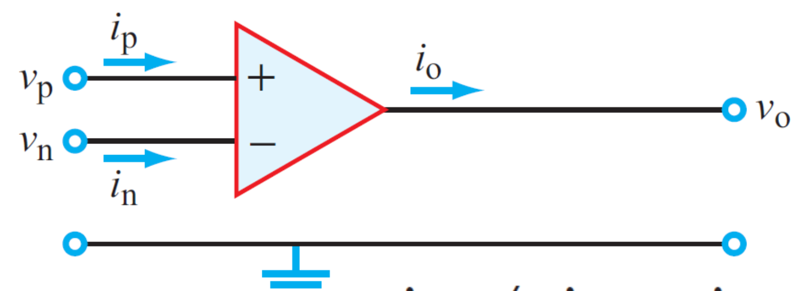


(b) Pin diagram

Op Amp showing power supply



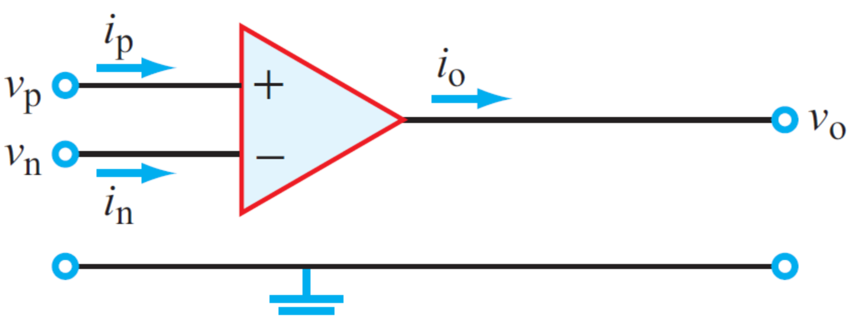
Op Amp with power supply not shown (which is how we usually display op amp circuits)

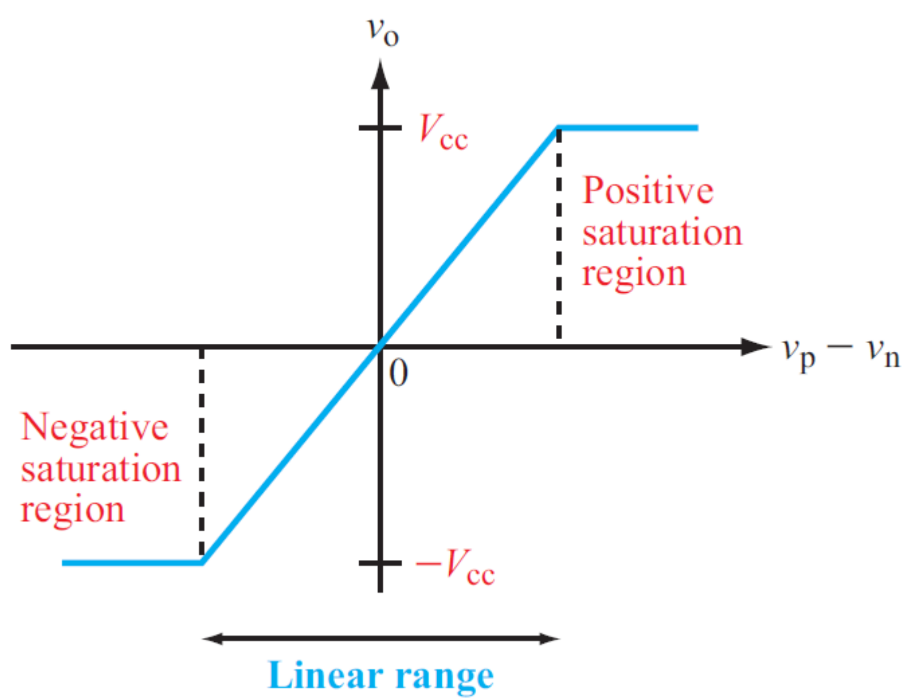


$$i_o \neq i_p + i_n$$

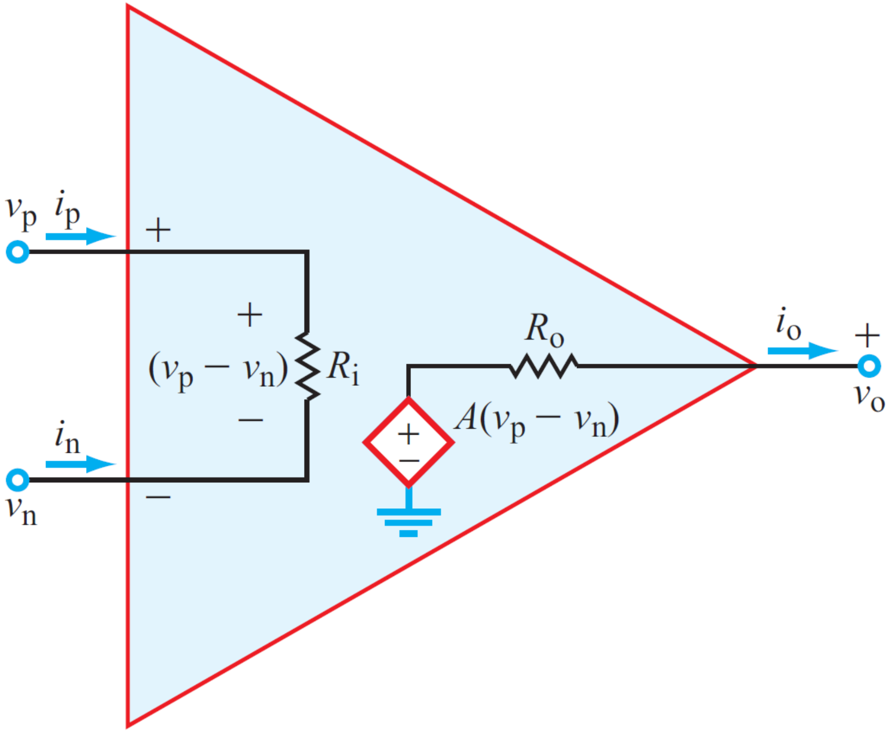
Gain

- Key important aspect of op amp: high voltage gain
- Output $v_o = A(v_p - v_n)$ op-amp gain (or open-loop gain) => different from circuit gain G *(closed loop gain)*
- Linear response





Equivalent Circuit

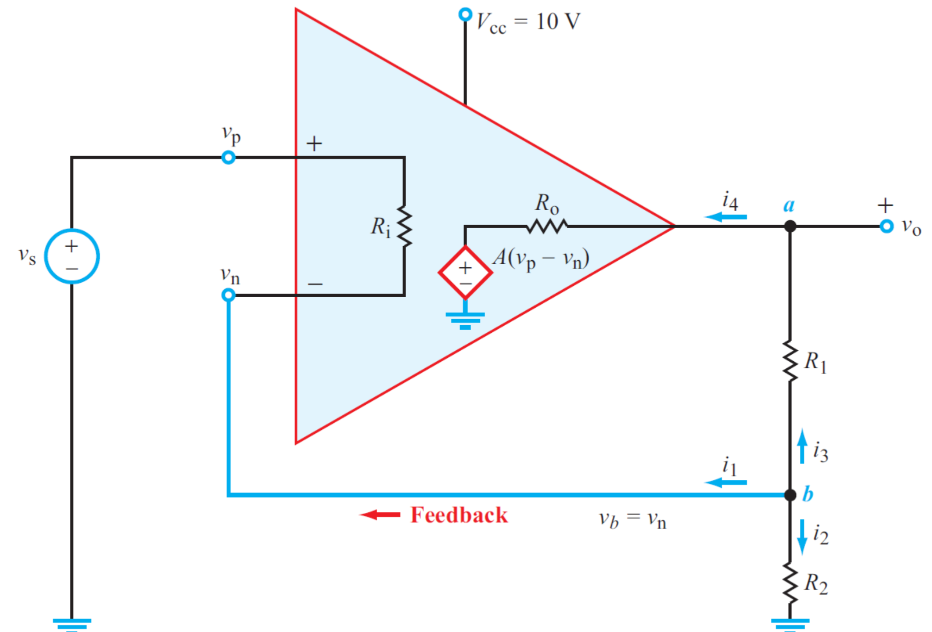


Op-Amp Characteristics	Parameter	Typical Range	Ideal Op Amp
• Linear input–output response	Open-loop gain A	10^4 to 10^8 (V/V)	∞
• High input resistance	Input resistance R_i	10^6 to 10^{13} Ω	∞ Ω
• Low output resistance	Output resistance R_o	1 to 100 Ω	0 Ω
• Very high gain	Supply voltage V_{cc}	5 to 24 V	As specified by manufacturer

Example 4-1: Op Amp Amplifier

- This op amp is used to amplify input signal voltage v_s. The circuit uses negative feedback to connect op-amp output (at node a) to the inverting input terminal v_n through R_1. Obtain an expression for the circuit gain $G = v_o/v_s$ and then evaluate it.

For $V_{cc} = 10$ V, $A = 10^6$, $R_i = 10^7$ Ω, $R_o = 10$ Ω, $R_1 = 80$ kΩ, and $R_2 = 20$ kΩ,



Example 4-1: Op Amp Amplifier

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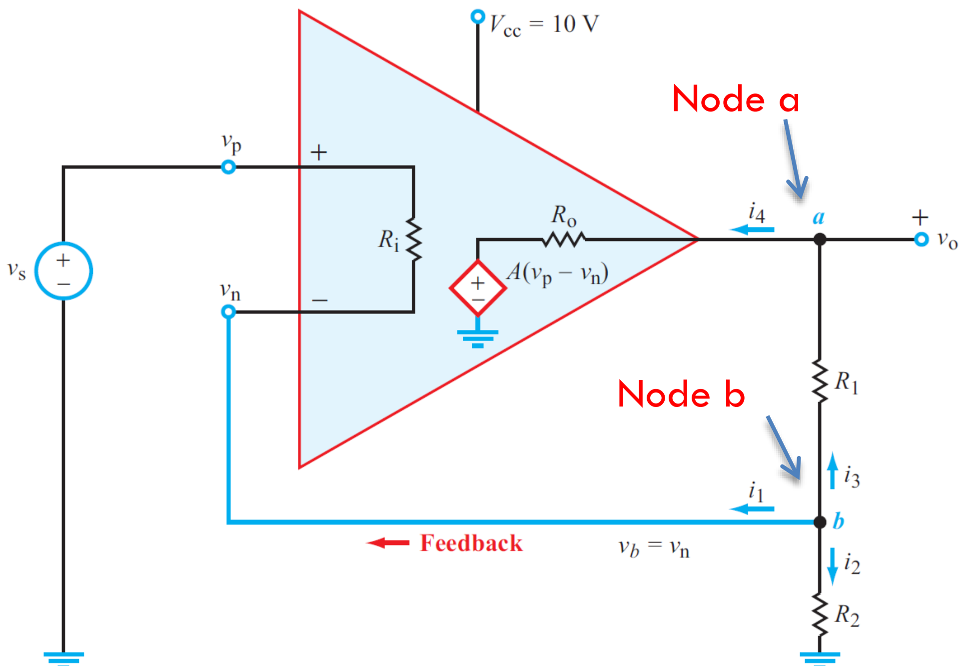


KCL at Node a:

$$\frac{v_n - v_o}{R_1} = \frac{v_o - A(v_p - v_n)}{R_o}$$

KCL at Node b:

$$\frac{v_n - v_p}{R_i} + \frac{v_n}{R_2} + \frac{v_n - v_o}{R_1} = 0$$

For infinite A:

$$G = \frac{v_0}{v_s} = \frac{R_1 + R_2}{R_2} = 5$$

Note that $v_p = v_s$ rearranging the two KCL, we have

$$G = \frac{v_o}{v_s} = \frac{[AR_i(R_1 + R_2) + R_2 R_o]}{AR_2 R_i + R_o(R_2 + R_i) + R_1 R_2 + R_i(R_1 + R_2)} = 4.999975$$

Op-Amp Switch

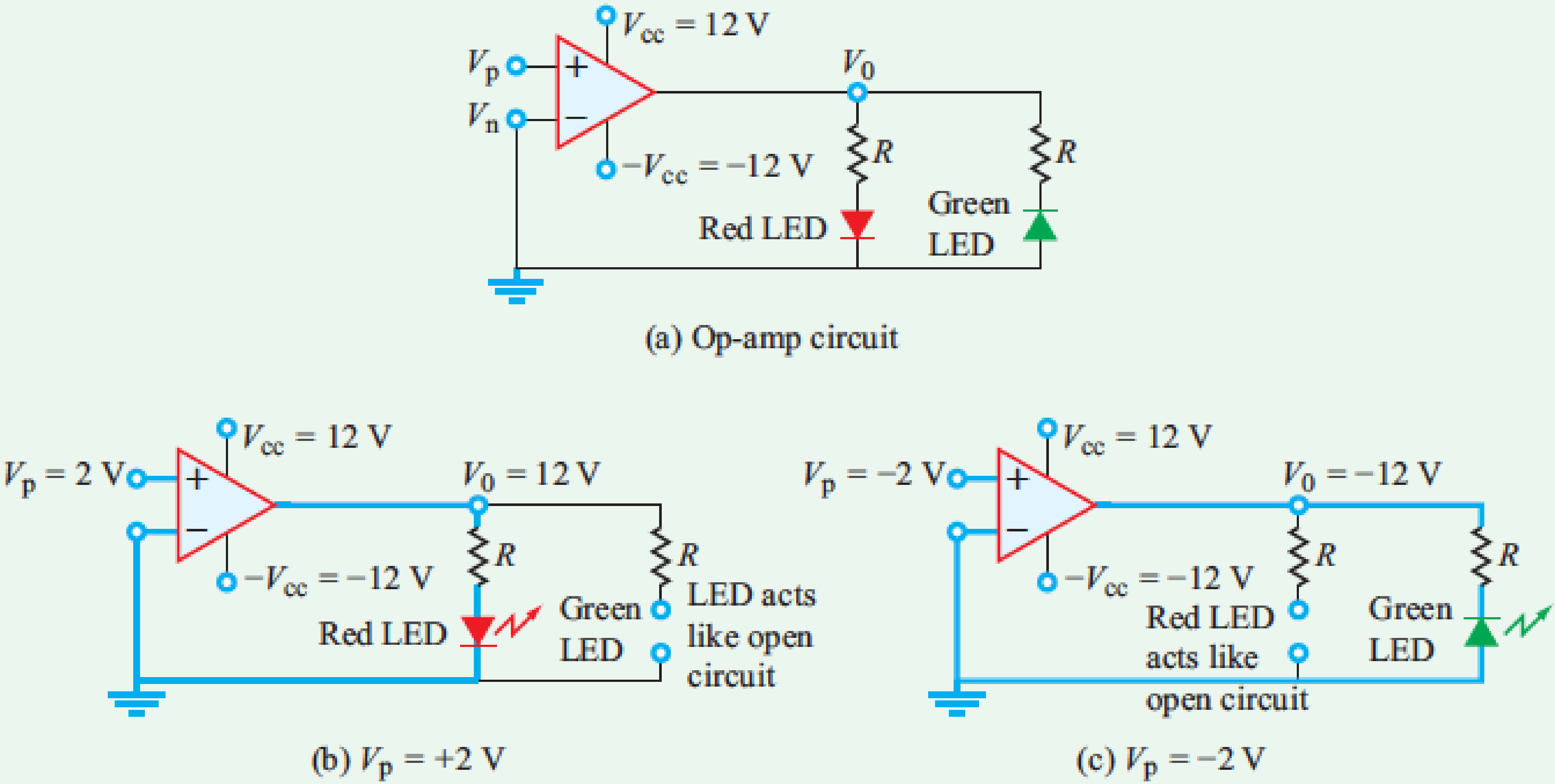
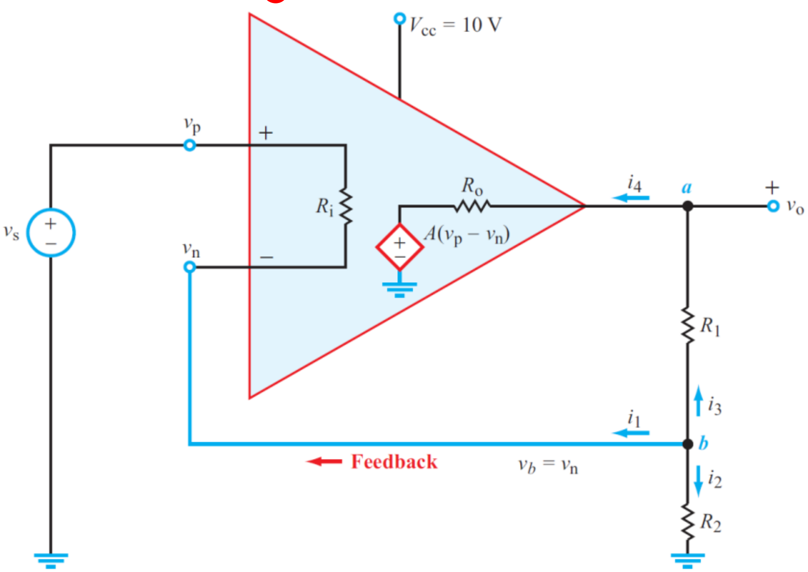


Figure 4-5: Op amp operated as a switch. The $\pm V_{cc}$ flags indicate the dc supply voltages connected to pins 7 and 4.

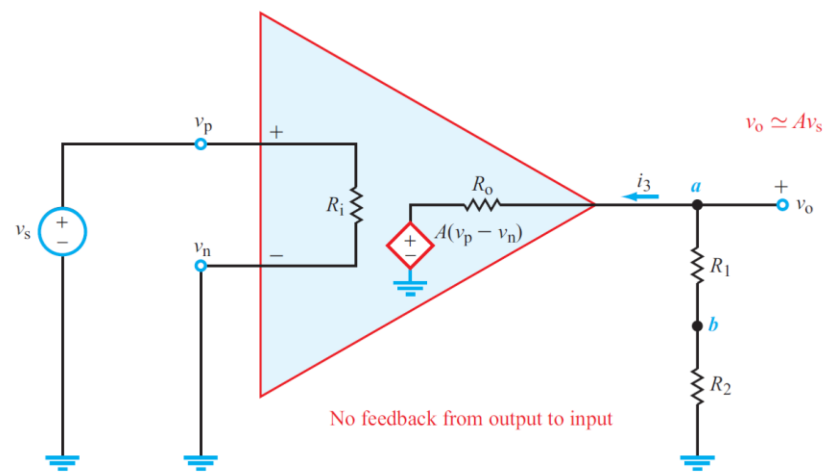
Negative Feedback

- Feedback: return some of the output to the input
- Negative feedback *decreases* input signal
- Achieves desired circuit gain, with wide range for input

Negative Feedback



No Feedback



$v_o \simeq 5\,v_s$

Gain = 5

Range of $v_s \Rightarrow \pm\dfrac{V_{CC}}{5}$

Range of v_s: −2 V to +2 V

$v_o \approx A\,v_s$

Gain = 1million

Range of $v_s \Rightarrow \pm\dfrac{V_{CC}}{A}$

Range of v_s: −10 mV to +10 mV

Negative Feedback

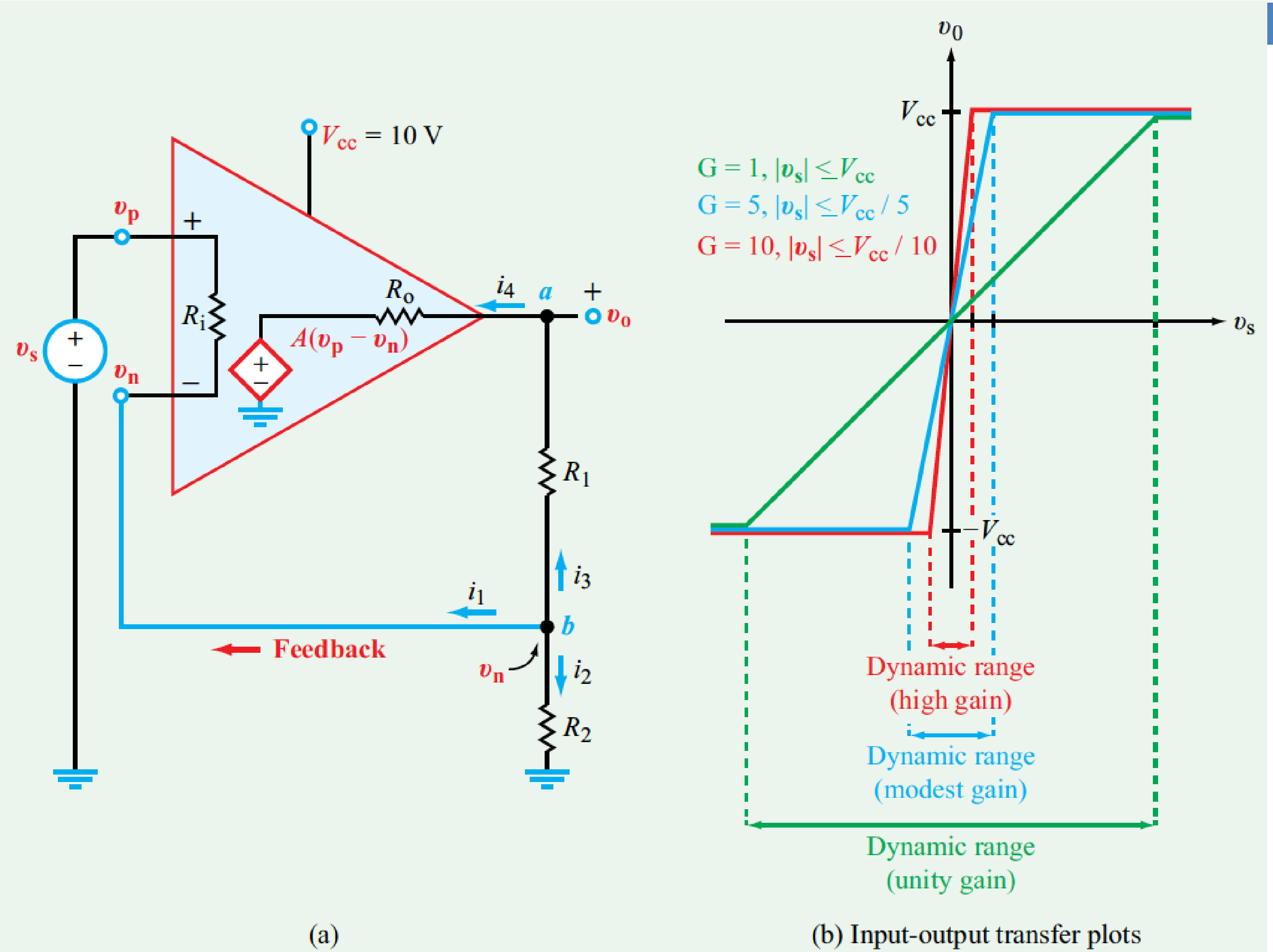
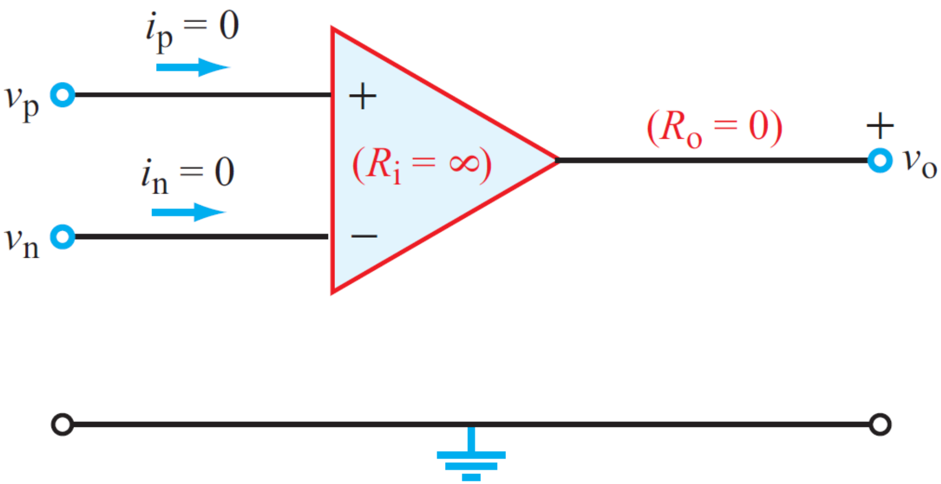


Figure 4-8: Trade-off between gain and dynamic range.

Circuit Analysis With Ideal Op Amps

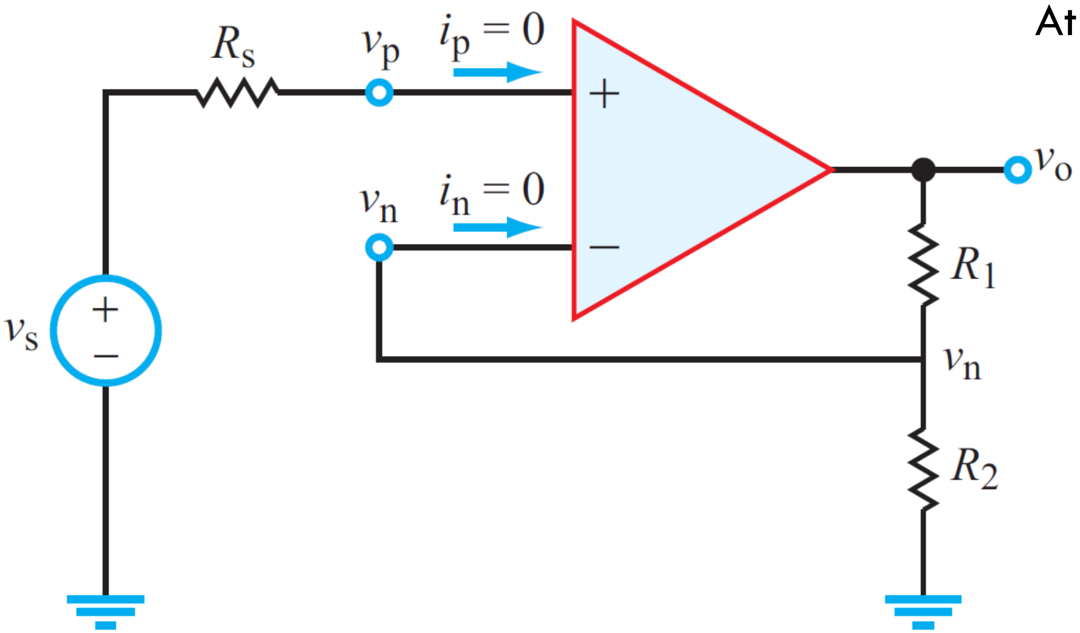


Ideal Op Amp

- Current constraint $i_p = i_n = 0$
- Voltage constraint $v_p = v_n$
- $A = \infty$ $R_i = \infty$ $R_o = 0$

- Use nodal analysis as before, but with "golden rules"
- $v_p = v_n$ (Ideal op-amp model). No voltage drop across op amp input
- $i_p = i_n = 0$ (Ideal op-amp model). No current into op amp
- Do not apply KCL at op amp output

Noninverting Amplifier

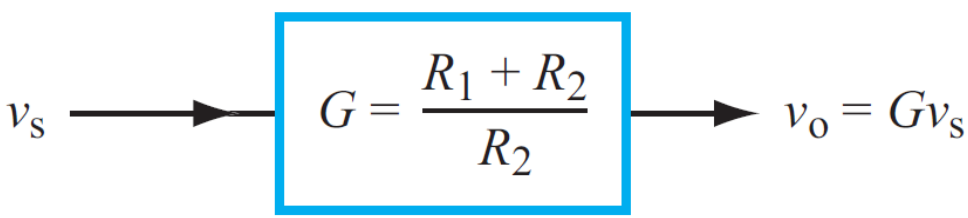


At node v_n

$$\frac{v_n - v_o}{R_1} + \frac{v_n}{R_2} = 0$$

$$v_n = v_p = v_s$$

$$\boxed{v_o = \frac{R_1 + R_2}{R_2} v_s}$$

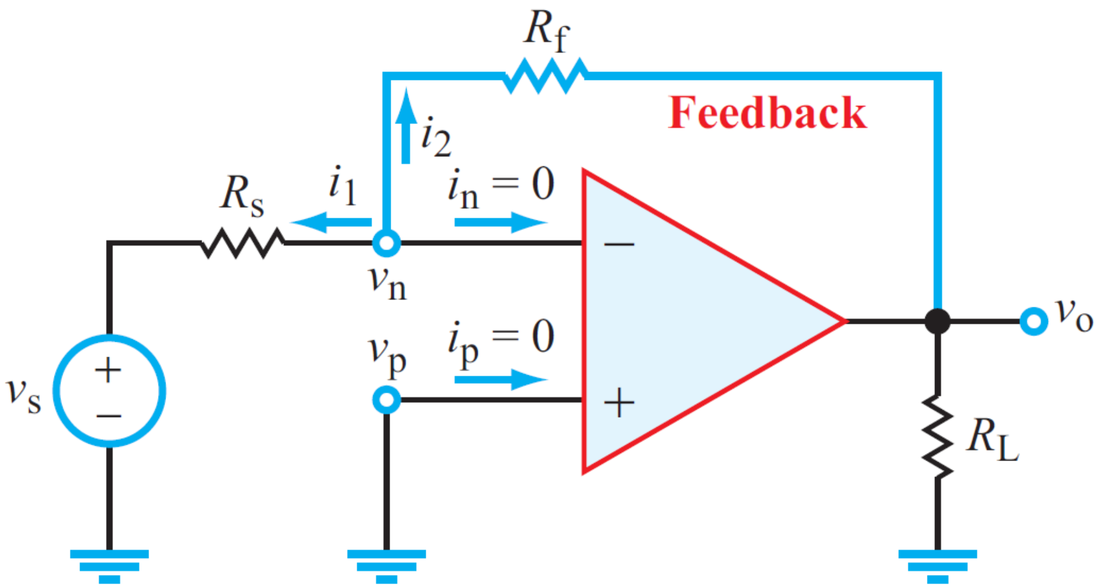


$$\boxed{V_o\ (\text{max}) = V_{cc}}$$

Inverting Amplifier

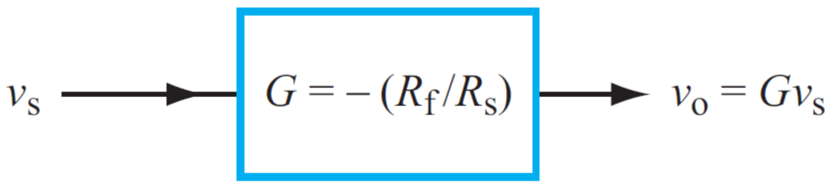
At node v_n

$$\frac{v_n - v_s}{R_s} + \frac{v_n - v_o}{R_f} + i_n = 0.$$



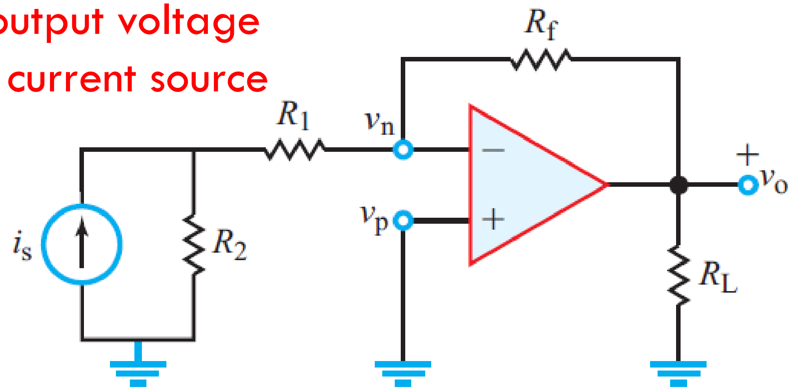
$$v_n = v_p = 0$$

$$G = \frac{v_o}{v_s} = -\left(\frac{R_f}{R_s}\right).$$

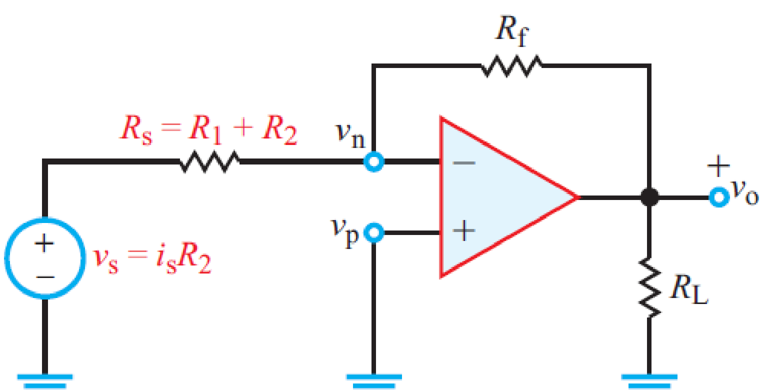


Example 4-2: Input Current Source

Relate output voltage
to input current source



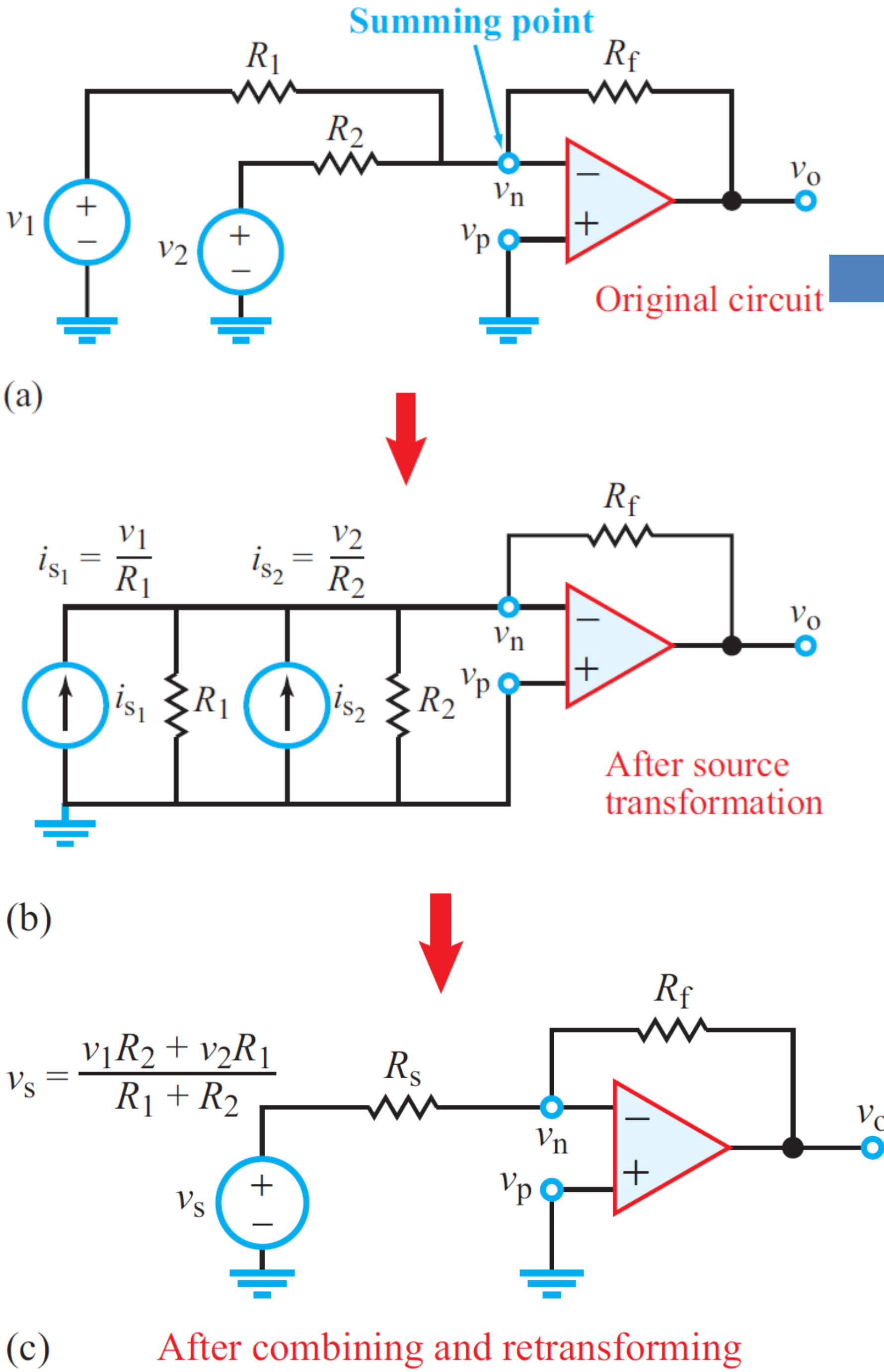
(a) Original circuit

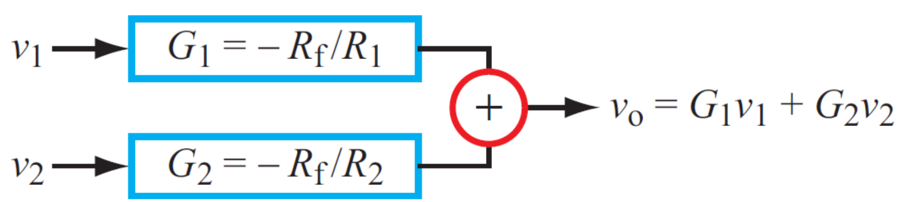


(b) After source transformation

$$v_o = -\left(\frac{R_f}{R_1 + R_2}\right) v_s$$
$$= -\left(\frac{R_f}{R_1 + R_2}\right) R_2 i_s$$

Inverting Summing Amplifier



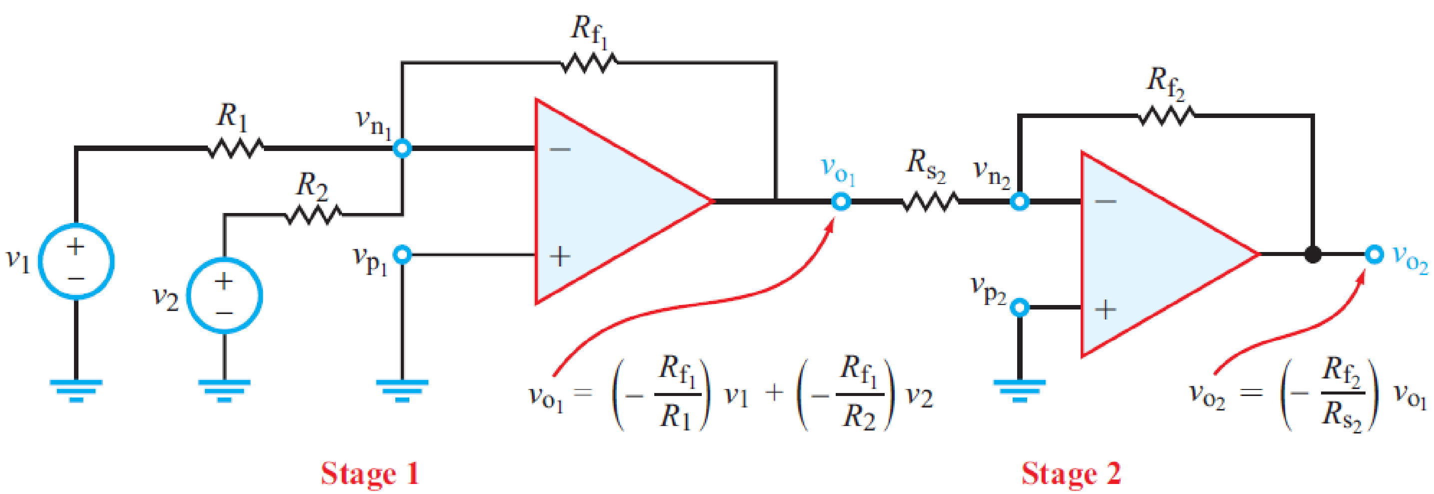


Example 4-3:

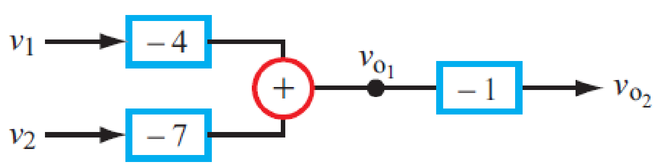
Design a circuit that performs the operation

$$v_o = 4v_1 + 7v_2.$$

Method one: Use 2 inverting op-amps, one to perform the inverting sum, and a second one to provide multiplication by (-1).



(a) Two-stage circuit



(b) Block diagram

Example 4-3:

Solution:

$$\frac{R_{f_1}}{R_1} = 4 \qquad \text{and} \qquad \frac{R_{f_1}}{R_2} = 7.$$

we will choose $R_{f_1} = 56\ \text{k}\Omega$, which then specifies the other resistors as

$$R_1 = 14\ \text{k}\Omega \qquad \text{and} \qquad R_2 = 8\ \text{k}\Omega.$$

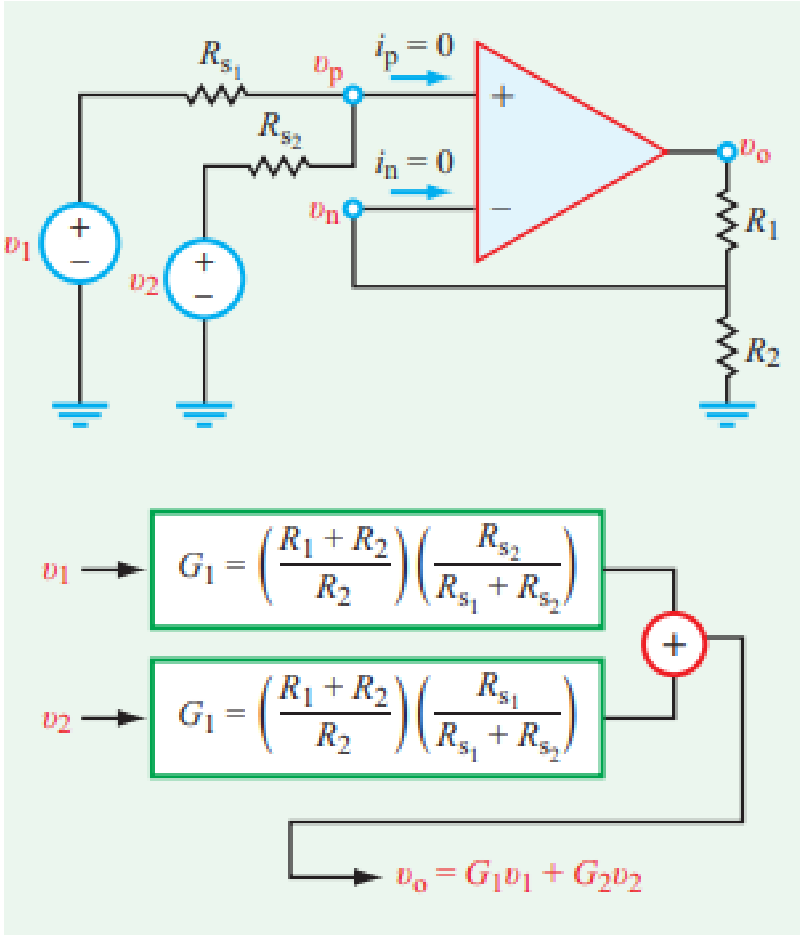
For the second stage, a gain of (-1) requires that

$$\frac{R_{f_2}}{R_{s_2}} = 1.$$

Arbitrarily, we choose $R_{f_2} = R_{s_2} = 20\ \text{k}\Omega$.

Example 4-3

Method two: Use a single op-amp in a noninverting amplifier circuit.



Example 4-3

Solution:

Gain of noninverting op amp is $$\frac{\upsilon_o}{\upsilon_p} = G = \frac{R_1 + R_2}{R_2}.$$

Given that op amp draws no current ($i_p = 0$), it is also straightforward to show that $$\upsilon_p = \frac{\upsilon_1 R_{s_2} + \upsilon_2 R_{s_1}}{R_{s_1} + R_{s_2}}.$$

Combining the two equations, we have $$\upsilon_o = G\left[\left(\frac{R_{s_2}}{R_{s_2} + R_{s_1}}\right)\upsilon_1 + \left(\frac{R_{s_1}}{R_{s_1} + R_{s_2}}\right)\upsilon_2\right].$$

To realise a coeffieint of 4 for v1 and 7 for v2, it is necessary that

$$\frac{G R_{s_2}}{R_{s_1} + R_{s_2}} = 4$$

and

$$\frac{G R_{s_1}}{R_{s_1} + R_{s_2}} = 7.$$

A possible solution that satisfies these two constraints is $R_{s_1} = 7\ \text{k}\Omega$, $R_{s_2} = 4\ \text{k}\Omega$, and $G = 11$. Furthermore, the specified value of G can be satisfied by choosing $R_1 = 50\ \text{k}\Omega$ and $R_2 = 5\ \text{k}\Omega$.

Multiple ways of building a system

(a) $v_o = (4v_1) + (7v_2)$: Multiply v_1 by 4 (noninverting amplifier with a gain of 4) and v_2 by 7 (noninverting amplifier with a gain of 7), and then add them together (noninverting summer with a gain of 1).

(b) $v_o = (-4v_1 - 7v_2)(-1)$: Multiply v_1 by -4 and v_2 by -7 and add them together (inverting summing amplifier with gains of -4 and -7), and then multiply the result by -1 (inverting amplifier with a gain of -1).

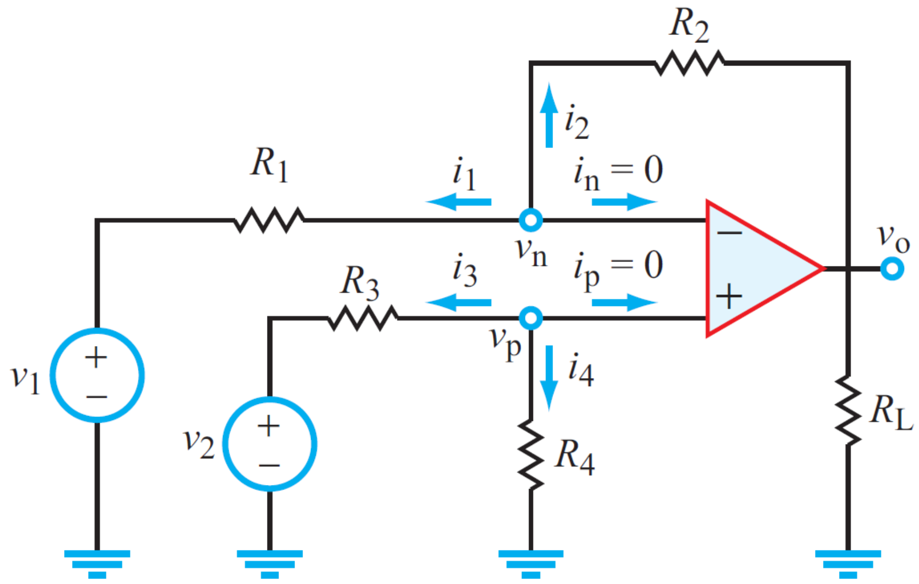
(c) $v_o = (4v_1 + 7v_2)$: Multiply v_1 by 4 and v_2 by 7 and add them together (noninverting summing amplifier with gains of 4 and 7).

(d) $v_o = [(2v_1) + (3.5v_2)] \times 2$: Multiply v_1 by 2 (noninverting amplifier with a gain of 2) and v_2 by 3.5 (noninverting amplifier with a gain of 3.5), and then add them (noninverting summer with a gain of 2).

Why choose a particular method?

- Several reasons:
  - To minimize the number of op amps (option c)
  - To meet gain limitations. An inverting amplifier can have a gain of less than 1, but a noninverting amplifier cannot.
  - To avoid saturation. The output voltage of any individual stage is limited by its V_{cc}. The order in which multiplication/summation is done must keep each individual stage from exceeding $+/-V_{cc}$.
  - Sensitivity when adding large and small values. Care is typically taken to add values that are similar in magnitude, so amplification is typically done prior to summation if two values have significantly different magnitudes.
  - Other considerations . . .

Difference Amplifier



At node v_n, $i_1 + i_2 + i_n = 0$,

$$\frac{v_n - v_1}{R_1} + \frac{v_n - v_o}{R_2} + i_n = 0 \qquad \textbf{(node } v_n\textbf{)}.$$

At v_p, $i_3 + i_4 + i_p = 0$, or

$$\frac{v_p - v_2}{R_3} + \frac{v_p}{R_4} + i_p = 0 \qquad \textbf{(node } v_p\textbf{)}.$$

Upon imposing the ideal op-amp constraints $i_p = i_n = 0$ and $v_p = v_n$, we end up with

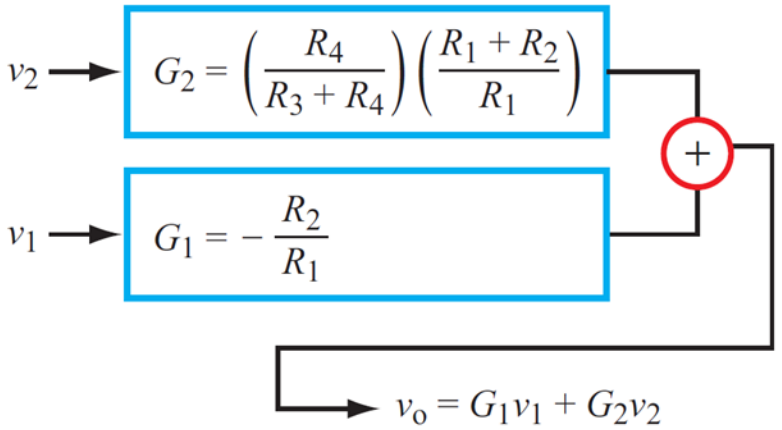
$$v_o = \left[\left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right)\right] v_2 - \left(\frac{R_2}{R_1}\right) v_1,$$

Note negative gain of channel 1

Difference Amplifier

which can be cast in the form

$$v_o = G_2 v_2 + G_1 v_1,$$



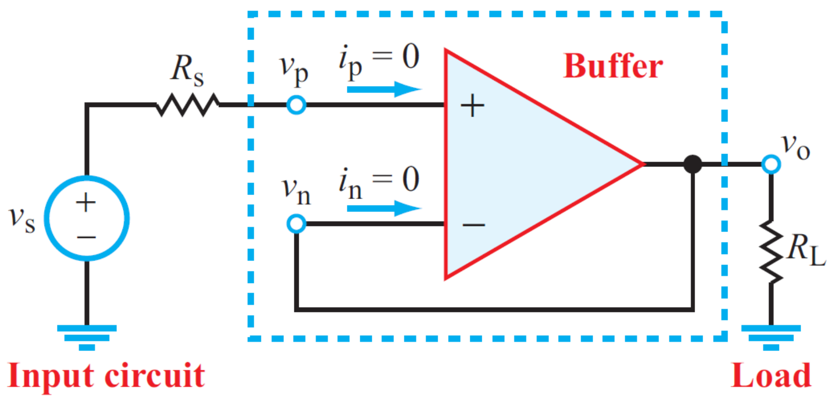
► The difference amplifier scales v_2 by positive gain G_2, v_1 by negative gain G_1 and adds them together. ◄

Voltage Follower

"Buffers" Sections of Circuit

$$v_o = \frac{v_s R_L}{R_s + R_L} \qquad \text{(without voltage follower)}$$

v_o depends on both input and load resistors



$$v_o = v_p = v_s \qquad \text{(with voltage follower)}$$

v_o is immune to input and load resistors

What is the op amp doing?

► The output of the voltage follower ***follows*** the input signal while remaining immune to changes in R_L because it has a high input resistance and low output resistance. ◄

Basic rule of Op Amp circuits

Basic Rules of Op-Amp Circuits

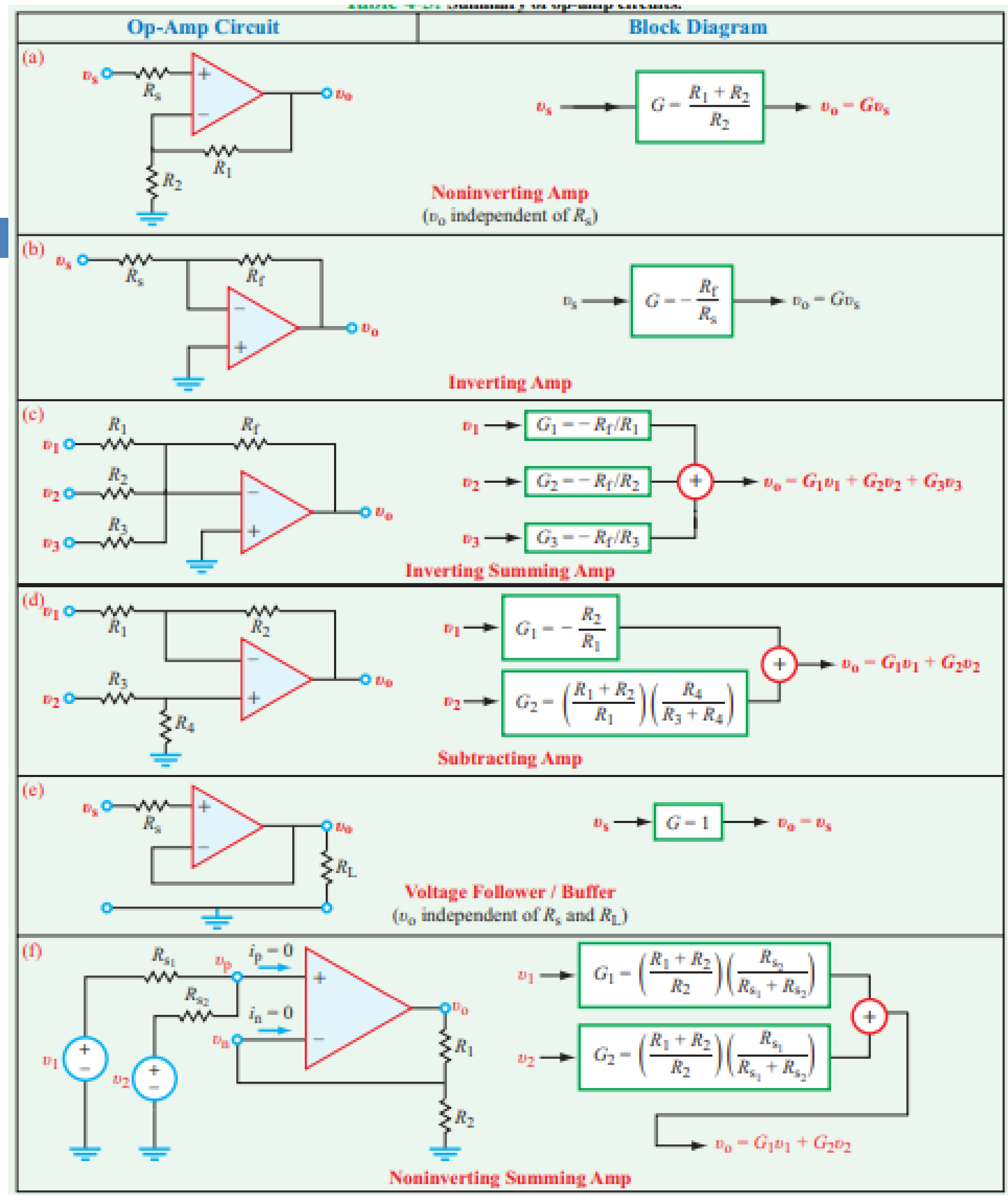
(1) KCL and KVL always apply everywhere in the circuit, but KCL is inapplicable at the output node when applying the ideal op-amp model. All other circuit-analysis tools can be applied to op-amp circuits.

(2) The op amp will operate in the linear range so long as $|\upsilon_o| < |V_{cc}|$.

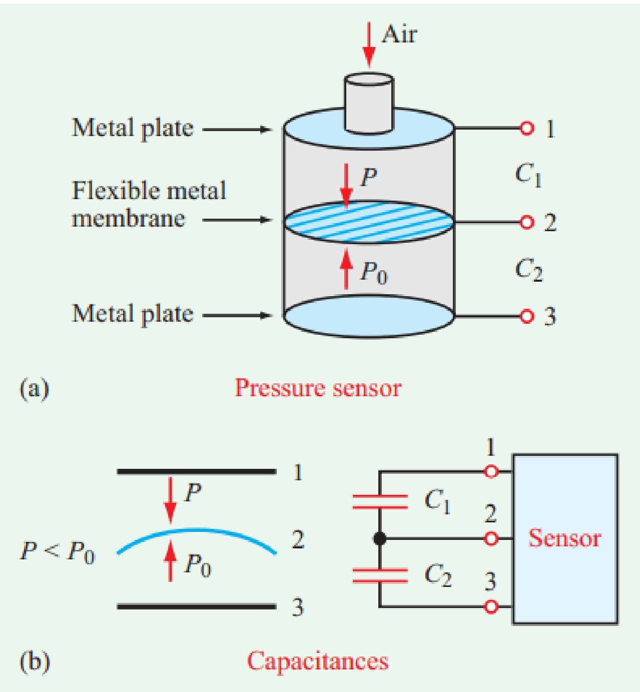
(3) The ideal op-amp model assumes that the source resistance R_s (connected to terminals υ_p or υ_n) is much smaller than the op-amp input resistance R_i (which usually is no less than 10 MΩ), and the load resistance R_L is much larger than the op-amp output resistance R_o (which is on the order of tens of ohms).

(4) The ideal op-amp constraints are $i_p = i_n = 0$ and $\upsilon_p = \upsilon_n$.

Summary of Op Amp Circuits



Example 4-5: Elevation Sensor



Pressure sensor

Capacitances

A hand-held elevation sensor uses a pair of capacitors separated by a flexible metallic membrane (Fig. (a)) to measure the height h above sea level. The lower chamber in Fig. (a) is sealed, and its pressure is P_0, which is the standard atmospheric pressure at sea level. The pressure in the upper chamber, which is open to the outside air, is P. When at sea level, $P = P_0$, so the membrane assumes a flat shape and the two capacitances are equal. Since atmospheric pressure decreases with elevation, a rise in altitude results in a change in the pressure P in the upper chamber, causing the membrane to bend upwards (Fig. (b)), thereby changing the capacitances of the two capacitors. The sensor measures a voltage v_s that is proportional to the change in capacitance. Based on measurements of v_s as a function of h, the data was found to exhibit an approximately linear variation given by

$v_s = 2 + 0.2h$ (V), where h is in km.

The sensor is designed to operate over the range $0 \le h \le 10$ km. Design a circuit whose output voltage υ_o (in volts) is an exact indicator of the height h (in km).

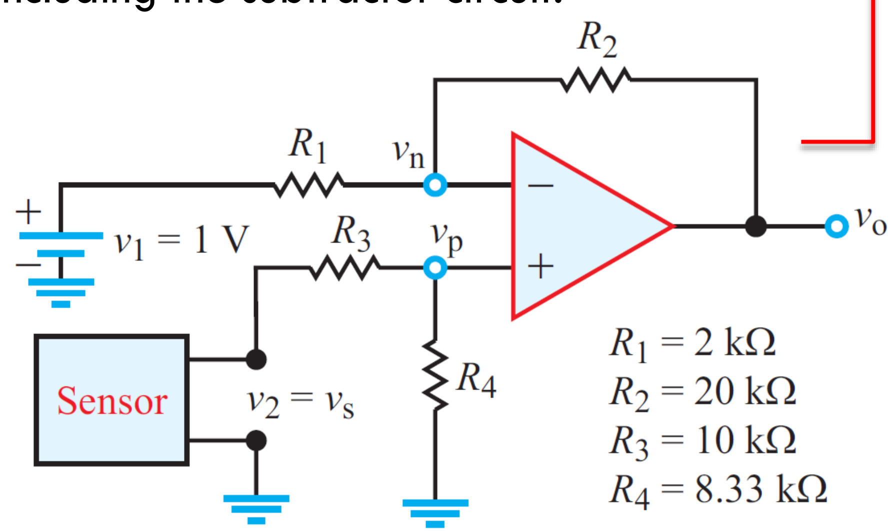
Example 4-5: Elevation Sensor

h = elevation, inversely proportional to air pressure

Sensor Response $\quad v_s = 2 + 0.2h \qquad \text{(V)},$

Desired Output $\quad v_o = h = \dfrac{1}{0.2}\, v_s - \dfrac{2}{0.2} = 5v_s - 10,$

There are multiple circuit configurations that can achieve the desired operation, including the subtractor circuit:



(c) Circuit realization

$$v_o = \left[\left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right)\right] v_2 - \left(\frac{R_2}{R_1}\right) v_1. \qquad (4.49)$$

Equation (4.49) can be made to correspond to Eq. (4.48) if we select the following

(a) $v_s = v_2$

(b) v_1 as a dc voltage source such that $(R_2/R_1)v_1 = 10$ V, which can be satisfied by arbitrarily selecting $v_1 = 1$ V and $(R_2/R_1) = 10$

(c) values for R_1 through R_4 that simultaneously satisfy the conditions

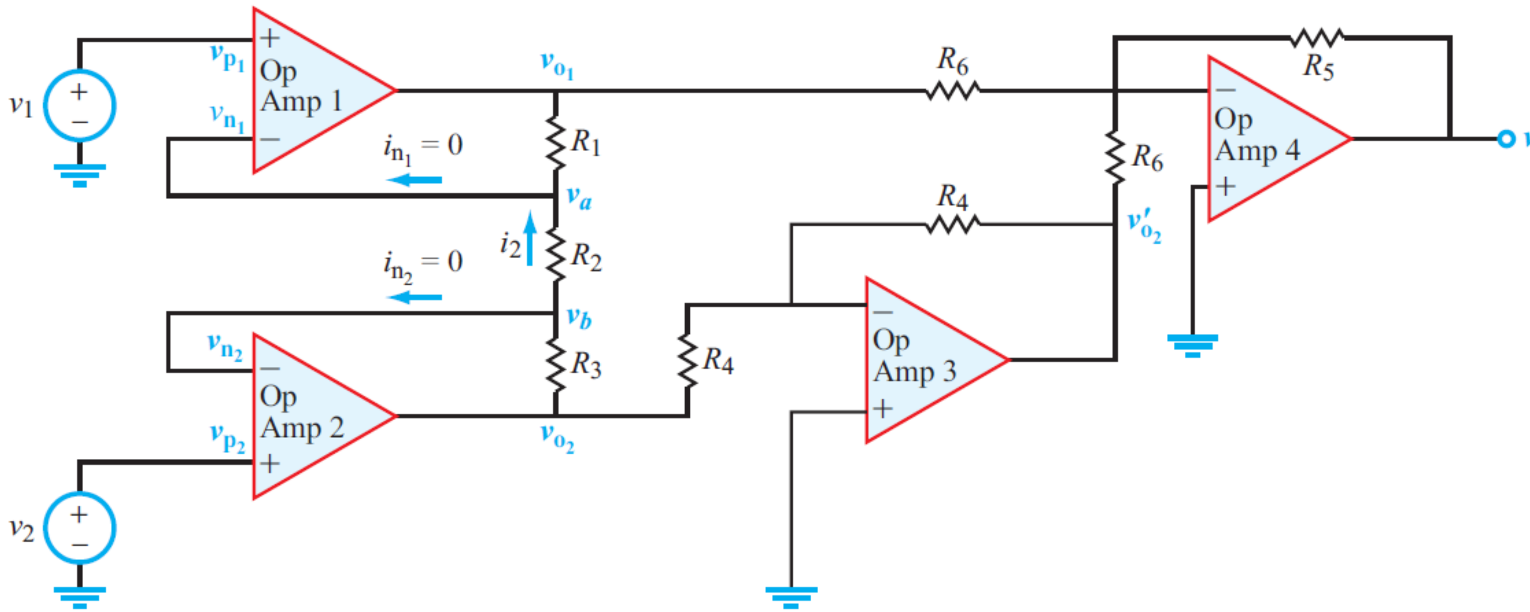
$$\frac{R_2}{R_1} = 10 \qquad \text{and} \qquad \left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right) = 5$$

A possible set of values that meets these conditions is

$$R_1 = 2\ \text{k}\Omega, \quad R_2 = 20\ \text{k}\Omega, \quad R_3 = 10\ \text{k}\Omega, \quad R_4 = 8.33\ \text{k}\Omega.$$

Example 4-6: Multiple Op-Amp Circuit

Relate output voltage v_o to input voltages v_1 and v_2:



For op amp 1, $v_a = v_{n_1} = v_1.$

Similarly, for op amp 2,

$$v_b = v_{n_2} = v_2.$$

Since $i_{n_1} = i_{n_2} = 0$ (op amp current constraint),

$$i_2 = \frac{v_b - v_a}{R_2} = \frac{v_2 - v_1}{R_2},$$

and

$$v_{o_2} - v_{o_1} = i_2(R_1 + R_2 + R_3) = \left(\frac{R_1 + R_2 + R_3}{R_2}\right)(v_2 - v_1). \tag{4.50}$$

For op amp 3,

$$v'_{o_2} = -v_{o_2},$$

and for op amp 4,

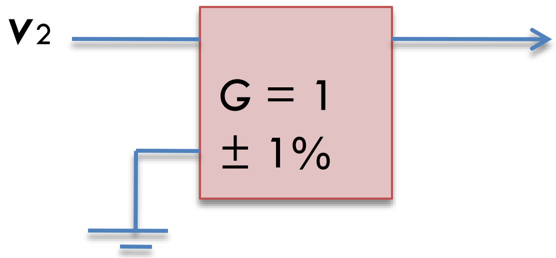
$$v_o = -\frac{R_5}{R_6}(v_{o_1} + v'_{o_2}) = -\frac{R_5}{R_6}(v_{o_1} - v_{o_2}) \quad = \frac{R_5}{R_6}(v_{o_2} - v_{o_1})$$

$$= R_5\left(\frac{R_1 + R_2 + R_3}{R_6 R_2}\right)(v_2 - v_1). \tag{4.51}$$

Measurement Uncertainty

Direct Measurement

Thermistor
(T = 21°C)

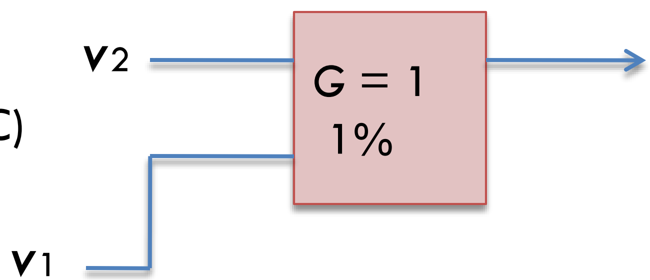


$V_0 = V_2 \pm 1\%$ of V_2

21°C ± 0.21°C

Differential Measurement

Thermistor
(T = 21°C)



$V_0 = (V_2 - V_1) \pm 1\%$ of $(V_2 - V_1)$

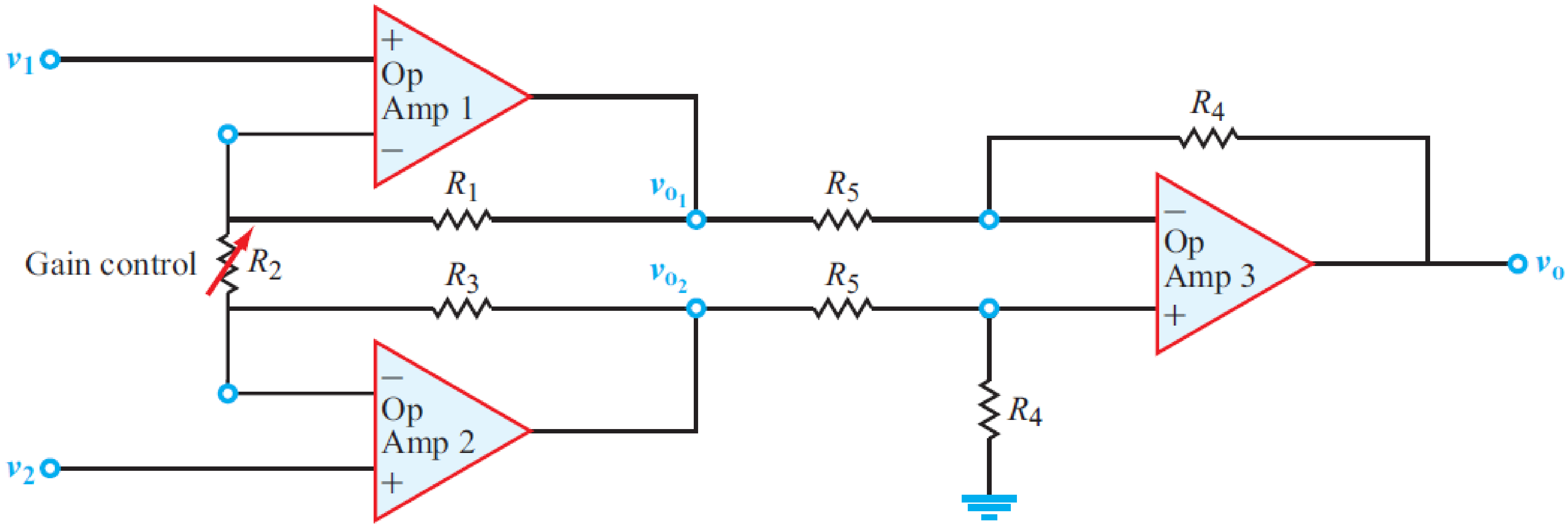
1°C ± 0.01°C

↑

Much better measurement uncertainty

Fixed Reference Temp = 20°C

Instrumentation Amplifier



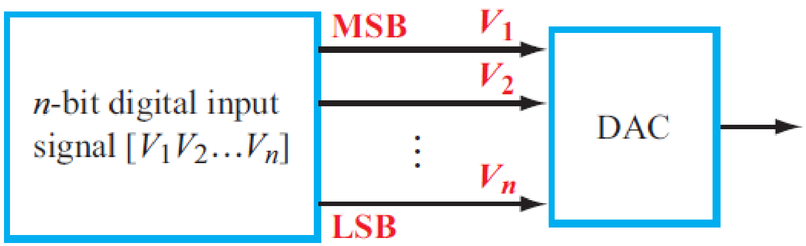
$$v_o = \left(\frac{R_4}{R_5}\right)\left(\frac{R_1 + R_2 + R_3}{R_2}\right)(v_2 - v_1)$$

If we set $R_1 = R_3 = R_4 = R_5 = R$

$$v_o = \left(1 + \frac{2R}{R_2}\right)(v_2 - v_1).$$

Digital to Analog Converter

Converts digital value into analog voltage



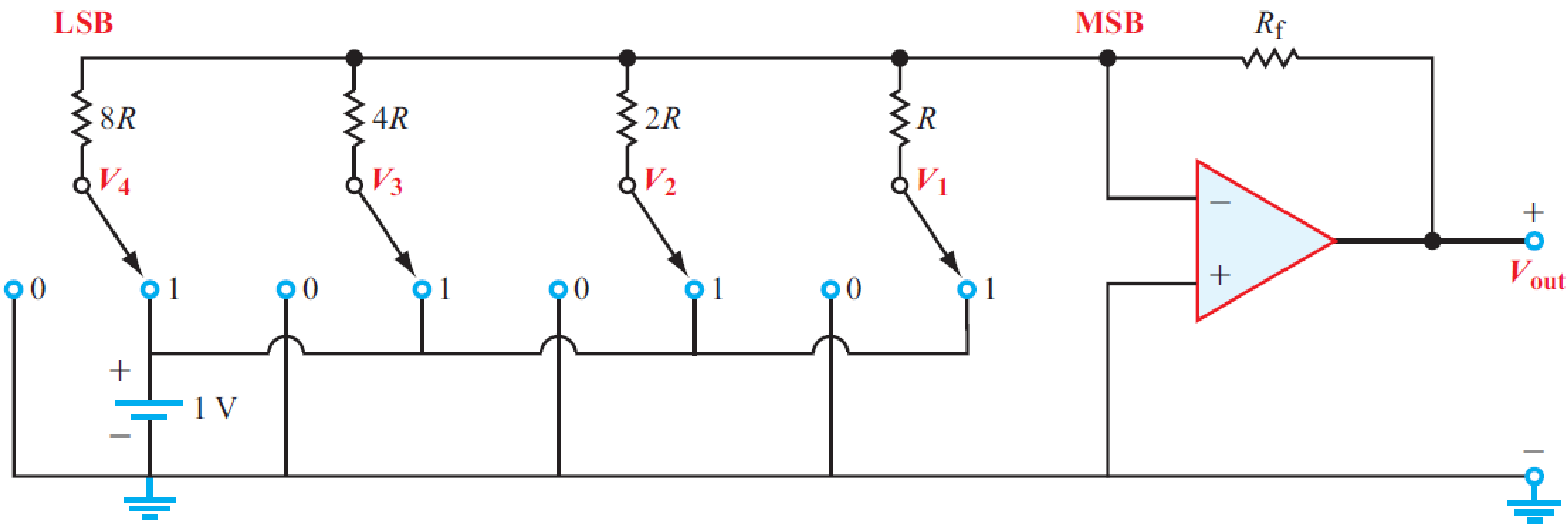
$$V_{out} = G(2^{n-1}V_1 + 2^{n-2}V_2 + \cdots + 2V_{n-1} + V_n)$$

Table 4-4: Correspondence between binary sequence and decimal value for a 4-bit digital signal and output of a DAC with $G = -0.5$.

4-digit example

$V_1V_2V_3V_4$	Decimal Value	DAC Output (V)
0000	0	0
0001	1	−0.5
0010	2	−1
0011	3	−1.5
0100	4	−2
0101	5	−2.5
0110	6	−3
0111	7	−3.5
1000	8	−4
1001	9	−4.5
1010	10	−5
1011	11	−5.5
1100	12	−6
1101	13	−6.5
1110	14	−7
1111	15	−7.5

Digital to Analog Converter



$$V_{out} = -\frac{R_f}{R}V_1 - \frac{R_f}{2R}V_2 - \frac{R_f}{4R}V_3 - \frac{R_f}{8R}V_4$$

$$= \frac{-R_f}{8R}(8V_1 + 4V_2 + 2V_3 + V_4), \qquad G = -\frac{R_f}{8R}$$

$$V_{out} = G(2^{n-1}V_1 + 2^{n-2}V_2 + \cdots + 2V_{n-1} + V_n)$$

Summary

Mathematical and Physical Models

Ideal op amp

$$v_p = v_n$$
$$i_p = i_n = 0$$

Noninverting amp*

$$G = \frac{v_o}{v_s} = \frac{R_1 + R_2}{R_2}$$

Inverting amp*

$$G = \frac{v_o}{v_s} = -\left(\frac{R_f}{R_s}\right)$$

Summing amp*

$$v_o = -R_f\left(\frac{v_1}{R_1} + \frac{v_2}{R_2}\right)$$

Difference amp*

$$v_o = G_2 v_2 + G_1 v_2$$

Voltage follower*

$$v_o = v_s$$

Instrumentation amp

$$v_o = \left(1 + \frac{2R}{R_2}\right)(v_2 - v_1)$$

(with gain-control resistor R_2)

MOSFET

$$V_{out} = V_{DD} - g R_D V_{in}$$

*See Table 4-3.