

VICTORIA UNIVERSITY OF
WELLINGTON
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Introduction to Electrical Power

XMUT203 Analogue Circuits and Systems

Topics

Topics

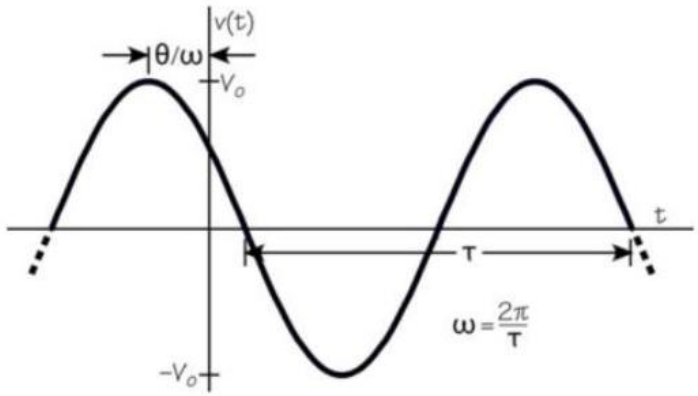
- Fundamentals of electrical power.
- Electrical power parameters.
- Electrical powers (P, Q, and S) and power factor.
- Electrical power circuits.
- Applications of electrical power.
- Electricity consumption and pricing.

Sine and Cosine

- Alternating current (AC) is sines and cosines that move large blocks of energy from one place to another.

$$f(t) = A\cos(\omega t + \theta)$$

- Cosine starts at t = 0.
- The period of cosine is τ.
- We say that phase shift is a leading phase angle θ.



- Frequency of the sinusoidal waveform is:

$$f = \frac{1}{T} \qquad \text{or} \qquad \omega = 2\pi f$$

Sine and Cosine

- Sinusoid voltage

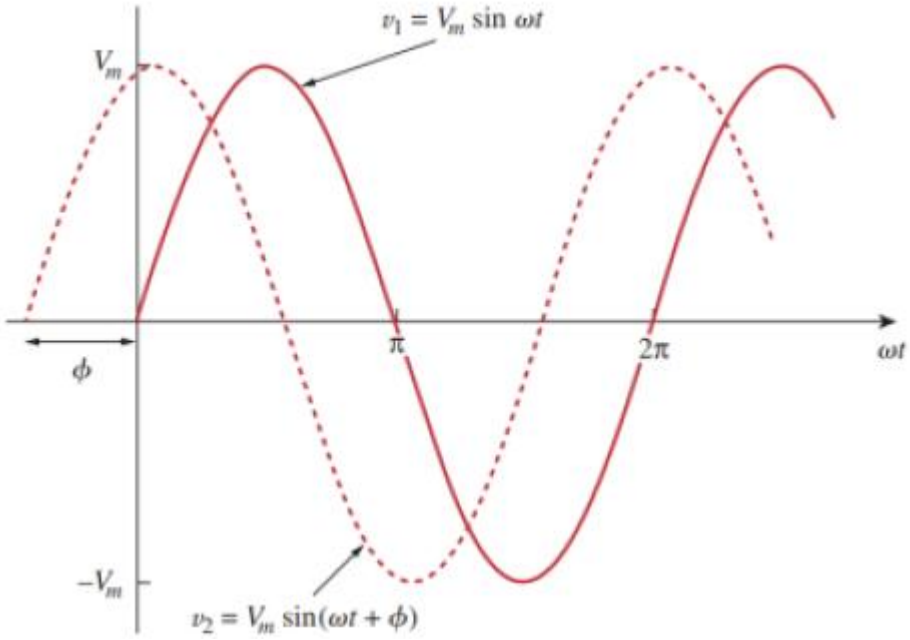
$$v(t) = V_m \sin \omega t$$

Where:

V_m = Amplitude of sinusoid

ω = Angular frequency in radian/s

ωt = argument of the sinusoid



- Let's consider a signal as given below:

$$v(t) = V_m \sin(\omega t + \phi)$$

Example Parameters of Sinusoidal Waveform

Find amplitude, phase, period, and frequency of the following signal: [10 marks]

$$v(t) = 12\cos(50t + 10°)$$

Solution

- The amplitude is:

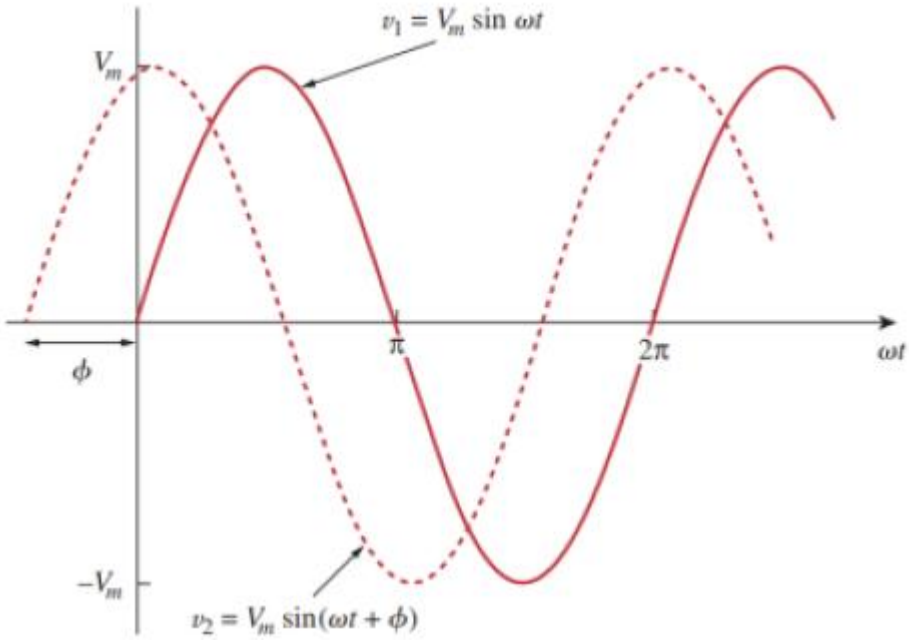
$$V_m = 12 \text{ V}$$

- The phase is:

$$\phi = 10°$$

- The angular frequency is:

$$\omega = 50 \text{ rad/s}$$



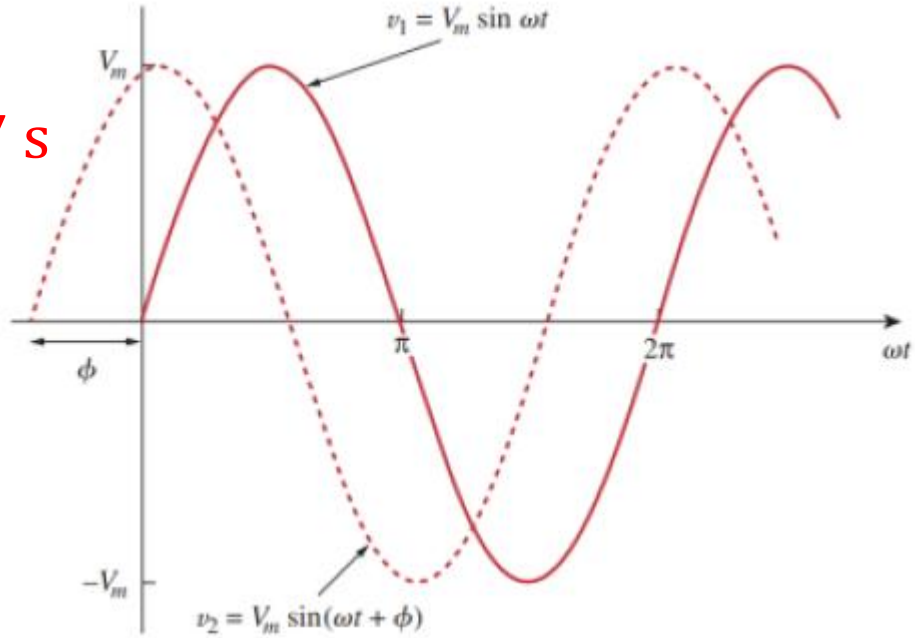
Example Parameters of Sinusoidal Waveform

- The period of the signal is:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257 \text{ s}$$

- The frequency is:

$$f = \frac{1}{T} = 7.958 \text{ Hz}$$



Self-Exercise:

- Now work backwards for 50 Hz keeping V_m and ϕ the same (i.e. $\omega = 2\pi f = 2\pi(50) = 314$ rad/s, thus $v(t) = 12\cos(314t + 10°)$).

AC Power

- We can treat AC power as a function of time (in sine and cosine):

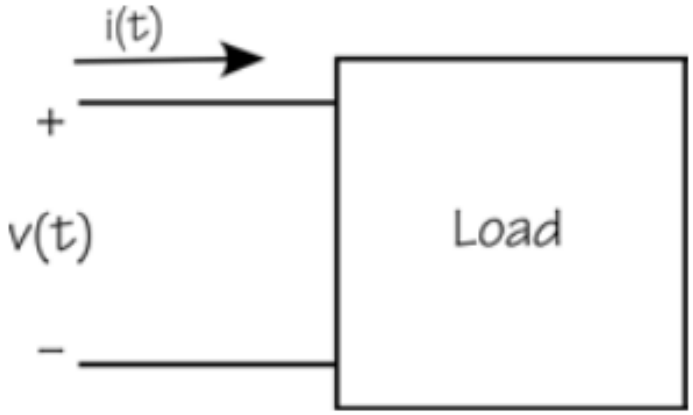
$$v(t) = V_m \cos(2\pi f t + \theta_v)$$

$$i(t) = I_m \cos(2\pi f t + \theta_i)$$

- Thus

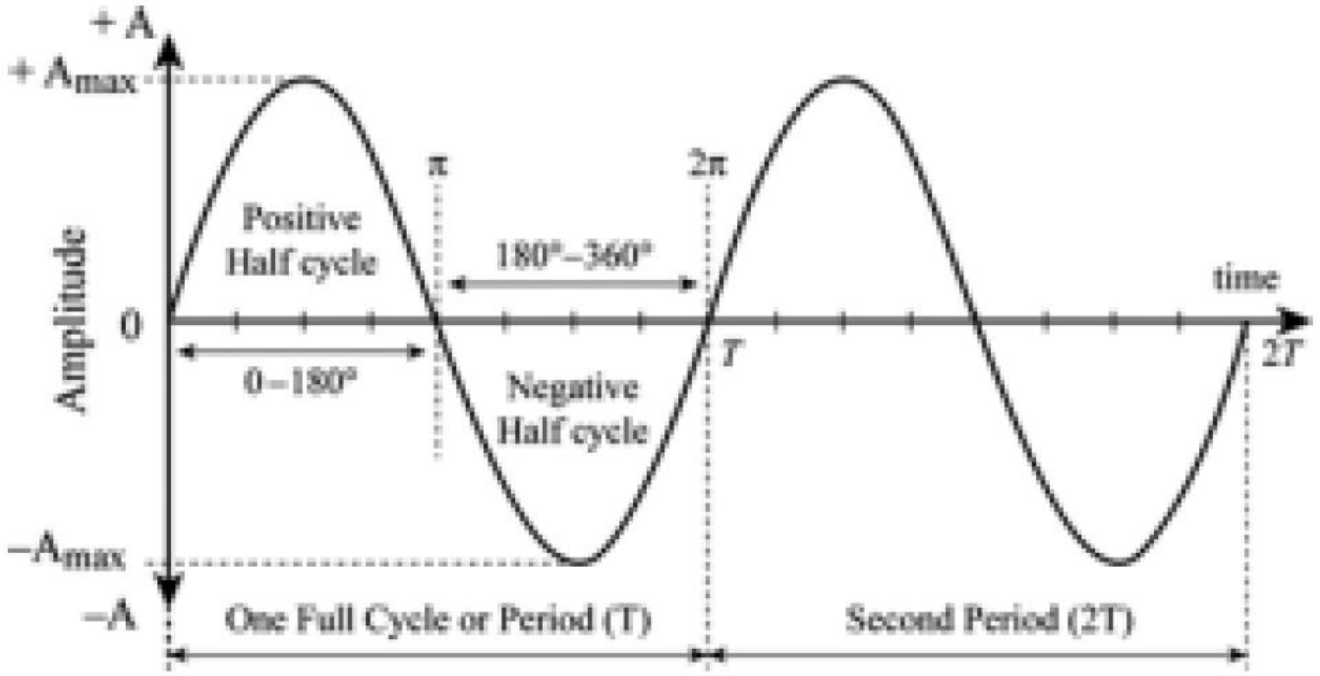
$$p(t) = v(t)i(t)$$

- Note that $p(t)$ is instantaneous power.



AC Power

- But this is not very useful, because we really need to know how much power is being delivered on the average.
- More importantly, how much energy is being moved (i.e. what power company want us to pay for).

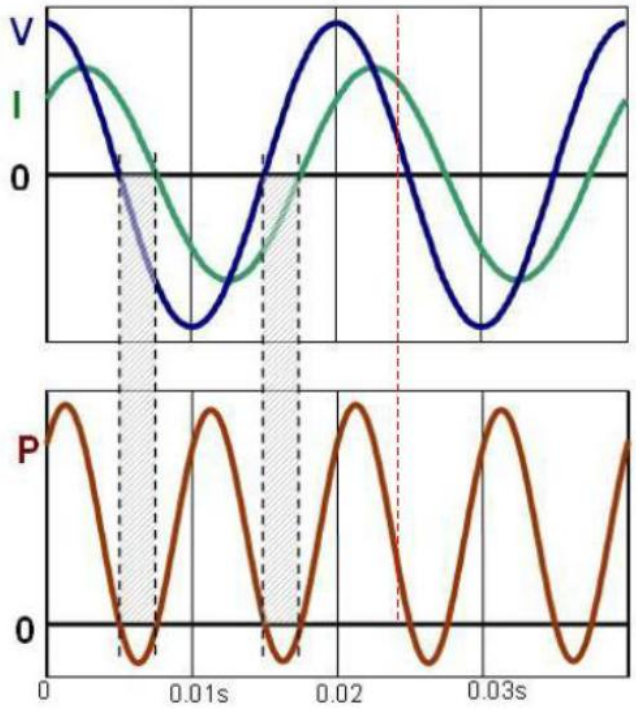


Instantaneous Power

- Instantaneous power is the power at an instant of time:

$$p(t) = v(t)i(t)$$

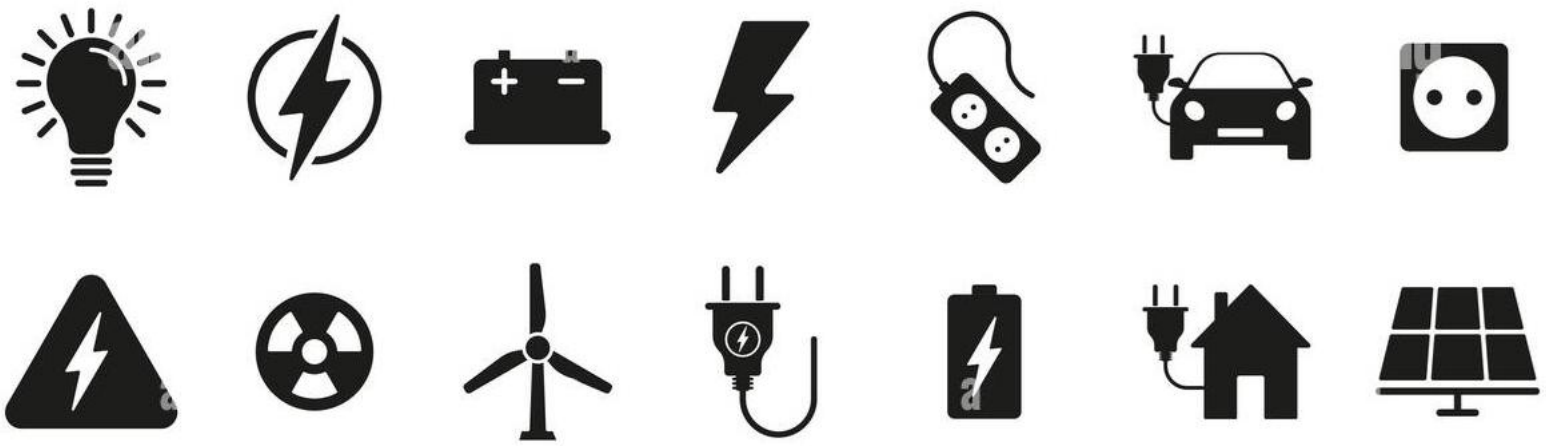
- If $p(t)$ has positive value (i.e. current in same direction as positive voltage polarity), it is source of power.
- if negative then it is sink of power (i.e. dissipate).



Energy (W)

- Energy is the time integral of power often use symbol W (but units are Joules).
- Do not confuse with unit for power Watts, W.

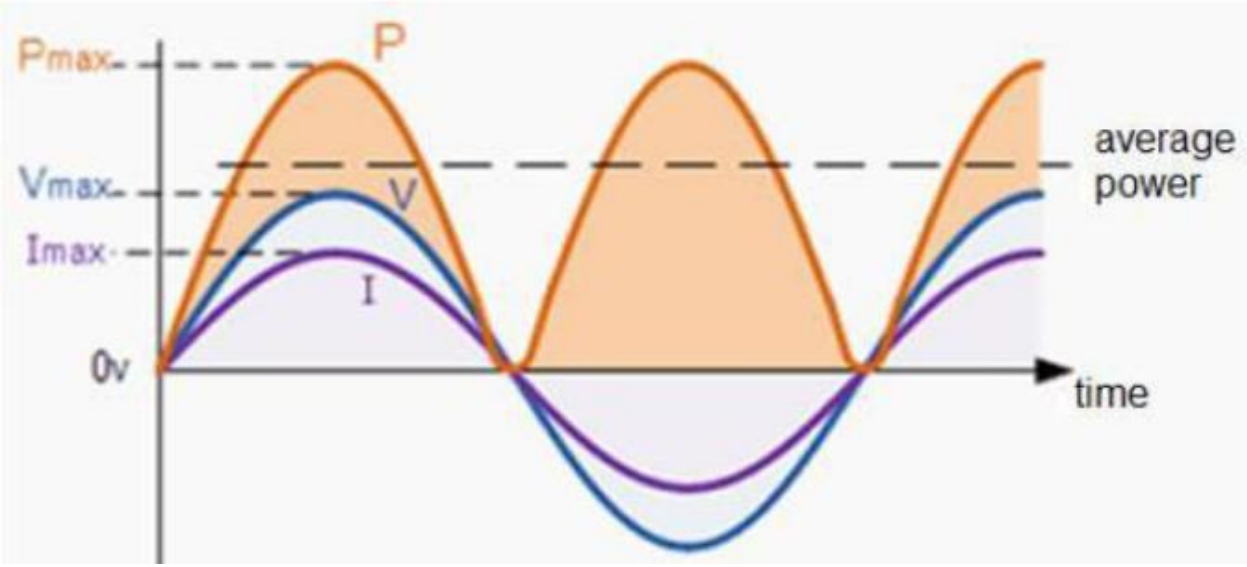
$$W = \int_{t_1}^{t_2} p(t)\, dt$$



Average Power

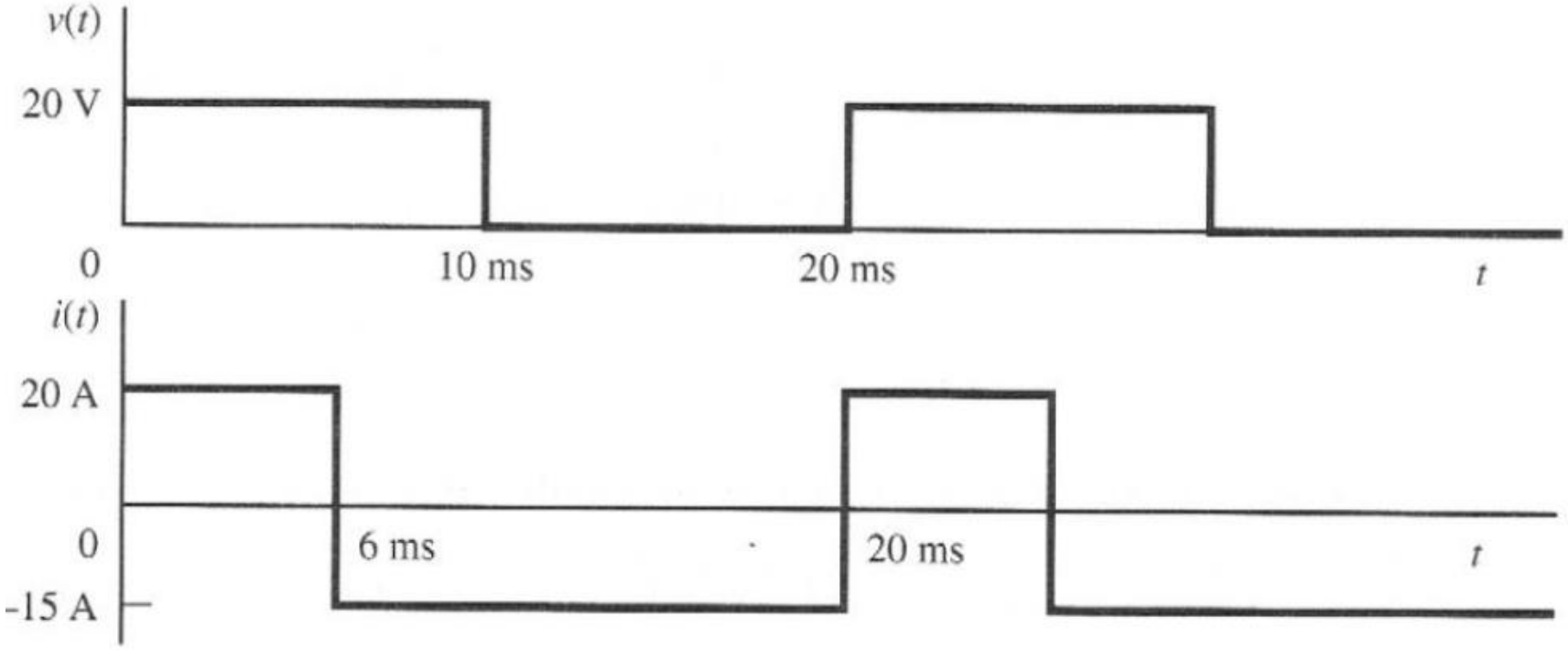
- It is the power averaged over entire cycle.
- Also known as real power or active power.

$$P_{ave} = \frac{1}{T}\int_0^t p(t)\,dt = \frac{1}{T}\int_0^t v(t)\,i(t)dt = \frac{W}{T}$$



Example Instantaneous Power

Given voltage and current waveforms as shown in the figures below.



Example Instantaneous Power

Find:

a. Expressions for the voltage and current waveforms. [4 marks]

b. The instantaneous power. [2 marks]

c. Energy absorbed by the device in one period. [4 marks]

d. Average power absorbed by the device. [4 marks]

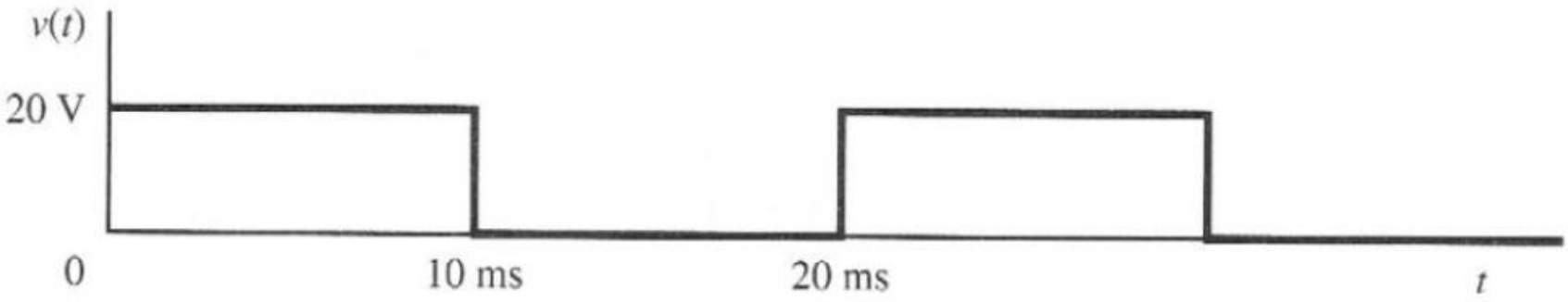
Example Instantaneous Power

Solution

a. The equations for the voltage and current waveforms are:

Voltage is:

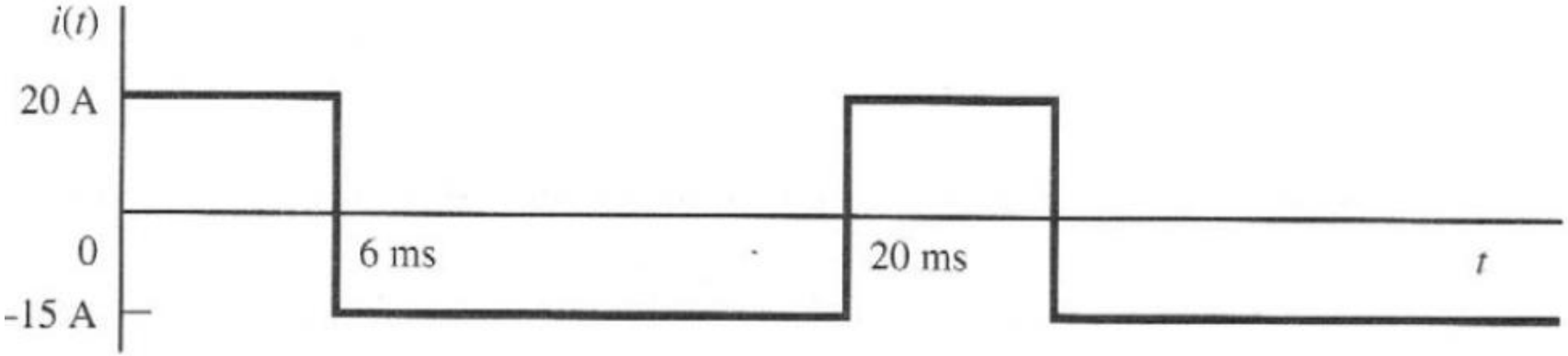
$$v(t) = \begin{cases} 20\ \text{V} & 0 < t < 10\ \text{ms} \\ 0\ \text{V} & 10\ \text{ms} < t < 20\ \text{ms} \end{cases}$$



Example Instantaneous Power

Current is:

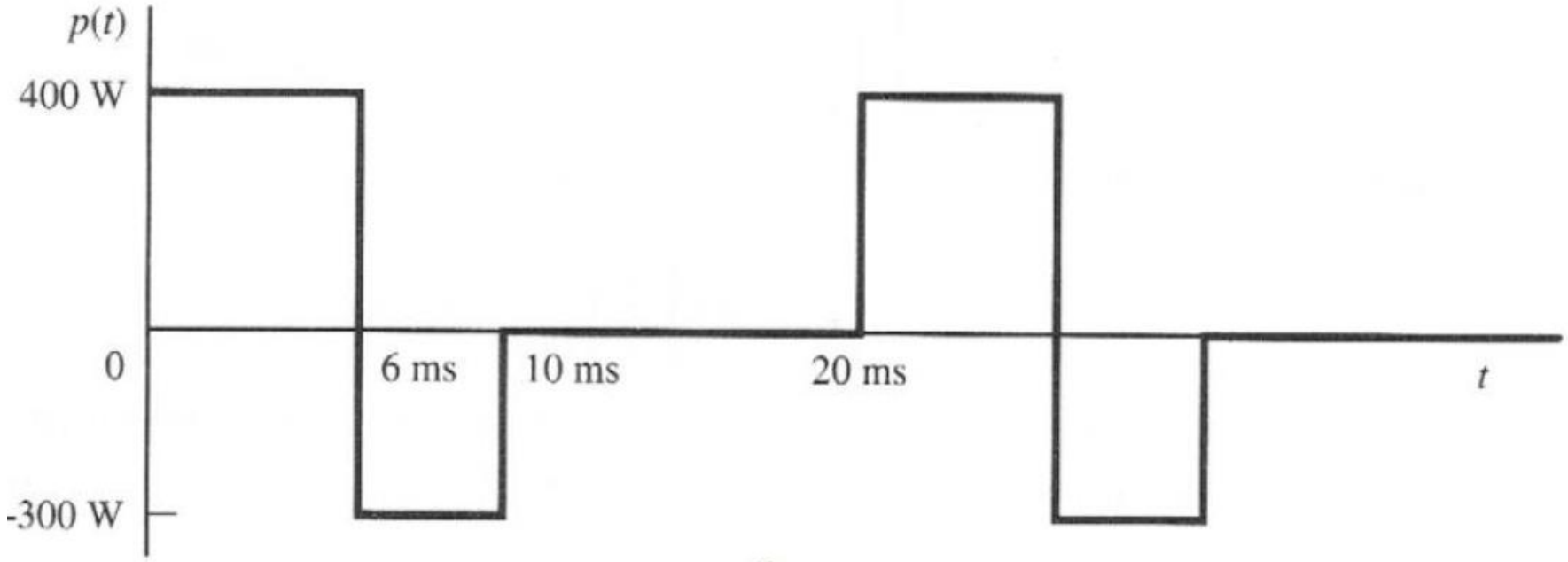
$$i(t) = \begin{cases} 20\ \text{A} & 0 < t < 6\ \text{ms} \\ -15\ \text{A} & 6\ \text{ms} < t < 20\ \text{ms} \end{cases}$$



Example Instantaneous Power

b. The instantaneous power is simply:

$$p(t) = \begin{cases} 400\ \text{W} & 0 < t < 6\ \text{ms} \\ -300\ \text{W} & 6\ \text{ms} < t < 10\ \text{ms} \\ 0 & 10\ \text{ms} < t < 20\ \text{ms} \end{cases}$$



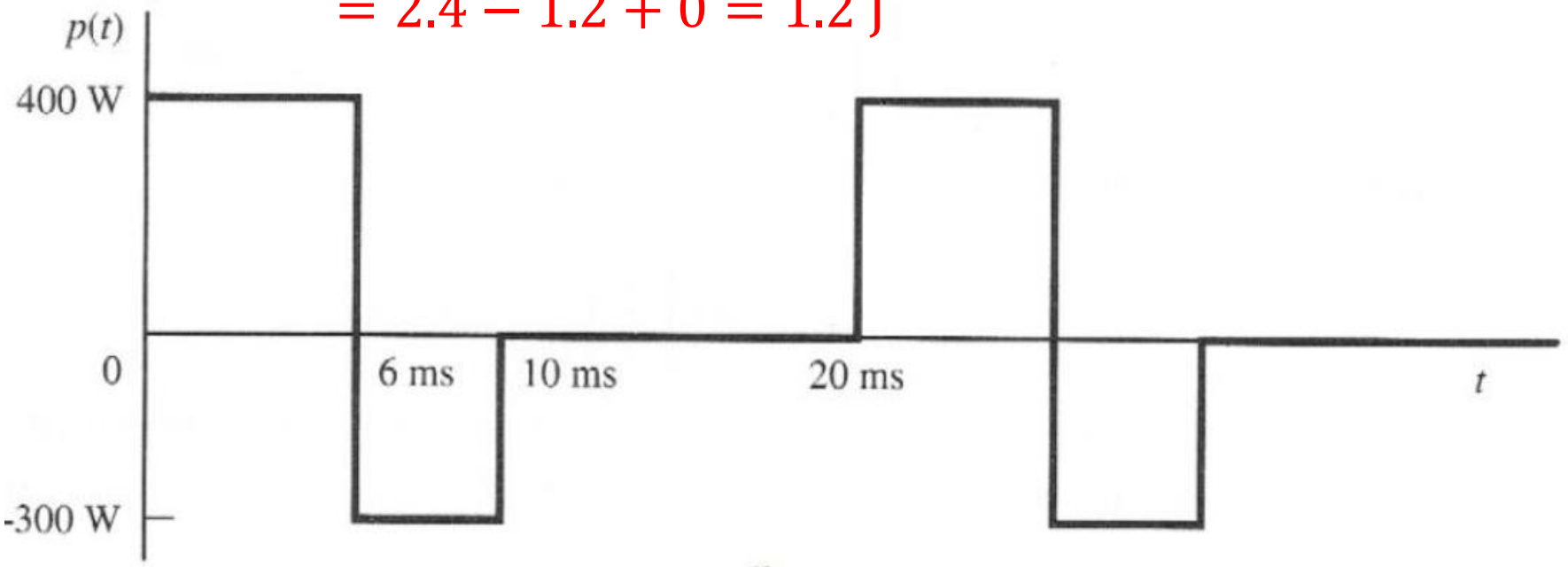
Example Instantaneous Power

c. The energy absorbed by the device in one period is:

$$W = \int_0^t p(t)\,dt$$

$$= \int_0^{0.006} 400\,dt + \int_{0.006}^{0.01} (-300)\,dt + \int_{0.01}^{0.02} (0)\,dt$$

$$= 2.4 - 1.2 + 0 = 1.2 \text{ J}$$



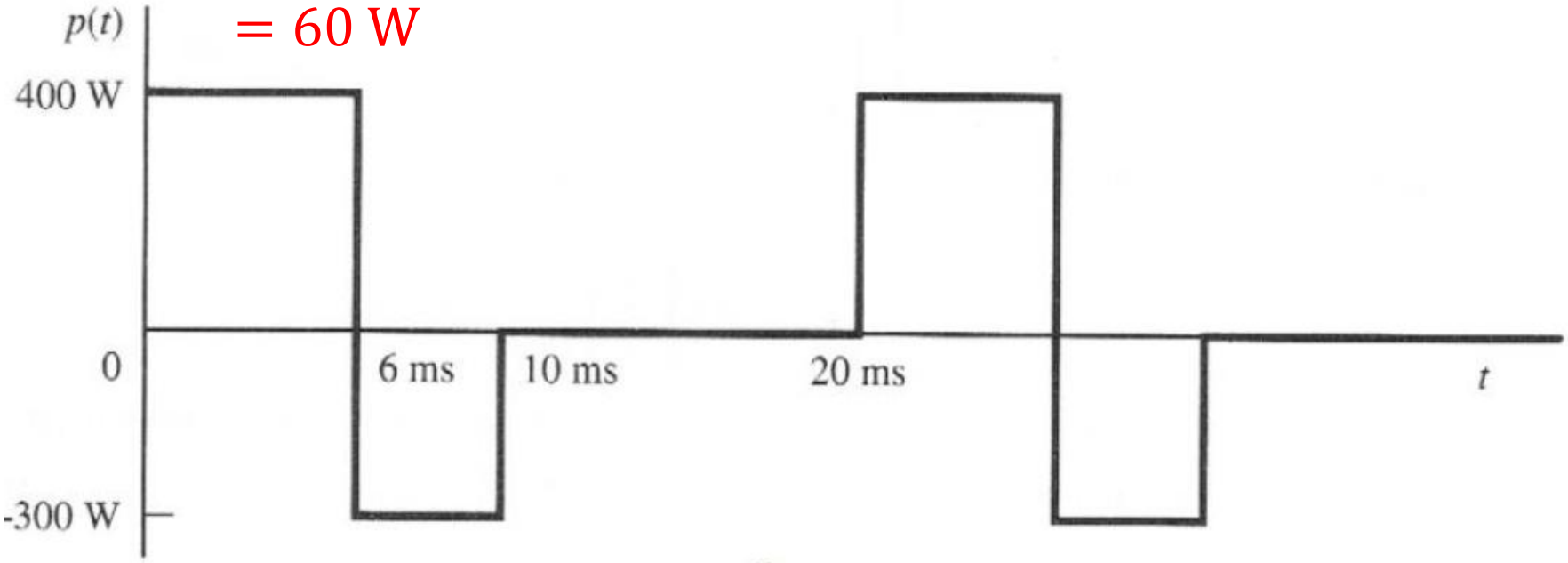
Example Instantaneous Power

d. The average power is:

$$P_{ave} = \frac{W}{T} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= \frac{1}{0.02}\left[\int_0^{0.006} 400\,dt + \int_{0.006}^{0.01} (-300)\,dt + \int_{0.01}^{0.02} (0)\,dt\right]$$

$$= 60 \text{ W}$$



Example AC Power

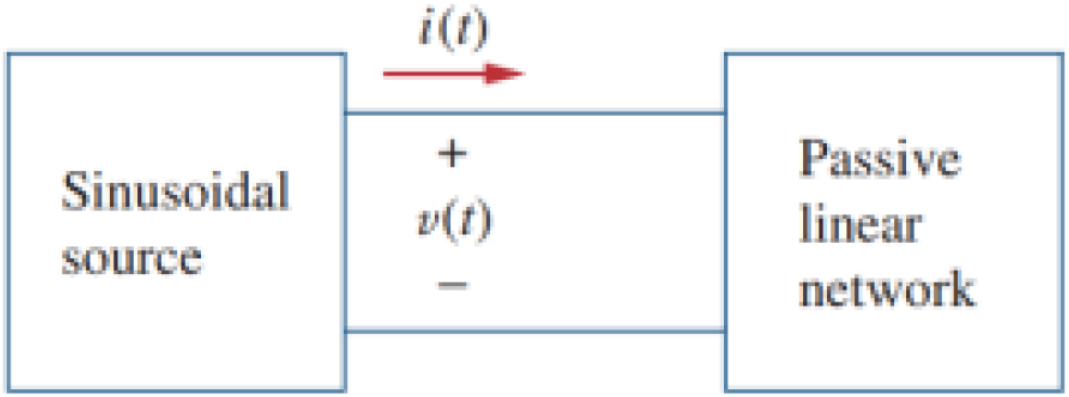
Given the following equations for voltage and current in the circuit:

$$v(t) = 120\cos(377t + 45°)\ \text{V}$$

And

$$i(t) = 10\cos(377t - 10°)\ \text{A}$$

Find the instantaneous power and the average power absorbed by the passive linear network. [10 marks]



Example AC Power

Solution

The instantaneous power is given by:

$$p(t) = v(t)i(t)$$
$$= 120\cos(377t + 45°)\, 10\cos(377t - 10°)$$
$$= 1200\cos(377t + 45°)\cos(377t - 10°)$$

Applying trigonometry identity:

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

This gives:

$$p(t)$$
$$= \frac{1200}{2}[\cos(377t + 45° + 377t - 10°)\cos(377t + 45°$$
$$- (377t - 10°))] = 600[\cos(754t + 35°) + \cos 55°]$$

Example AC Power

Or

$$p(t) = 344.2 + 600\cos(754t + 35°)\ \text{W}$$

The average power is:

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2}(120)(10)\cos[45° - (-10°)]$$

$$= 600\cos 55° = 344.2\ \text{W}$$

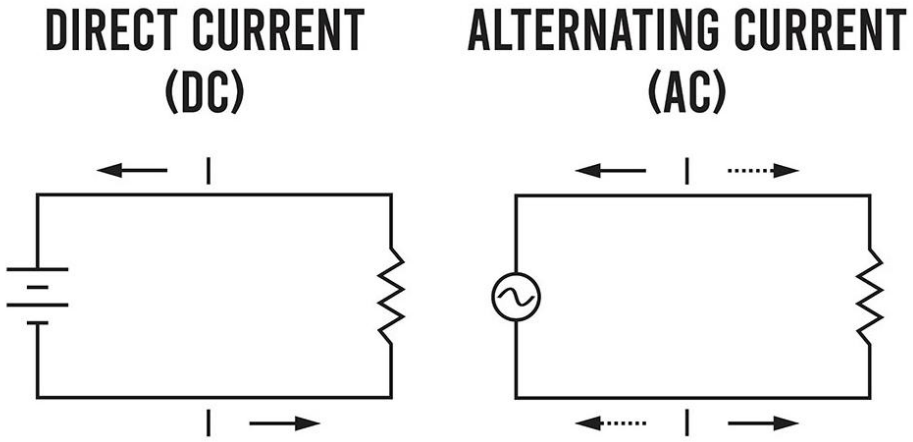
DC Source

- DC voltage with average current:

$$P_{dc} = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\,dt = \frac{1}{T}\int_{t_0}^{t_0+T} V_{dc}\,i(t)dt$$

- If the circuit is purely resistive $v = Ri$

$$P_{ave} = \frac{R}{T}\int_{0}^{T} i^2(t)\,dt$$



Root Mean Square (RMS) Power

- For the RMS current I, the average power is:

$$P_{ave} = RI_{RMS}^2$$

- Hence the term 'RMS':

$$P_{ave} = \frac{R}{T}\int_0^T i^2(t)\,dt$$

- Thus:

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

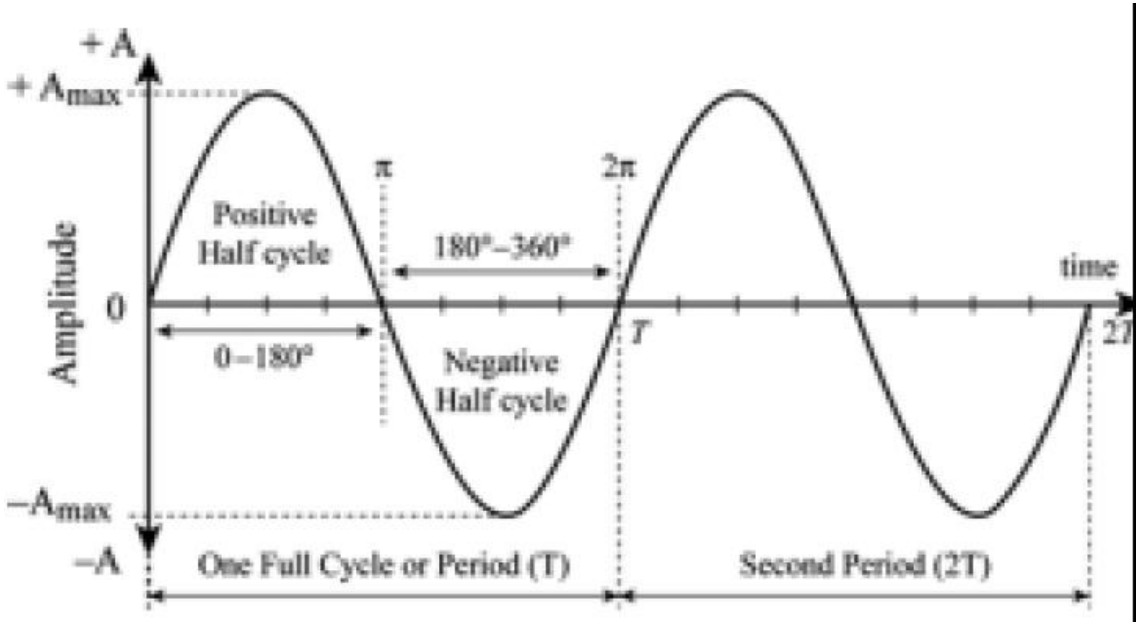
AC Power Quantities - Averages

- Average power comes from averaging the instantaneous power $p(t)$ over one period of the AC waveform.

$$p(t) = [V_m \cos(2\pi f t + \theta_v)][I_m \cos(2\pi f t + \theta_i)]$$

- Thus

$$p(t) = \frac{V_m I_m}{2}\left[\cos(\theta_v - \theta_i) + \cos\left(4\pi f t + (\theta_v - \theta_i)\right)\right]$$



AC Power Quantities - Averages

- As a result, the average power is:

$$P_{ave} = \frac{V_m I_m}{2}[\cos(\theta_v - \theta_i)]$$

  It is good for mathematicians.

- We do not use the peak values in AC systems.
- We use RMS value.
- For sine wave $V_{RMS} = V_m/\sqrt{2} = 0.707V_m$, thus:

$$P_{ave} = V_{RMS} I_{RMS} \cos(\theta_v - \theta_i)$$

RMS from Peak Values

The following table shows the conversion of the peak quantities to the RMS value.

Waveform	**Equation**
Rectangular pulse (duty cycle D)	$V_{RMS} = V_m\sqrt{D}$
Sinusoids	$V_{RMS} = V_m/\sqrt{2}$
Full-wave rectified sinusoids	$V_{RMS} = V_m/\sqrt{2}$
Half-wave rectified sinusoids	$V_{RMS} = V_m/2$
Sum of different frequency sinusoids	$V_{RMS} = \sqrt{V_{1,RMS}^2 + V_{2,RMS}^2 + \cdots}$
Triangular wave	$V_{RMS} = V_m/\sqrt{3}$

Note: Substitute I for V to determine currents.

Sum of Periodic Waveforms

- If periodic voltage is the sum of two periodic voltage waveforms $v_1(t) + v_2(t)$:

$$V_{RMS}^2 = \frac{1}{T}\int_0^T (v_1 + v_2)^2 dt$$

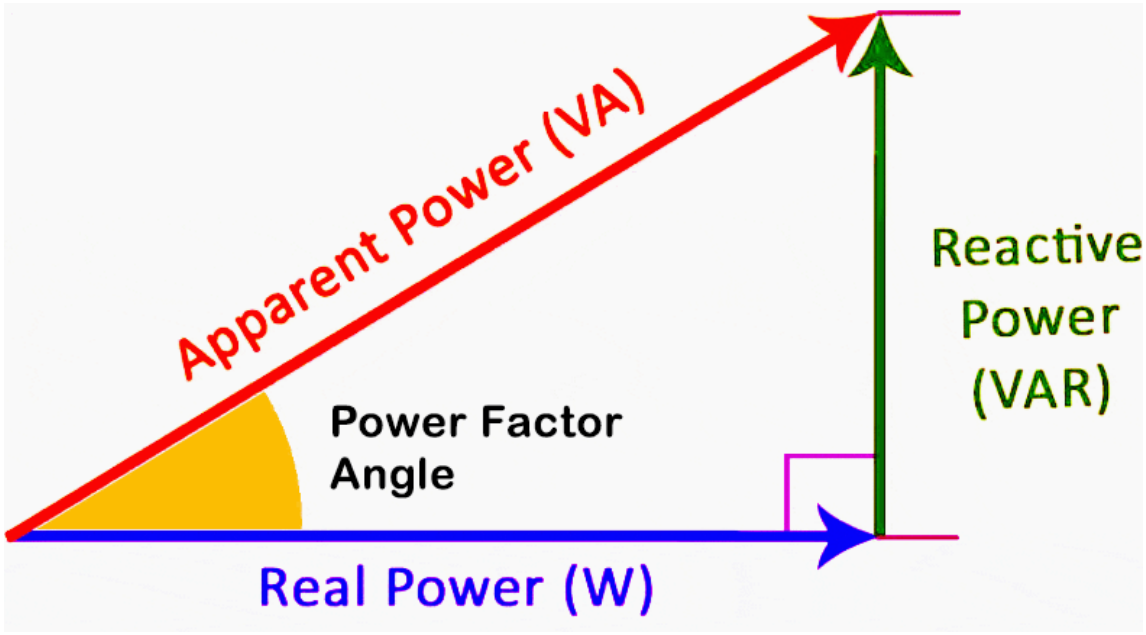
$$= \frac{1}{T}\int_0^T v_1^2\, dt + \frac{1}{T}\int_0^T 2v_1 v_2\, dt + \frac{1}{T}\int_0^T v_2^2\, dt$$

- If the voltages are of two different frequencies (orthogonal), then the term $v_1 v_2$ in the equation above is zero.

Introduction to Electrical Power

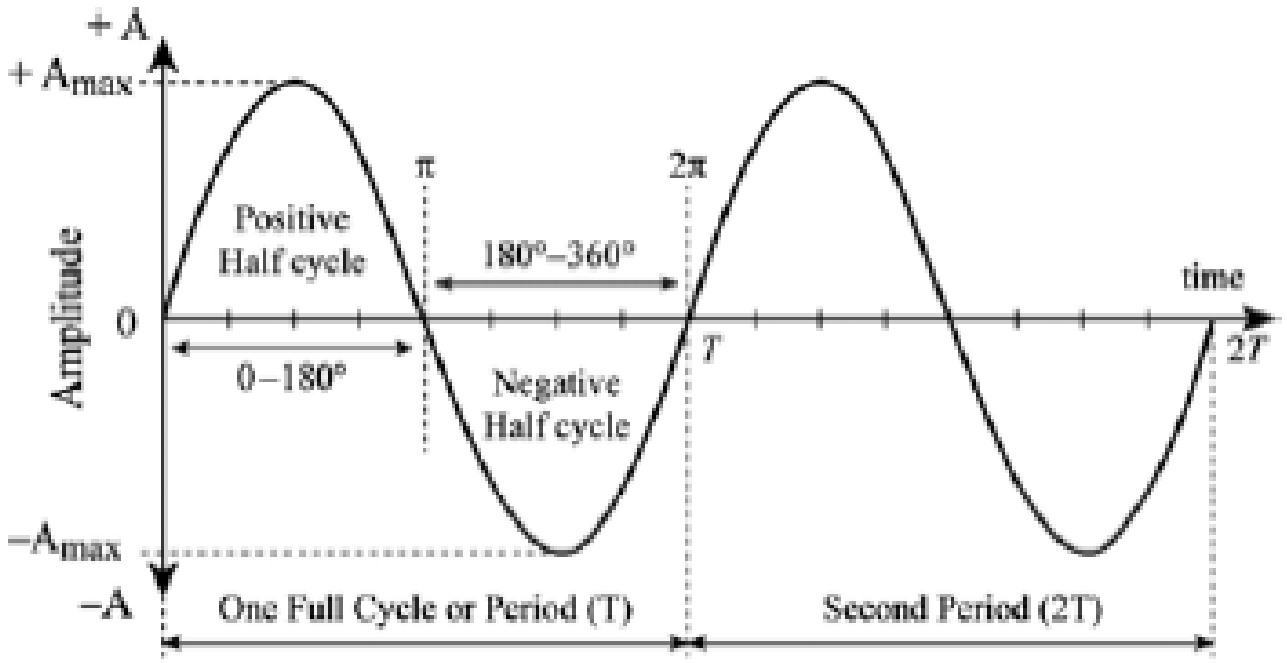
- In practice, we recognise that there are real power (P), reactive power (Q), and apparent power (S).

$$S = \sqrt{P^2 + Q^2}$$



Real Electrical Power (P)

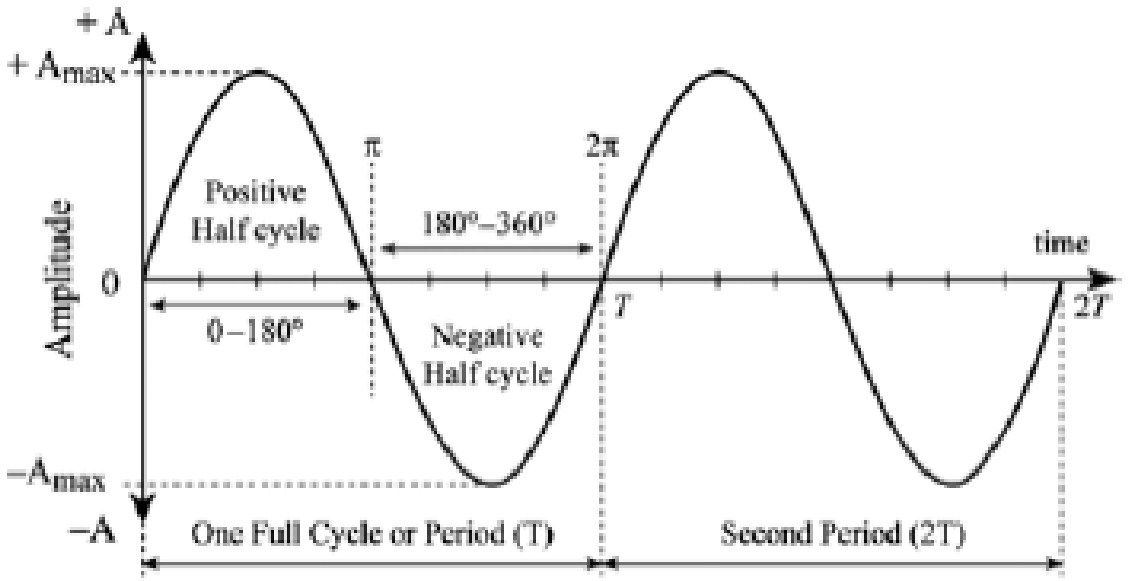
- The P has the unit of Watts and the unit symbol W.
- The P is sometimes referred to as average power or real power, but much more commonly, it's just plain power.
- If someone says simply, "power," it means P.



Real Electrical Power (P)

$$p(t) = \frac{V_m I_m}{2}\left[\cos\theta_v - \theta_i + \cos\left(4\pi f t + (\theta_v - \theta_i)\right)\right]$$

- This second term is still power, even though its average is zero.
- This is the power that flows one way for a quarter of a cycle, then the other way for a quarter of a cycle.
- From this second term, we define a term called the reactive power Q.



Reactive Electrical Power (Q)

- The Q delivers no energy on average.
- Represents the energy that flows forwards and backwards.
- No net flow of energy but accounts for part of the current flowing in the system – hence we need to include in our calculations.
- The Q is positive for loads that are primarily inductive and negative if the loads are primarily capacitive.
- Its units are VAR (volt-amp-reactive):

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) = V_{RMS} I_{RMS} \sin(\theta_v - \theta_i)$$

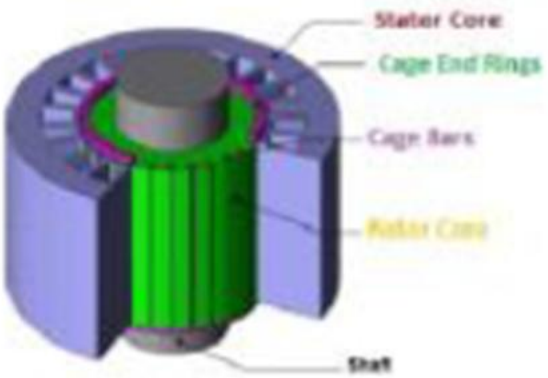
YouTube video link:

https://www.youtube.com/watch?v=VFPwgjGHqFQ

Why Q is Important?

- Take induction motor for example: Needs current to supply the magnetic field that makes the motor work.

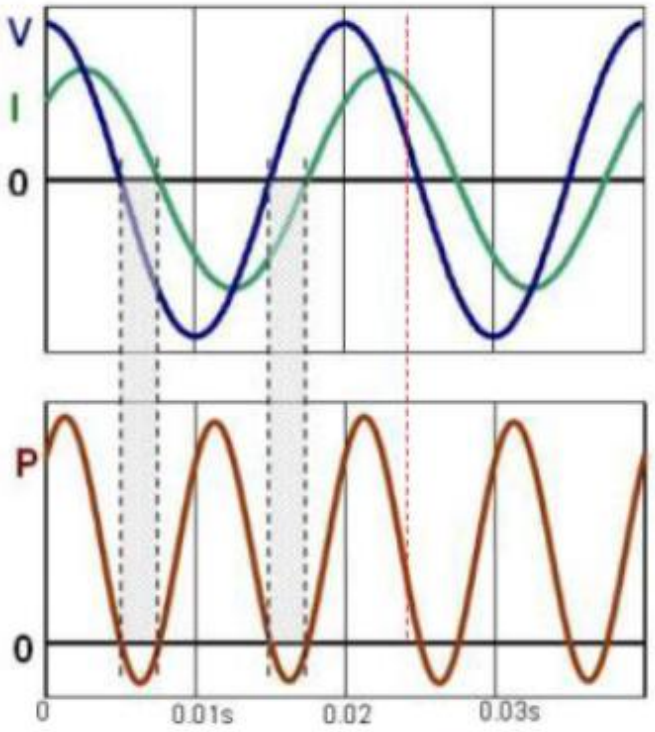




- Just P cannot supply this current i.e. hence we need Q.
- The increase in current will increase losses. Hence, we need to quantify Q.

Why Q is Important?

- If you include an inductor or a capacitor, your circuit is consuming Q (i.e. through the tracks/wires and coils).
- More on topic of the reactive power Q in later courses.

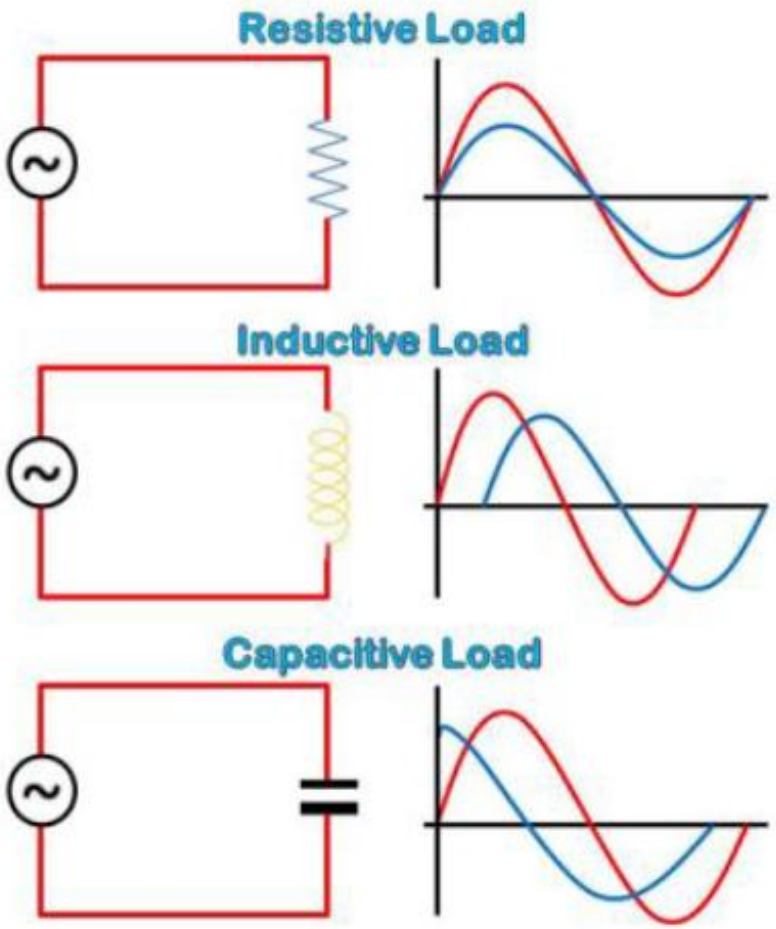


Power Factor (PF)

- Defined as the cosine of the angle between the voltage and the current.

$$pf = \cos(\theta_v - \theta_i)$$

- Usually in the form of ‘leading’ or ‘lagging’ as the cosine value will always produce a positive value (i.e. 0 to 1).



Power Factor (PF)

- How do we know lead or lag?
- If the load is inductive, current lags the voltage in time.
- As a result, angle of current will be less than voltage angle.
- Then, the current lags voltage.
- Thus, power factor is lagging.
- So, if load is capacitive, the power factor is leading as current leads.

Complex or Apparent Electrical Power (S)

- Another power quantity that is useful is complex power S:

$$S = P + jQ$$

- Unit of complex power is volt-ampere (VA).
- The S is a complex number with a real part of P and an imaginary part of Q.
- The S is also the product of phasor voltage and phasor current:

$$S = VI^*$$

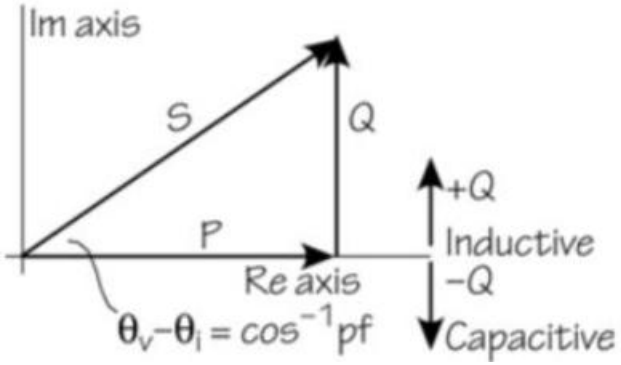
Note: Asterisk indicates the complex conjugate.

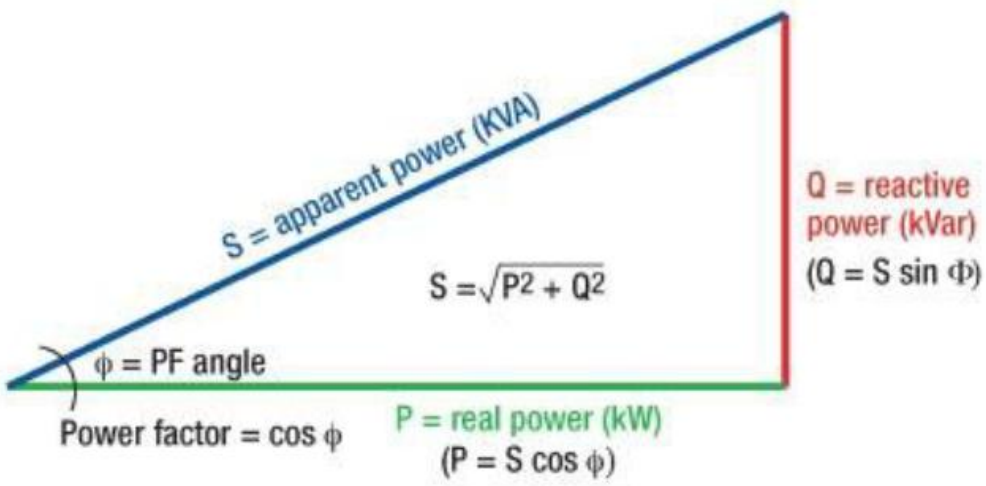
- Power factor (pf) is defined as the ratio of average power to apparent power:

$$pf = \frac{P}{|S|} \quad \text{or} \quad pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

Power Triangle

- The figure showing below shows an easy way to visualize the relationship among P, Q, S, and power factor.
- Apparent power is calculated from real pow and reactive power.





$$\phi = \theta_v - \theta_i$$

Apparent Power (S) – Summary

- Apparent Power S is magnitude of the complex power:

$$S = P + jQ$$

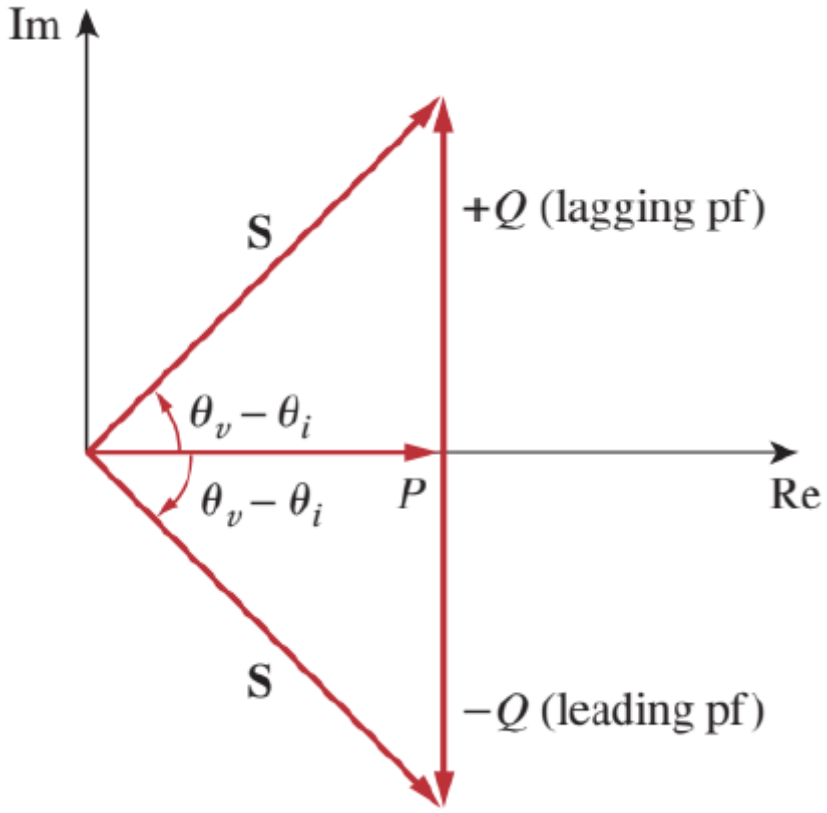
Where:

$$S = |S| = \sqrt{P^2 + Q^2}$$

- Thus

$$S = V_{RMS} I_{RMS}$$

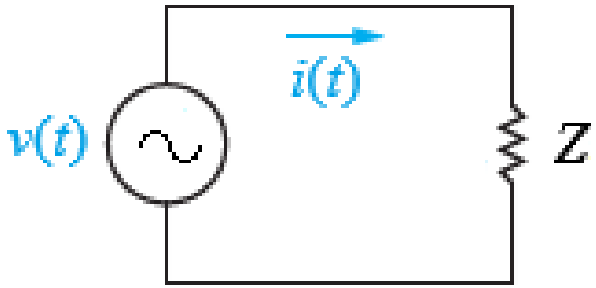
$$\angle S = \tan^{-1}\left(\frac{Q}{P}\right) = pf$$



- Unit of apparent power S is VA, reactive power Q is Var, and real power P is Watt.

Example of Apparent Power and PF

A series-connected load draws a current $i(t) = 4\cos(100\pi t + 10°)$ A when the applied voltage is $v(t) = 120\cos(100\pi t - 20°)$ V.



a. Find the apparent power and the power factor of the load.

[4 marks]

b. Determine the power factor using the load impedance.

[4 marks]

c. Determine the element values that form the series-connected load.

[4 marks]

Example of Apparent Power and PF

Solution

a. The apparent power is:

$$S = V_{RMS} I_{RMS} = \left(\frac{120}{\sqrt{2}}\right)\left(\frac{4}{\sqrt{2}}\right) = 240 \text{ VA}$$

The power factor is:

$$\begin{aligned} pf &= \cos(\theta_v - \theta_i) \\ &= \cos(-20° - 10°) \\ &= 0.866 \text{ (leading)} \end{aligned}$$

The pf is leading because the current leads the voltage.

Example of Apparent Power and PF

b. The pf may also be obtained from the load impedance:

$$Z = \frac{V}{I} = \frac{120\angle -20°}{4\angle 10°} = 30\angle -30° = 25.98 - j15\ \Omega$$

Thus

$$pf = \cos(-30°) = 0.866\ (\text{leading})$$

c. The load impedance Z can be modelled by a 25.98 Ω resistor in series with a capacitor with:

$$X_C = -\frac{1}{\omega C} = -15$$

Assuming the line frequency is 50 Hz, thus:

$$C = \frac{1}{15(2\pi f)} = \frac{1}{15(100\pi)} = 212.2\ \mu\text{F}$$

Reactive Power

- For a given electrical system

$$P = V_{RMS} I_{RMS} \cos(\theta_v - \theta_i)$$

- And

$$Q = V_{RMS} I_{RMS} \sin(\theta_v - \theta_i)$$

- The P is the actual power dissipated by the load i.e. the useful power.
- The Q is energy exchange between source and reactive part of the load.
  - The $Q = 0$ for resistive loads (unity pf).
  - The $Q < 0$ for capacitive loads (leading pf).
  - The $Q > 0$ for inductive loads (lagging pf).

Summary of Electrical Powers

- Complex power:

$$S = P + jQ = V_{RMS}(I_{RMS})^* = |V_{RMS}||I_{RMS}|\angle(\theta_v - \theta_i)$$

- Apparent power:

$$S = |S| = |V_{RMS}||I_{RMS}| = \sqrt{P^2 + Q^2}$$

- Real power:

$$P = \mathrm{Re}(S) = S\cos(\theta_v - \theta_i)$$

- Reactive power:

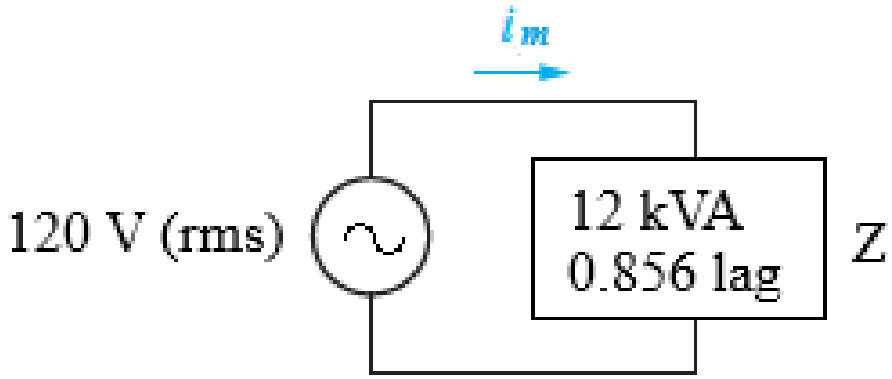
$$Q = \mathrm{Im}(S) = S\sin(\theta_v - \theta_i)$$

- Power factor:

$$\frac{P}{S} = \cos(\theta_v - \theta_i)$$

Example of Load Power and PF

A load Z draws 12 kVA at a power factor of 0.856 lagging from a 120-V rms sinusoidal source.



Calculate:

a. the average and reactive powers delivered to the load, [6 marks]

b. the peak current, and [6 marks]

c. the load impedance. [2 marks]

Example of Load Power and PF

Solution

a. Given that $pf = \cos\theta = 0.856$, we obtain the power angle as:

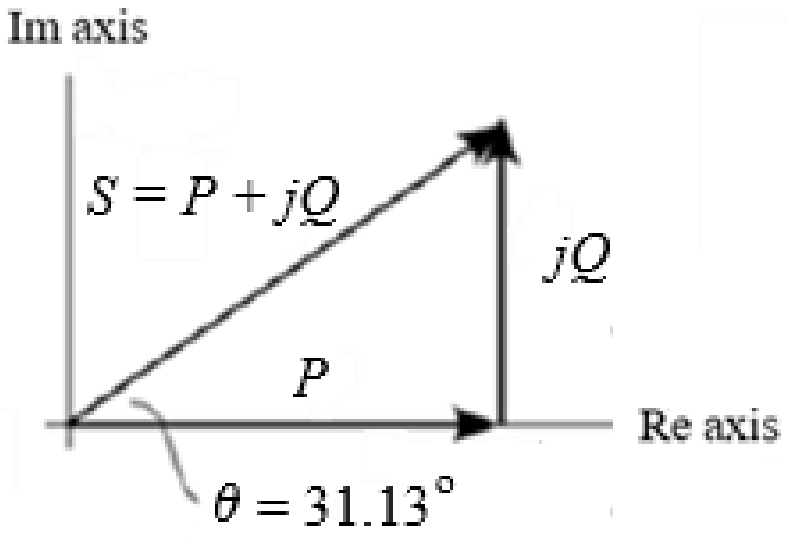
$$\theta = \cos^{-1} 0.856 = 31.13°$$

If the apparent power is S = 12,000 VA, then the average or real power is:

$$P = S\cos\theta$$
$$= (12 \times 10^3)(0.856)$$
$$= 10,272 \text{ kW}$$

While the reactive power is:

$$Q = S\sin\theta$$
$$= (12 \times 10^3)(0.517)$$
$$= 6.204 \text{ kVA}$$

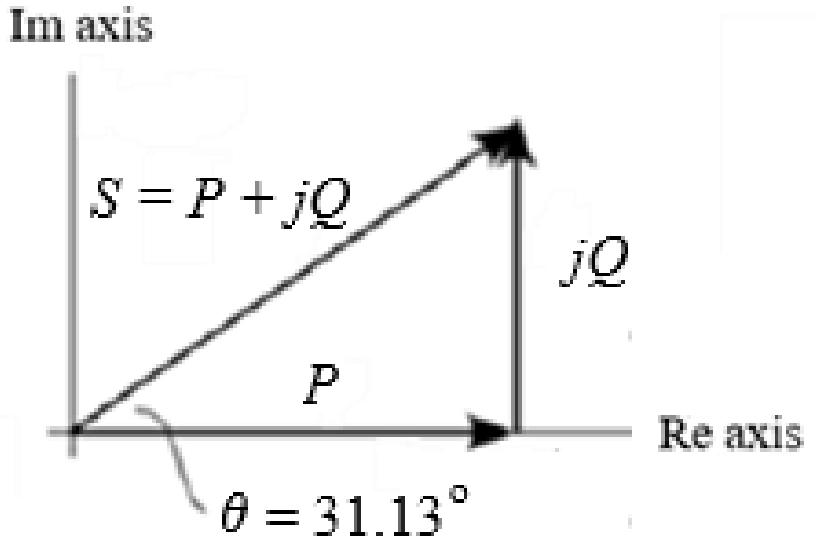


Example of Load Power and PF

b. Since the pf is lagging, the complex power is:

$$S = P + jQ$$

$$= 10.272 + j6.204 \text{ kVA}$$



From $S = V_{RMS}I_{RMS}^*$, we obtain:

$$I_{RMS}^* = \frac{S}{V_{RMS}}$$

$$= \frac{10{,}272 + j6{,}204}{120\angle 0°}$$

$$= 85.6 + j51.7 \text{ A} = 100\angle 31.13° \text{ A}$$

Thus, $I_{RMS} = 100\angle -31.13°$ and the peak current is:

$$I_m = \sqrt{2}I_{RMS} = \sqrt{2}(100) = 141.4 \text{ A}$$

Example of Load Power and PF

c. The load impedance is:

$$Z = \frac{V_{RMS}}{I_{RMS}}$$

$$= \frac{120\angle 0°}{100\angle -31.13°} = 1.2\angle 31.13° \ \Omega$$

Thus, this is an inductive impedance.

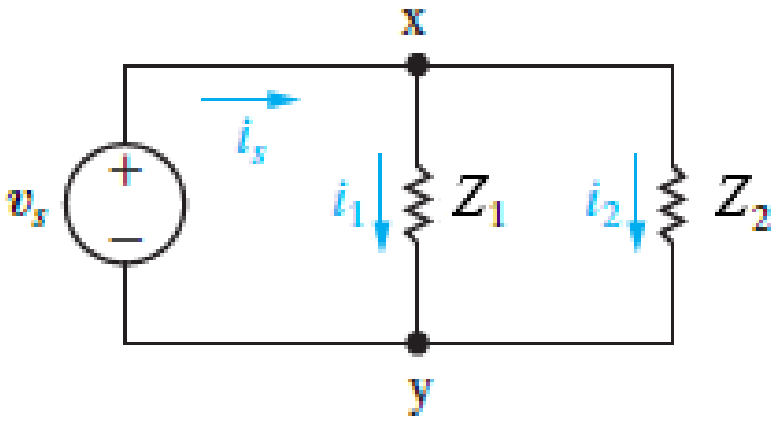
Principle of Power Conservation

- Principle of conservation of power applies to AC systems as well.
- For the parallel configuration as shown below:

$$I = I_1 + I_2$$

- Thus

$$S = VI^* = V(I_1^* + I_2^*) = VI_1^* + VI_2^* = S_1 + S_2$$



Principle of Power Conservation

- For series configuration as shown below:

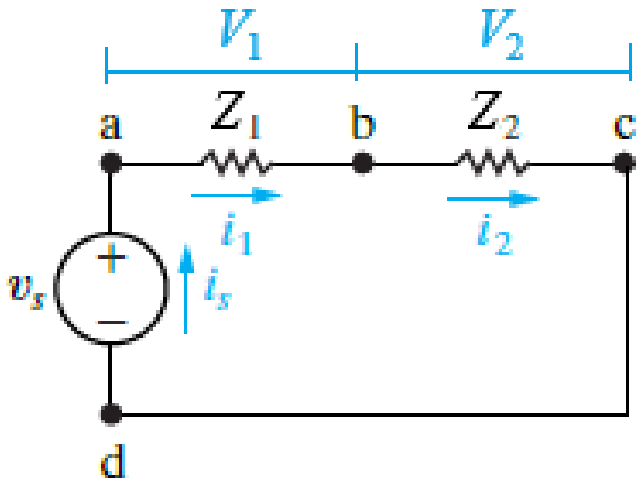
$$V = V_1 + V_2$$

- Thus

$$S = VI^* = (V_1 + V_2)I^* = V_1 I^* + V_2 I^* = S_1 + S_2$$

- Whether the loads are connected in series or in parallel, the total power supplied by the source equals the total power delivered to the load:

$$S = S_1 + S_2 + \cdots + S_N$$

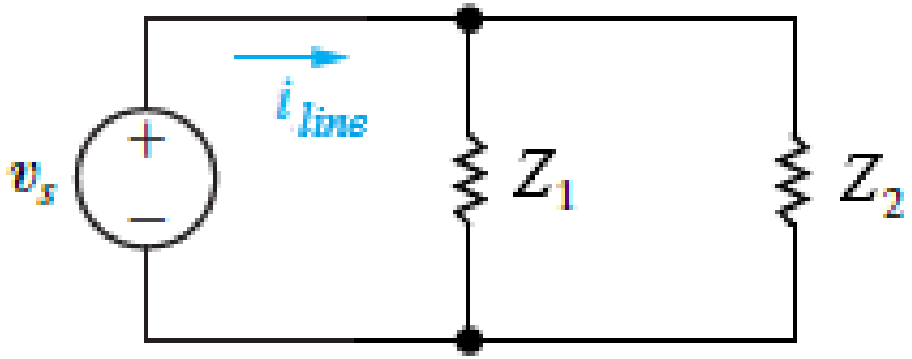


Example of Electrical Distribution Load

As shown in the figures below, the electricity system supplied from a source of 480 V has two loads:

- Load 1, Z_1: P = 20 kW (0.86 lagging).
- Load 2, Z_2: S = 20 kVA (0.72 lagging).

Find total power, reactive power, apparent power, and the line current. [20 marks]



Example of Electrical Distribution Load

Solution

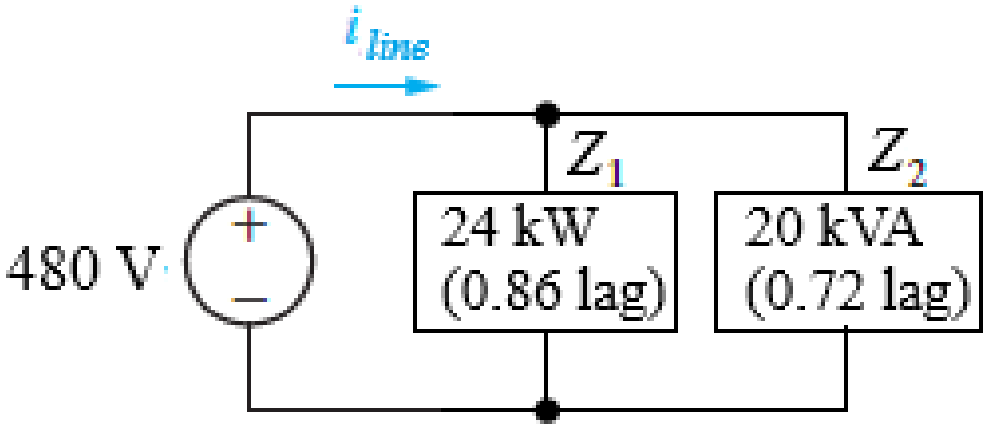
Load 1:

The power in the first load is:

$$P_1 = 24 \text{ kW (with a power factor of 0.86 lagging)}$$

If we look at the power triangle and remember that the tangent of the angle is Q/P, then the reactive power is:

$$Q_1 = P_1 \tan(\cos^{-1} pf) = 24 \tan(\cos^{-1} 0.86) = 14.24 \text{ kVAR}$$



Example of Electrical Distribution Load

Load 2:

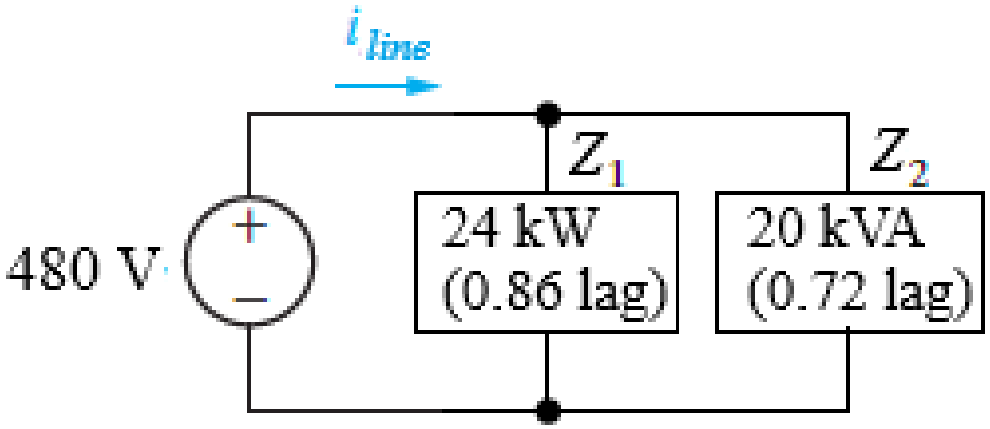
The power in the second load is:

$$P_2 = |S_2| pf_2 = (20)(0.72) = 14.4 \text{ kW}$$

And

$$Q_2 = |S_2| \sin(\cos^{-1} pf_2) = (20)(0.694) = 13.88 \text{ kVar}$$

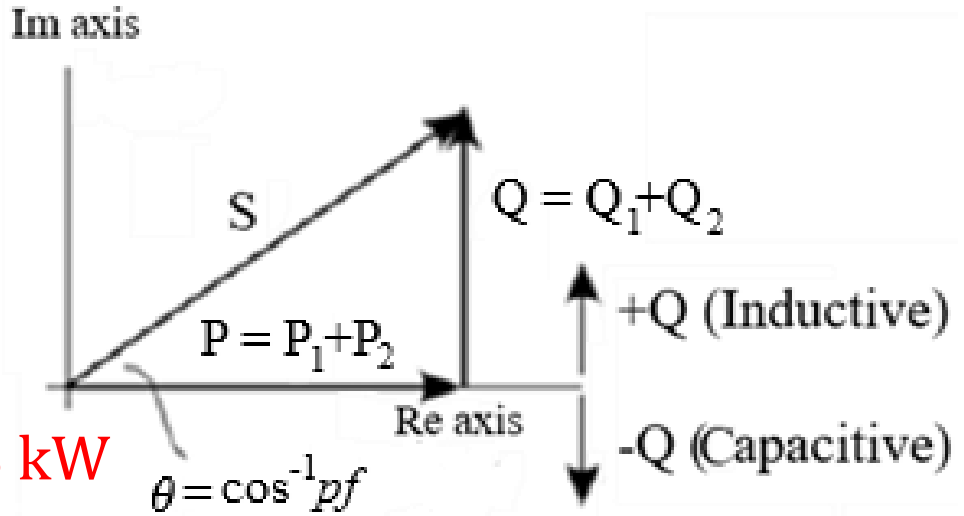
The power triangle tells us that Q and S are related by the sine of the power-factor angle.



Example of Electrical Distribution Load

Conservation principle for P, S and Q:

$$P_{total} = P_1 + P_2$$
$$= 24\ \text{kW} + 14.4\ \text{kW}$$
$$= 38.4\ \text{kW}$$



$$Q_{total} = Q_1 + Q_2$$
$$= 14.24\ \text{kW} + 13.88\ \text{kW}$$
$$= 28.12\ \text{kVar}$$

$$S_{total} = P_{total} + jQ_{total} = 38.4\ \text{kW} + \text{j}28.12\ \text{kVAR}$$
$$= 47.6\angle 36.2°\ \text{kVA}$$

$$pf_{total} = \frac{P_{total}}{|S_{total}|} = \frac{38.4\ \text{kW}}{47.6\ \text{kVA}} = 0.807\ \text{(lagging)}$$

Example of Electrical Distribution Load

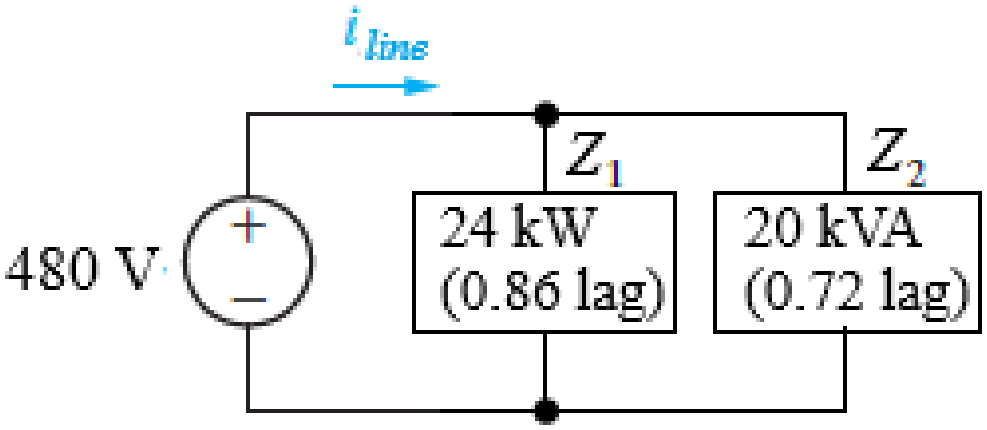
The current is calculated from $S = VI^*$:

$$I_{line}^* = \frac{S_{total}}{V_{line}}$$

$$= \frac{47.6 \times 10^3 \angle 36.2°}{480} = 99.2 \angle 36.2° \text{ A}$$

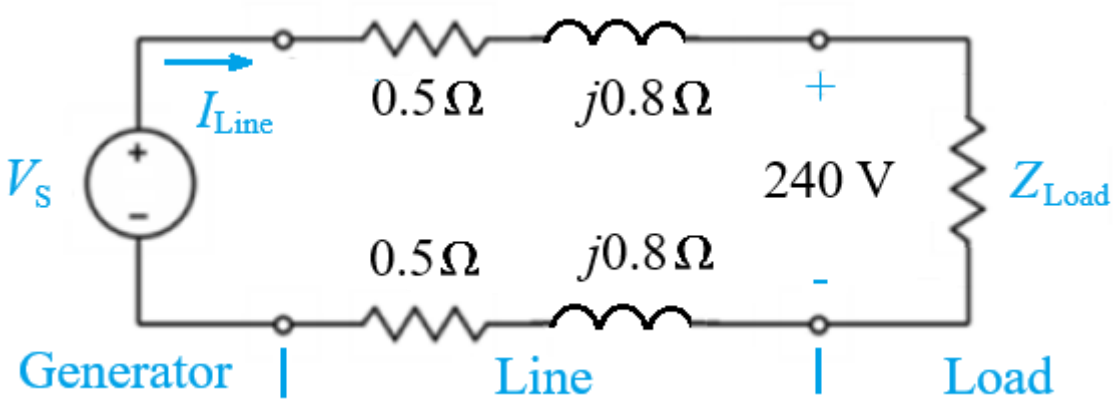
Note the sign, thus

$$I_{line} = 99.2 \angle -36.2° \text{ A}$$



Example of Electrical Motor Load

For a 2 HP motor load with 0.78 lagging power factor and 85% efficiency, find the power delivered to the motor, the line current, the power lost in the line, the total P, Q, and S, the power factor at the generator, the percent voltage regulation, and the percent efficiency. [24 marks]



Example of Electrical Motor Load

Solution

If 1 HP is 746 W and the motor is 85% efficient, its input power is:

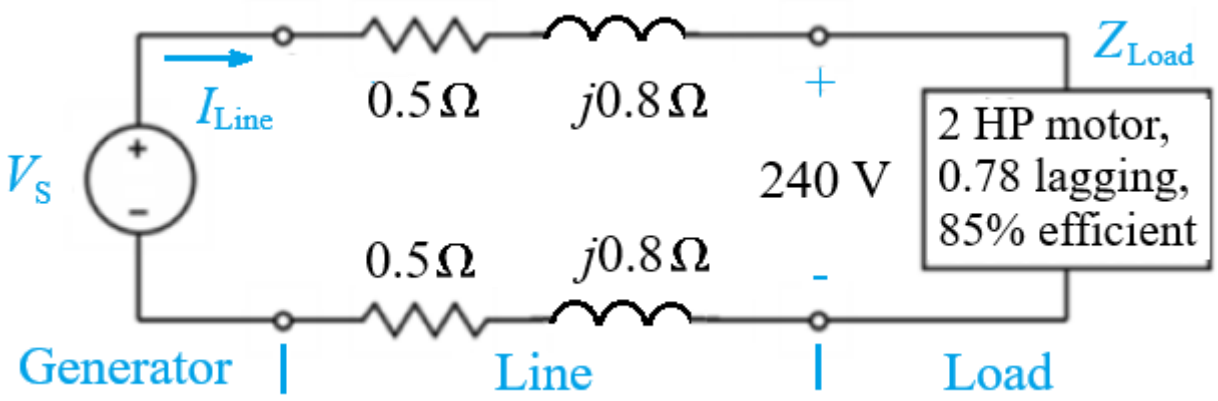
$$P_{load} = \frac{P}{\eta} = \frac{2(746)}{0.85} = 1.755\ \text{kW}$$

The reactive power input to the motor is:

$$Q_{load} = 1.755 \tan(\cos^{-1} 0.78) = 1.408\ \text{kVar}$$

The line current is calculated from:

$$P_{load} = |V||I_{line}|(pf_{load})$$

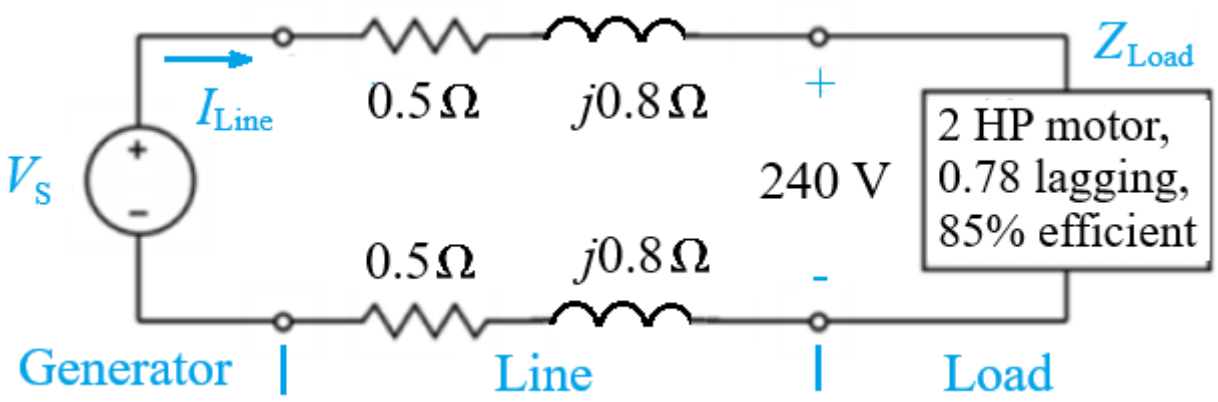


Example of Electrical Motor Load

So

$$I_{line} = \frac{P_{Load}}{V_{load}(pf_{load})} \angle -\cos^{-1} pf$$

$$= \frac{1.755 \times 10^3}{(240)(0.78)} \angle -\cos^{-1} 0.78 = 9.375 \angle -38.74° \text{ A}$$



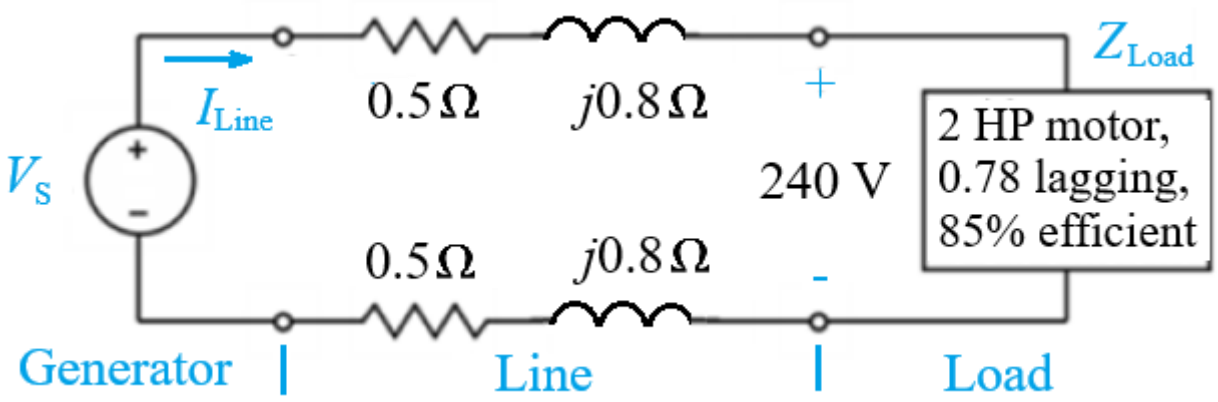
The line power loss is mainly resistive part:

$$P_{line} = R_{line}|I_{line}|^2 = (2 \times 0.5\ \Omega)(9.375\ \text{A})^2 = 87.9\ \text{W}$$

Example of Electrical Motor Load

The generator's voltage using KVL:

$$V_s = V_{load} + V_{line}$$
$$= V_{load} + (R_{line} + jX_{line})I_{line}$$
$$= 240 + (0.5 + j0.8 + 0.5 + j0.8)(9.375\angle -38.74°)$$
$$= 240 + (1 + j1.6)(9.375\angle -38.74°)$$
$$= 240 + (1.887\angle 40°)(9.375\angle -38.74°)$$
$$= 256.8\angle 1.26° \text{ V}$$



Example of Electrical Motor Load

We need to get Q and S. Beginning with apparent power:

$$S = VI^*$$

Thus

$$S_s = V_s I_s = (256.8\angle 1.26°)(9.375\angle 38.74°) = 2.408\ \text{kVA}\angle 40.0°$$

$$P_s = \text{Re}[S_s]$$

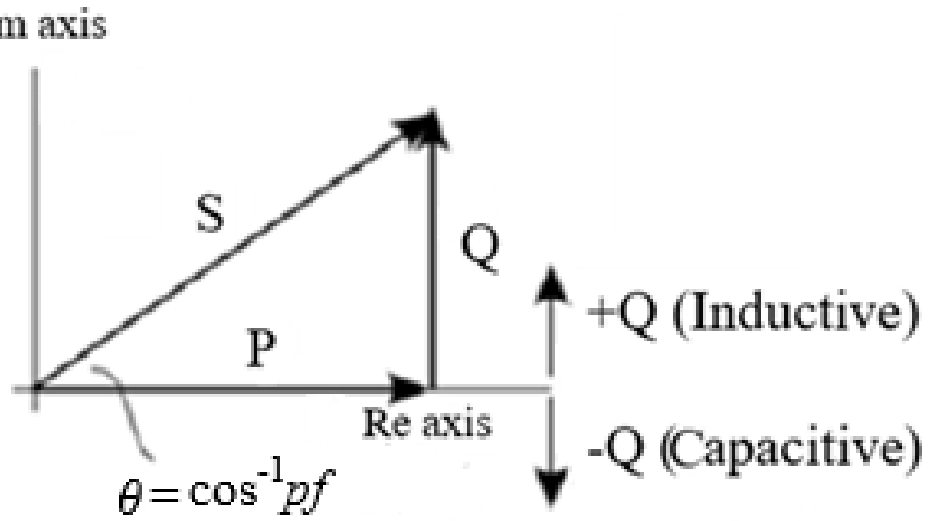
$$= 2.408\ \text{kVA} \times \cos 40°$$

$$= 1.844\ \text{kW}$$

$$Q_s = \text{Im}[S_s]$$

$$= 2.408\ \text{kVA} \times \sin 40°$$

$$= 1.548\ \text{kVar}$$



$$pf_s = \cos(40.0°) = 0.766\ \text{(lagging)}$$

Example of Electrical Motor Load

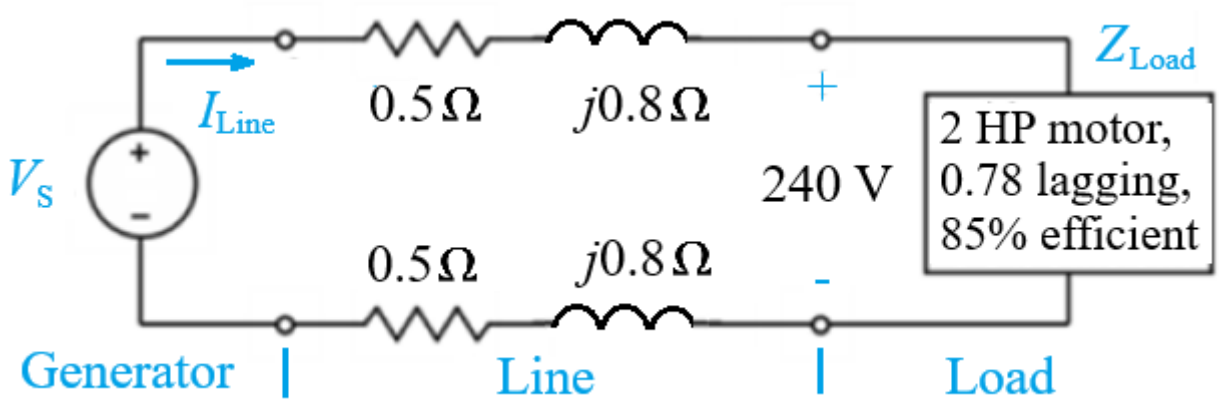
Now, the voltage regulation (i.e. voltage increases as loss of load):

$$\%VR = \left(\frac{|V_s| - |V_{load}|}{|V_{load}|}\right) \times 100\% = \left(\frac{256.8 - 240}{240}\right) \times 100\%$$

$$= 7.0\%$$

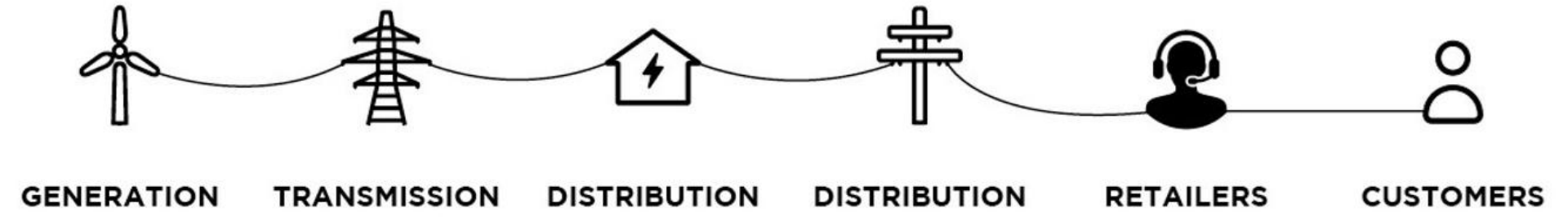
The percent efficiency is:

$$\%\eta = \left(\frac{P_{load}}{P_s}\right) \times 100\% = \left(\frac{1.755}{1.844}\right) \times 100\% = 95.2\%$$



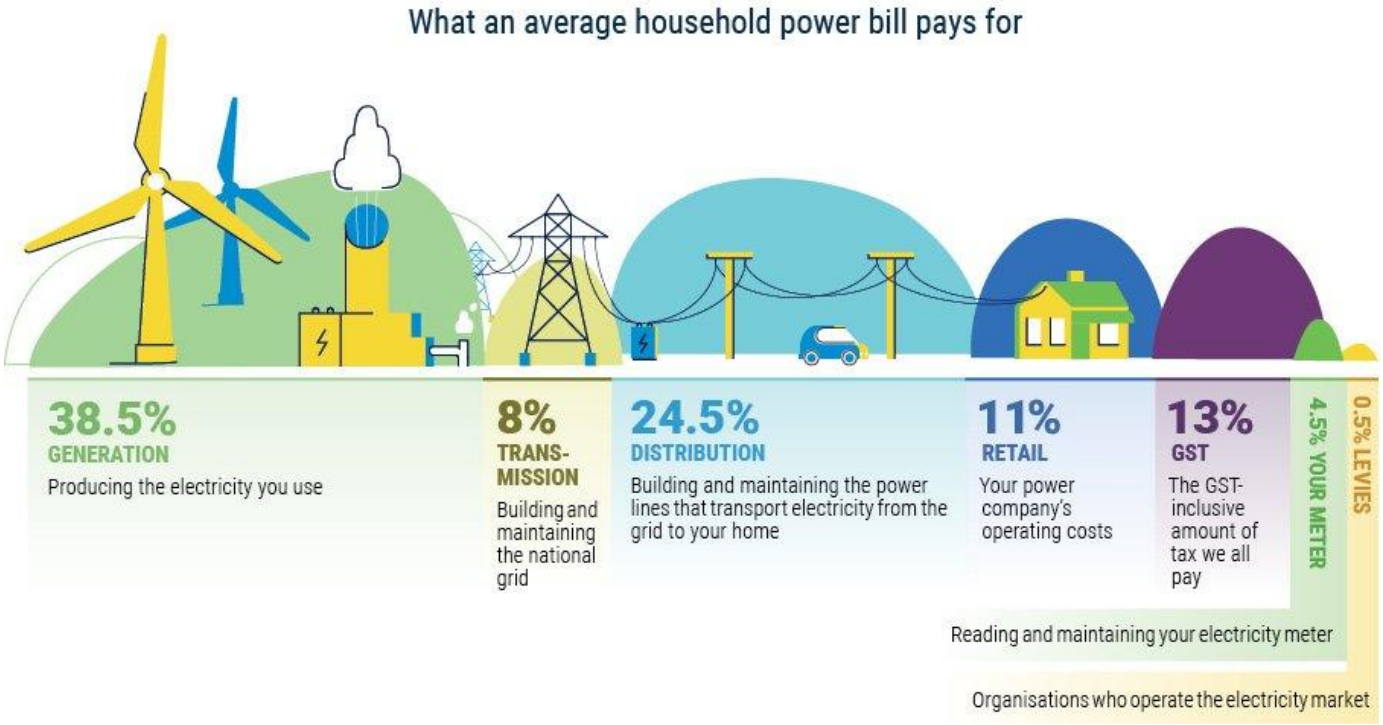
Electricity Consumption Cost

- Costing of electricity depends on generation, transmission and distribution costs and retail costs.
- The cost is also heavily influenced by fluctuating fuel prices (coal, gas), supply/demand imbalances, and weather.
- Key items considered in calculating the power bills include infrastructure investment, maintenance, regulatory policies, and the mix of energy sources, such as hydropower or fossil fuels.

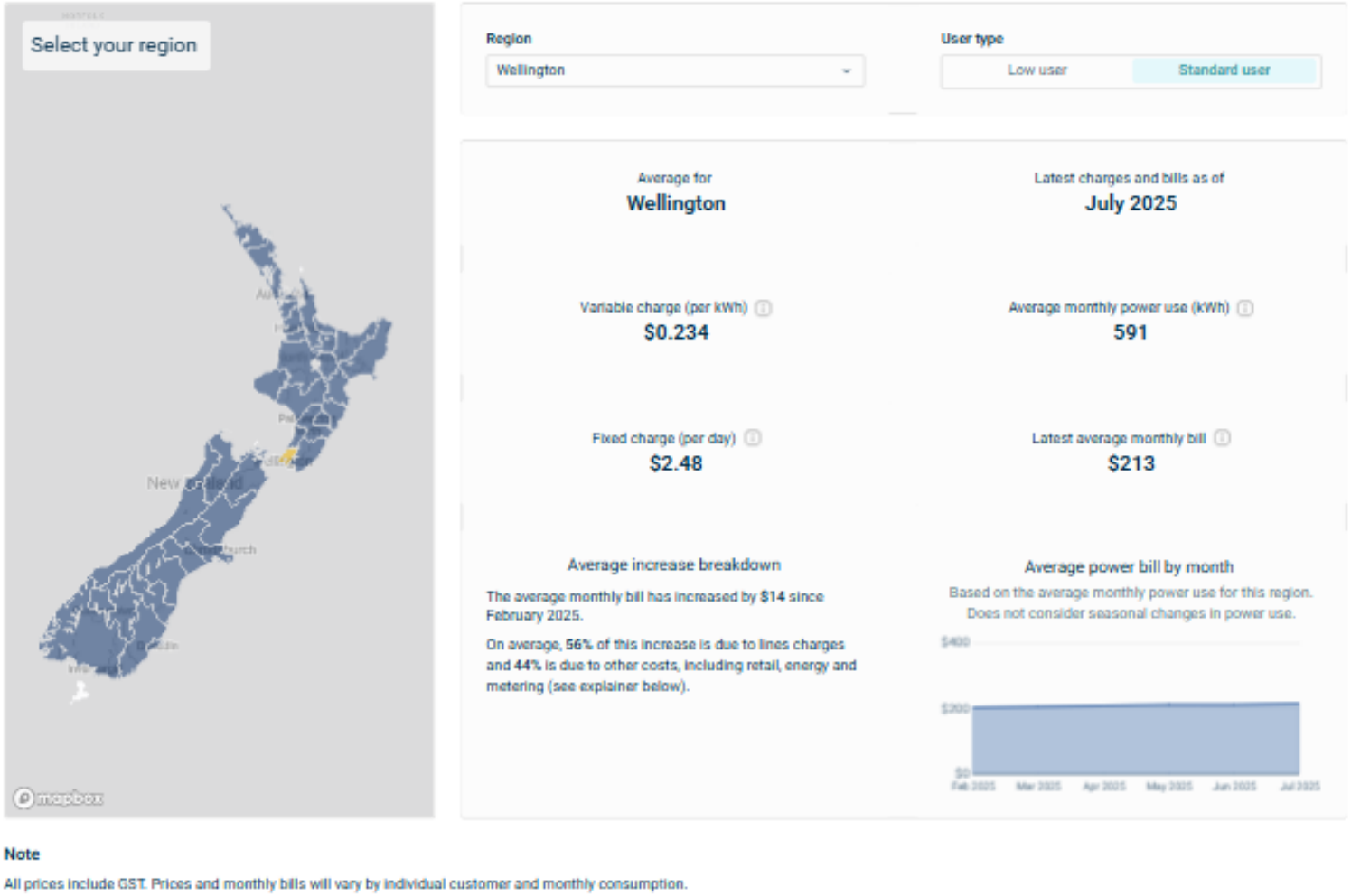


Electricity Consumption Cost

- Factors considered for costing the electricity:
  - Investment in generation and transmission.
  - Total energy consumed.
  - Efficiency of the use and power factor (i.e. leads to penalties when lacking).



Electricity Consumption Cost



Note

All prices include GST. Prices and monthly bills will vary by individual customer and monthly consumption.

https://www.ea.govt.nz/data-and-insights/charts-and-dashboards/regional-power-prices/

Example of Electricity Consumption

A manufacturing industry consumes 200 MWh in one month. If the maximum demand is 1,600 kW, calculate the electricity bill based on the following two-part rate: [10 marks]

- Demand charge: $5.00 per month per kW of billing demand.
- Energy charge: 8 cents per kWh for the first 50,000 kWh, 5 cents per kWh for the remaining energy.

Solution

The demand charge is:

$$\text{Max. Demand Cost} = (\$\ 5.00/\text{kW})(1{,}600\ \text{kW}) = \$\ 8{,}000$$

The energy charge for the first 50,000 kWh is:

$$\text{Initial Charge Cost} = (\$\ 0.08/\text{kWh})(50{,}000\ \text{kWh}) = \$\ 4{,}000$$

Example of Electricity Consumption

The remaining energy is:

$$W = 200{,}000\ \text{kWh} - 50{,}000\ \text{kWh} = 150{,}000\ \text{kWh}$$

The corresponding energy charge is:

$$\text{Energy Cost} = \$\ \text{Charge} \times W = (\$\ 0.05)(150{,}000\ \text{kWh})$$
$$= \$\ 7{,}500$$

Total bill of the month:

$$\text{Total Cost}$$
$$= \text{Demand Cost} + \text{Initial Charge Cost} + \text{Energy Cost}$$
$$= \$\ 8{,}000 + \$\ 4{,}000 + \$\ 7{,}500 = \$\ 19{,}50$$

Role of Load in Electricity Cost

- Loads with low-power factors are costly to serve because they require large currents.
- A load with nonzero-reactive power Q means that energy flows back and forth between the load and the source, giving rise to additional power losses.
- Hence, power companies penalise customers with power factor issues.

Example of Energy Costing

A 300-kW load supplied at 13 kV (RMS) operates 520 hours a month at 80 percent power factor. Calculate the average cost per month based on this simplified tariff: [8 marks]

- Energy charge: 6 cents per kWh
- Power-factor penalty: 0.1 percent of energy charge for every 0.01 that pf falls below 0.85.
- Power-factor credit: 0.1 percent of energy charge for every 0.01 that pf exceeds 0.85.

Example of Energy Costing

Solution

The energy consumed is:

$$W = P \times h = (300\ \text{kW})(520\ \text{h}) = 156{,}000\ \text{kWh}$$

The operating power factor, pf = 80% = 0.8, is 5 x (0.01) below the prescribed power factor of 0.85.

Since there is 0.1 percent energy charge for every 0.01, there is a power-factor penalty charge of 0.5 percent.

This amounts to an energy charge of:

$$\Delta W = W \times \%\text{charge}$$

$$= 156{,}000\left(\frac{0.85 - 0.80}{0.01}\right)\left(\frac{0.1}{100}\right) = 780\ \text{kWh}$$

Example of Energy Costing

The total energy is:

$$W_T = W + \Delta W$$
$$= 156{,}000\ \text{kWh} + 780\ \text{kWh}$$
$$= 156{,}780\ \text{kWh}$$

The cost per month is given by:

$$\text{Cost per month} = \text{UnitPrice} \times W_T$$
$$= (\$\ 0.06)(156{,}780)$$
$$= \$\ 9{,}406.80$$