XMUT203 Analogue Circuits and Systems

Note 6a: Introduction to Second-Order Circuits

Topics:

- Introduction to second-order RLC circuits.
- Resonance in the second-order RLC circuit.
- Analysis of resonant RLC circuit.
- Parallel RLC circuits.
- Series RLC circuits.
- Natural and step responses of parallel RLC circuit.
- Natural and step responses of series RLC circuit.

1. Introduction to Second Order RLC Circuits

In this section we will start to investigate the characteristics of transient responses of second-order RLC circuits.

1.1. Second-Order RLC Circuit

An RLC circuit consists of devices such as resistor, inductor and capacitor. Based on its response, the RLC circuit is a class of second order system. To simplify the analysis to be performed, we limit ourselves to two simple structures: the parallel RLC circuit and the series RLC circuit.

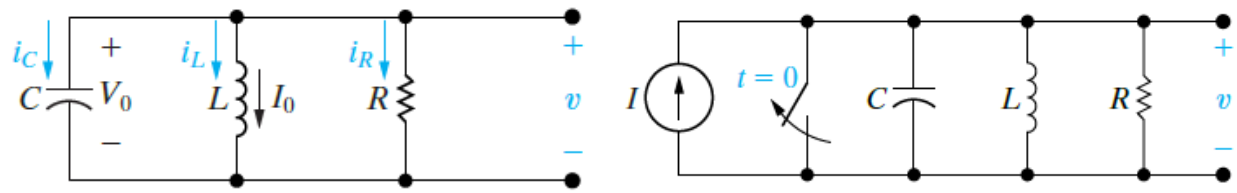


Figure 1: Parallel RLC circuit

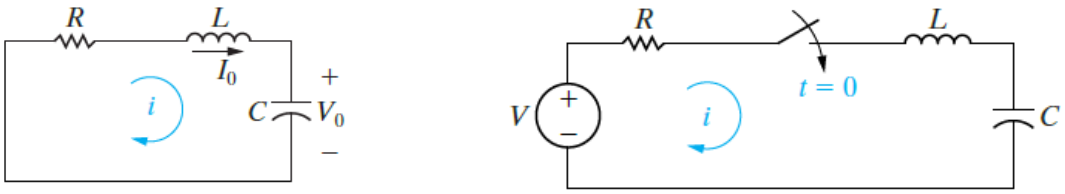


Figure 2: Series RLC circuit

1.2. Analysis of Resonant RLC Circuit

The RLC circuit is mathematically represented as a second order differential equation. We would analyse natural response (e.g. disconnected from source) and step response (e.g. connected with a source) of circuits containing both inductors and capacitors.

Then, we perform derivation of the equations using ordinary differential equation for second order systems. We would start with solutions for natural and step responses of a parallel RLC circuit and analyse three distinct forms of the solutions. Along the way, we summarise solutions for natural and step responses of a series RLC circuit.

1.3. Resonance in RLC Circuits

Resonance is a phenomenon where a system responds preferentially at a particular frequency. It can only occur in systems of at least second order, as it requires energy to be shuffled back and forth between different energy storage locations.

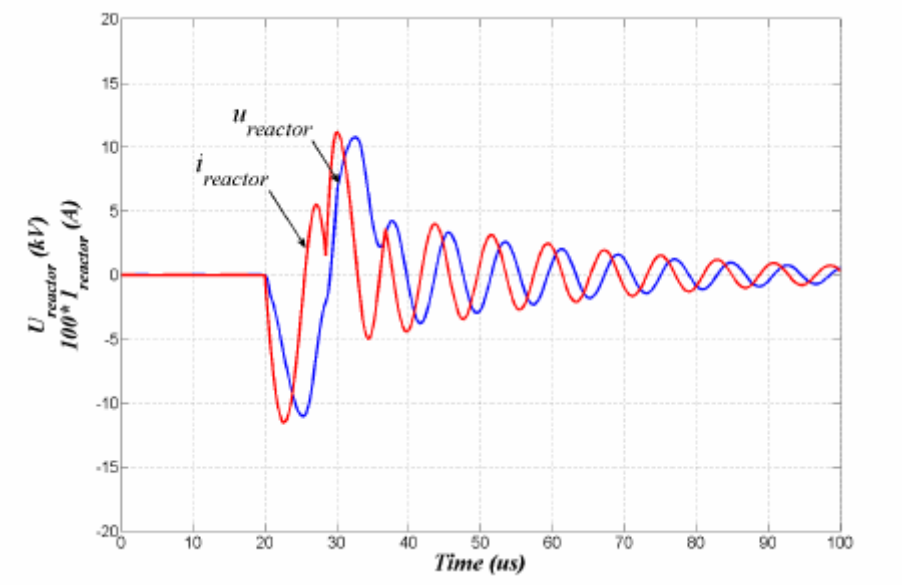


Figure 3: Transient response of a resonance

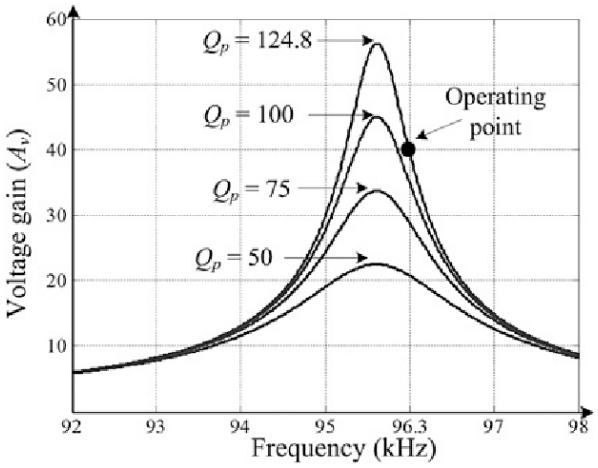


Figure 4: Frequency response of resonance

1.4. Forms of Resonance

Resonance is one of the recurring concepts of physical science and are governed by the same mathematical treatment. We study resonance in electrical systems, but you will already have seen it in mechanical systems (e.g. mass-spring-damper systems, pendulum systems, etc.). Resonance also occurs in less familiar scenarios, such as in fluids, lasers, or nuclear magnetic resonance.

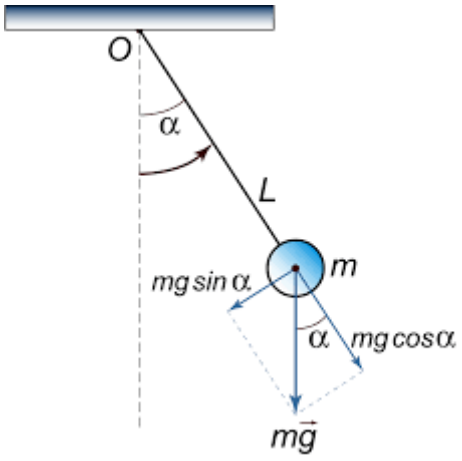


Figure 5: Pendulum system

Notice that in the pendulum system, the system experience exchange of kinetic energy and gravitational energy and vice versa as the pendulum swinging. Whereas in the mass-damper spring system, energy keeps getting stored and released from the spring to the damper and vice versa.

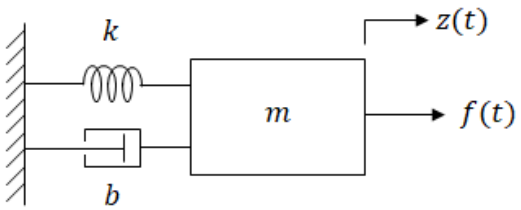


Figure 6: Mass-damper and spring system

1.5. Parallel RLC Circuit

In this section, we try to evaluate the natural and step responses of the parallel RLC circuit.

1.5.1. Natural Response of Parallel RLC Circuit

Natural response of a parallel RLC circuit consists of finding the voltage created across the parallel branches by the release of energy stored in the inductor or capacitor or both. The task is defined in terms of the parallel RLC circuit shown in the figure below.

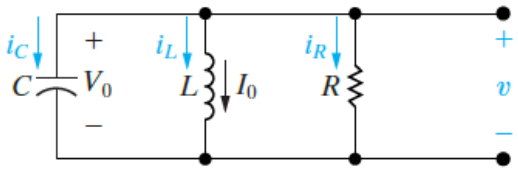


Figure 7: Circuit for realising natural response of parallel RLC circuit

Initial voltage on the capacitor, V_0, represents the initial energy stored in the capacitor. The initial current through the inductor, I_0, represents the initial energy stored in the inductor. If the individual branch currents are of interest, you can find them after determining the terminal voltage.

1.5.2. Step Response of Parallel RLC Circuit

For a step response analysis of a parallel RLC circuit, we use the circuit given below.

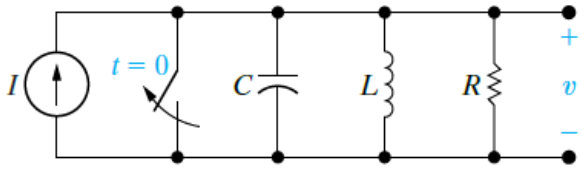


Figure 8: Circuit for realising step response of parallel RLC circuit

We are interested in the voltage, v that appears across the parallel branches because of the sudden application of a DC current source. Energy may or may not be stored in the circuit when the current source is applied.

1.6. Series RLC Circuit

Like its parallel RLC circuit, we analyse the natural and step responses of the series RLC circuit.

1.6.1. Natural Response of Series RLC Circuit

Natural response of a series RLC circuit consists of finding the current generated in the series-connected elements by the release of initially stored energy in the inductor, capacitor, or both. The task is defined by the series RLC circuit as shown in the figure below.

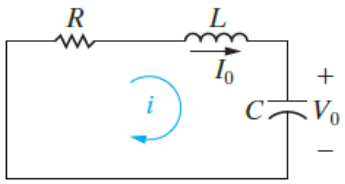


Figure 9: Circuit for realising natural response of series RLC circuit

As before, the initial inductor current, I_0, and the initial capacitor voltage, V_0, represent the initially stored energy. If any of the individual element voltages are of interest, you can find them after determining the current.

1.6.2. Step Response of Series RLC Circuit

For a step response of a series RLC circuit, we use the circuit given below.

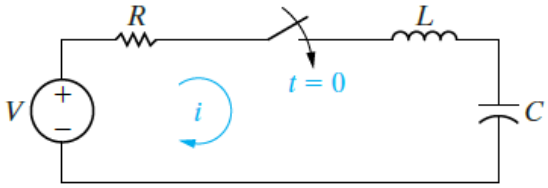


Figure 10: Circuit for realising step response of series RLC circuit

We analyse current resulting from the sudden application of the dc voltage source. Energy may or may not be stored in the circuit when the switch is closed.

2. Analysis of Parallel and Series RLC Circuit

Forms of analysis of parallel and series RLC circuit:

- Parallel RLC circuit:
    - Natural response of parallel RLC circuit.
    - Step response of parallel RLC circuit.
- Series RLC circuit:
    - Natural response of series RLC circuit.
    - Step response of series RLC circuit.

2.1. Natural Response of Parallel Circuit

Differential equation for the voltage is derived by summing the currents away from the top node through KCL, where each current is expressed as a function of the unknown voltage v:

$$i_R + i_L + i_C + I_0 = 0 \qquad (Eq.1)$$

Or

$$\frac{v}{R} + \frac{1}{L}\int_0^t v\,d\tau + C\frac{dv}{dt} + I_0 = 0 \qquad (Eq.2)$$

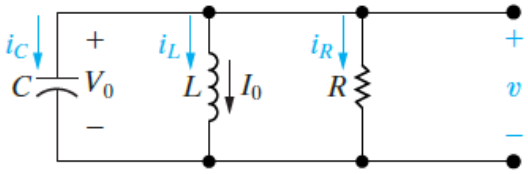


Figure 11: Natural response of parallel RLC circuit

By differentiating with respect to t, and, because I_0 is a constant, we get:

$$\left(\frac{1}{R}\right)\frac{dv}{dt} + \frac{v}{L} + C\frac{d^2v}{dt^2} = 0 \qquad (Eq.3)$$

Divide through the equation above by the capacitance C and arrange the derivatives in descending order:

$$\frac{d^2v}{dt^2} + \left(\frac{1}{RC}\right)\frac{dv}{dt} + \frac{v}{LC} = 0 \qquad (Eq.4)$$

Compared with first-order circuit, the equation above differs by the presence of the term involving the second derivative e.g. ordinary second-order differential equation with constant coefficients.

To obtain a general solution for second-order differential equation, assume that the solution is of exponential form of voltage:

$$v = Ae^{j\omega t} \qquad (Eq.5)$$

Where: A and $j\omega$ are unknown constants.

Substituting the exponential form of voltage into the second order equation:

$$A(j\omega)^2 e^{j\omega t} + \frac{A}{RC}(j\omega)e^{j\omega t} + \frac{A}{LC}e^{j\omega t} = 0 \qquad (Eq.6)$$

Or

$$Ae^{j\omega t}\left((j\omega)^2 + \frac{(j\omega)}{RC} + \frac{1}{LC}\right) = 0 \qquad (Eq.7)$$

The following equation is a characteristic equation of the differential equation of a parallel RLC circuit.

$$(j\omega)^2 + \frac{j\omega}{RC} + \frac{1}{LC} = 0 \qquad (Eq.8)$$

Because the roots of this quadratic equation, we determine the mathematical character of $v(t)$. Using solution for the standard quadratic equation, the two roots of the equation are:

$$j\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \qquad (Eq.9a)$$

$$j\omega_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \qquad (Eq.9b)$$

Knowing the roots of the secondary equation are:

$$v_1 = A_1 e^{j\omega_1 t} \qquad \text{and} \qquad v_2 = A_2 e^{j\omega_2 t} \qquad (Eq.10)$$

If we let:

$$v = v_1 + v_2 = A_1 e^{j\omega_1 t} + A_2 e^{j\omega_2 t} \qquad (Eq.11)$$

Then

$$\frac{dv}{dt} = A_1 j\omega_1 e^{j\omega_1 t} + A_2 j\omega_2 e^{j\omega_2 t} \qquad (Eq.12)$$

And

$$\frac{d^2v}{dt^2} = A_1 (j\omega_1)^2 e^{j\omega_1 t} + A_2 (j\omega_2)^2 e^{j\omega_2 t} \qquad (Eq.13)$$

Substituting the derivatives given above to the circuit equation:

$$A_1 e^{j\omega_1 t}\left((j\omega_1)^2 + \frac{j\omega_1}{RC} + \frac{1}{LC}\right) + A_2 e^{j\omega_2 t}\left((j\omega_2)^2 + \frac{j\omega_2}{RC} + \frac{1}{LC}\right) = 0 \qquad (Eq.14)$$

Hence, the general solution to the natural response of parallel RLC circuit is of the form:

$$v = A_1 e^{j\omega_1 t} + A_2 e^{j\omega_2 t} \qquad (Eq.15)$$

The roots of the characteristic equation ($j\omega_1$ and $j\omega_2$) are determined by the circuit parameters R, L, and C. The initial conditions determine the values of the constants A_1 and A_2.

Furthermore, the behaviour of $v(t)$ depends on the values of $j\omega_1$ and $j\omega_2$.

$$v = A_1 e^{j\omega_1 t} + A_2 e^{j\omega_2 t} \qquad (Eq.16)$$

Therefore, the first step in finding the natural response is to determine the roots of the characteristic equation. Alternatively, the equation for the RLC circuit can be derived as:

$$j\omega_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad (Eq.17a)$$

And

$$j\omega_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \qquad (Eq.17b)$$

Where:

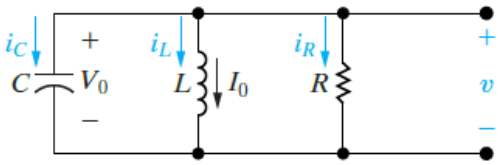
$$\alpha = \frac{1}{2RC} \qquad \text{and} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad (Eq.18)$$

Note that α is neper frequency and ω_0 is the resonant radian frequency. There are three possible outcomes:

- $\omega_0^2 < \alpha^2$ in this example both roots are real and distinct (overdamped response).
- $\omega_0^2 > \alpha^2$ in this example both roots are complex (underdamped response).
- $\omega_0^2 = \alpha^2$ in this example both roots are real and equal (critically damped response).

Example 1: Natural Response of Parallel RLC

For the RLC circuit given in the figure below, answer the following questions:



a. Find the roots of the characteristic equation that governs the transient behaviour of the voltage shown in the figure above if R = 200 Ω, L = 50 mH and C = 0.2 μF. [6 marks]

b. Will the response be overdamped, underdamped, or critically damped? [2 marks]

c. Repeat parts (a) and (b) for R = 312.5 Ω. [6 marks]

d. What value of R causes the response to be critically damped? [4 marks]

Solution

a. For the given values of R, L, and C, the neper and resonant frequencies are:

$$\alpha = \frac{1}{2RC} = \frac{10^6}{(400)(0.2)} = 1.25 \times 10^4 \text{ rad/s}$$

And

$$\omega_0^2 = \frac{1}{LC} = \frac{(10^3)(10^6)}{(50)(0.2)} = 10^8 \text{ rad}^2/\text{s}^2$$

From the equations given below, we can calculate the roots of the equation:

$$j\omega_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$= -1.25 \times 10^4 + \sqrt{1.5625 \times 10^8 - 10^8}$$

$$= -12{,}500 + 7500 = -5000 \text{ rad/s}$$

And

$$j\omega_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$= -1.25 \times 10^4 - \sqrt{1.5625 \times 10^8 - 10^8}$$

b. The voltage response is overdamped because $\omega_0^2 < \alpha^2$.

c. For R = 312.5 Ω, the neper frequency is:

$$\alpha = \frac{10^6}{(625)(0.2)} = 8000 \text{ rad/s}$$

So

$$\alpha^2 = 64 \times 10^6 = 0.64 \times 10^8 \text{ rad}^2/\text{s}^2$$

As ω_0^2 remains at $10^8 \text{ rad}^2/\text{s}^2$

$$j\omega_1 = -8000 + j6000 \text{ rad/s}$$

And

$$j\omega_2 = -8000 - j6000 \text{ rad/s}$$

In this case, the voltage response is underdamped since $\omega_0^2 > \alpha^2$.

Notice that in electrical engineering, the imaginary number $\sqrt{-1}$ is represented by the letter j, because the letter i represents current.

d. For critical damping, $\alpha^2 = \omega_0^2$, so:

$$\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = 10^8$$

or

$$\frac{1}{2RC} = 10^4$$

and

$$R = \frac{10^6}{(10^4)(0.2)} = 250\ \Omega$$

2.2. Step Response of Parallel RLC Circuit

Finding the step response of a parallel RLC circuit involves finding the voltage across the parallel branches or the current in the individual branches as a result of the sudden application of a dc

current source. There may or may not be energy stored in the circuit when the current source is applied. The task is represented by the circuit shown in the figure below.

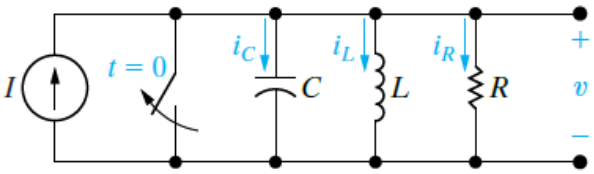


Figure 12: Step response of parallel RLC circuit

To develop a general approach to finding the step response of a second-order circuit, we focus on finding the current in the inductive branch (i_L). This current is of particular interest because it does not approach zero as t increases. Rather, after the switch has been open for a long time, the inductor current equals the dc source current I.

Because we want to focus on the technique for finding the step response, we assume that the initial energy stored in the circuit is zero. This assumption simplifies the calculations and does not alter the basic process involved. To find the inductor current i_L, we must solve a second-order differential equation equated to the forcing function I, which we derive as follows.

From Kirchhoff's current law, we have:

$$i_L + i_R + i_C = I \qquad (Eq.19)$$

or

$$i_L + \frac{v}{R} + C\frac{dv}{dt} = I \qquad (Eq.20)$$

Because

$$v = L\frac{di_L}{dt} \qquad \text{and} \qquad \frac{dv}{dt} = L\frac{d^2i_L}{dt^2} \qquad (Eq.21)$$

Substituting the derivatives into the circuit equation gives:

$$i_L + \left(\frac{L}{R}\right)\frac{di_L}{dt} + (LC)\frac{d^2i_L}{dt^2} = I \qquad (Eq.22)$$

For convenience, we divide through by *LC* and rearrange terms:

$$\frac{d^2i_L}{dt^2} + \left(\frac{1}{RC}\right)\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I}{LC} \qquad (Eq.23)$$

Comparing the equation above with the equation that we obtained before reveals that the presence of a nonzero term on the right-hand side of the equation complicates the task for finding solution.

$$\frac{d^2v}{dt^2} + \left(\frac{1}{RC}\right)\frac{dv}{dt} + \frac{v}{LC} = 0 \qquad (Eq.24)$$

We can solve for i_L indirectly by first finding the voltage v. We do this with the techniques introduced in previous section, because the differential equation that v must satisfy is identical to:

$$\frac{d^2v}{dt^2} + \left(\frac{1}{RC}\right)\frac{dv}{dt} + \frac{v}{LC} = 0 \qquad (Eq.25)$$

To see this, we simply return to the equation as shown below and express i_L as a function of v.

$$i_L + \frac{v}{R} + C\frac{dv}{dt} = I \qquad (Eq.26)$$

Thus

$$\frac{1}{L}\int_0^t v\,d\tau + \frac{v}{R} + C\frac{dv}{dt} = I \qquad (Eq.27)$$

Differentiating the equation above with respect to t reduces the right-hand side to zero because I is a constant. Thus:

$$\frac{v}{L} + \left(\frac{1}{R}\right)\frac{dv}{dt} + C\frac{d^2v}{dt^2} = 0 \qquad (Eq.28)$$

or

$$\frac{d^2v}{dt^2} + \left(\frac{1}{RC}\right)\frac{dv}{dt} + \frac{v}{LC} = 0 \qquad (Eq.29)$$

As discussed in previous derivation, the solution for v depends on the roots of the characteristic equation. Thus, the three possible solutions are:

$$v = A_1e^{j\omega_1 t} + A_2e^{j\omega_2 t} \qquad (Eq.30a)$$

$$v = B_1e^{-\alpha t}\cos\omega_d t + B_2e^{-\alpha t}\sin\omega_d t \qquad (Eq.30b)$$

$$v = D_1te^{-\alpha t} + D_2e^{-\alpha t} \qquad (Eq.30c)$$

A word of caution: Because there is a source in the circuit for $t > 0$, you must take into account the value of the source current at $t = 0^+$ when you evaluate the coefficients in the equations above.

To find the three possible solutions for i_L, we substitute all the equations above into the equation below.

$$i_L + \frac{v}{R} + C\frac{dv}{dt} = I \qquad (Eq.31)$$

You should be able to verify, when this has been done, that the three solutions for i_L will be:

$$i_L = I + A_1'e^{j\omega_1 t} + A_2'e^{j\omega_2 t} \qquad (Eq.32a)$$

$$i_L = I + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t \qquad (Eq.32b)$$

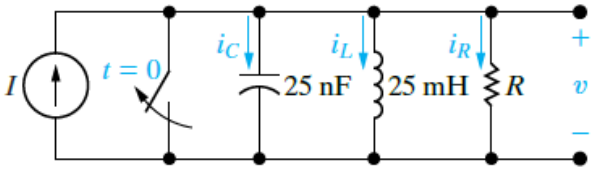
$$i_L = I + D_1'te^{-\alpha t} + D_2'e^{-\alpha t} \qquad (Eq.32c)$$

Where: A_1', A_2', B_1', B_2', D_1' and D_2' are arbitrary constants.

In each case, the primed constants can be found indirectly in terms of the arbitrary constants associated with the voltage solution.

Example 2: Step Response of Parallel RLC

The initial energy stored in the circuit in the figure below is zero. At $t = 0$, a dc current source of 24 mA is applied to the circuit. The value of the resistor is 400 Ω.



What are the roots of the characteristic equation? [10 marks]

Solution

From the circuit elements, we obtain:

$$\omega_0^2 = \frac{1}{LC} = \frac{10^{12}}{(25)(25)} = 16 \times 10^8$$

And

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(400)(25)} = 5 \times 10^4 \text{ rad/s}$$

or

$$\alpha^2 = 25 \times 10^8$$

Because $\omega_0^2 < \alpha^2$, the roots of the characteristic equation are real and distinct. Thus

$$j\omega_1 = -5 \times 10^4 + 3 \times 10^4 = -20{,}000 \text{ rad/s}$$

And

$$j\omega_2 = -5 \times 10^4 - 3 \times 10^4 = -80{,}000 \text{ rad/s}$$

Because the roots of the characteristic equation are real and distinct, the inductor current response will be overdamped.

$$i_L = I + A_1' e^{j\omega_1 t} + A_2' e^{j\omega_2 t}$$

Thus $i_L(t)$ takes the form of the equation above, namely,

$$i_L(t) = i_f(t) + A_1' e^{s_1 t} + A_2' e^{s_2 t}$$

Hence, from this solution, the two simultaneous equations that determine A_1' and A_2' are

$$i_L(t) = i_f(t) + A_1' + A_2' = 0$$

2.3. Natural Response of Series RLC Circuit

The procedures for finding the natural or step responses of a series RLC circuit are the same for the natural or step responses of a parallel RLC circuit. Both circuits are described by differential

equations that have the same form. The equivalent circuit for analysing natural response of series RLC circuit is shown below.

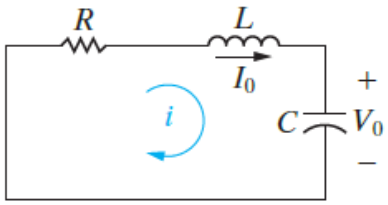


Figure 13: Natural response of series RLC circuit

We begin by applying KVL and summing the voltages around the closed path in the circuit. Thus

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,d\tau + V_0 = 0 \qquad (Eq.33)$$

We now differentiate the equation with respect to t to get:

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0 \qquad (Eq.34)$$

Which we can rearrange as:

$$\frac{d^2i}{dt^2} + \left(\frac{R}{L}\right)\frac{di}{dt} + \frac{i}{LC} = 0 \qquad (Eq.35)$$

Comparing the equations above reveals that they have the same form. Therefore, to find the solution, we follow the same process that led us to the solution of parallel RLC circuit. From the equation above, the characteristic equation for the series RLC circuit is:

$$(j\omega)^2 + \left(\frac{R}{L}\right)j\omega + \frac{1}{LC} = 0 \qquad (Eq.36)$$

The roots of the characteristic equation are:

$$j\omega_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad (Eq.37)$$

or

$$j\omega_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \qquad (Eq.38)$$

The neper frequency (α) for the series RLC circuit is:

$$\alpha = \frac{R}{2L}\ \text{rad/s} \qquad (Eq.39)$$

The expression for the resonant radian frequency is:

$$\omega_0 = -\frac{1}{\sqrt{LC}}\ \text{rad/s} \qquad (Eq.40)$$

Note that the equation for neper frequency of the series RLC circuit differs from that of the parallel RLC circuit. But, the equations for resonant and damped radian frequencies are the same. The current response will be overdamped, underdamped, or critically damped according to whether: $\omega_0^2 < \alpha^2$, $\omega_0^2 > \alpha^2$, or $\omega_0^2 = \alpha^2$, respectively. Thus, the three possible solutions for the current are as follows:

$$i(t) = A_1 e^{j\omega_1 t} + A_2 e^{j\omega_2 t} \qquad \text{(overdamped)} \qquad (Eq.41a)$$

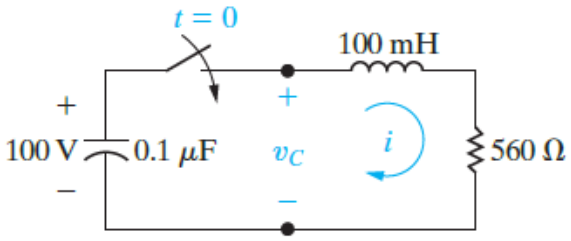
$$i(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t \qquad \text{(underdamped)} \qquad (Eq.41b)$$

$$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t} \qquad \text{(critically damped)} \qquad (Eq.41c)$$

When you have obtained the natural current response, you can find the natural voltage response across any circuit element.

Example 3: Natural Response of Series RLC

The 0.1 μF capacitor in the circuit shown in the figure below is charged to 100 V. At $t = 0$, the capacitor is discharged through a series combination of a 100 mH inductor and a 560 Ω resistor. Find $i(t)$ for $t \geq 0$. Find $v_C(t)$ for $t \geq 0$. [10 marks]



Solution

The first step to finding $i(t)$ is to calculate the roots of the characteristic equation. For the given element values,

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{(100\times10^{-3})(0.1\times10^{-6})} = 10^8$$

And

$$\alpha = \frac{R}{2L} = \frac{560}{2(100)}\times10^3 = 2800 \text{ rad/s}$$

Next, we compare ω_0^2 to α^2 and note that $\omega_0^2 > \alpha^2$ because:

$$\alpha^2 = 7.84\times10^6 = 0.0784\times10^8$$

At this point, we know that the response is underdamped and that the solution for $i(t)$ is of the form:

$$i_L(t) = i_f(t) + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$$

where α = 2800 rad/s and ω_d = 9600 rad/s.

2.4. Step Response of Series RLC Circuit

The circuit for analysing the step response of a series RLC circuit is shown below. Applying Kirchhoff's voltage law to the circuit around closed circuit, the equation for the voltages in the circuit is:

$$V = Ri + L\frac{di}{dt} + v_C \qquad (Eq.42)$$

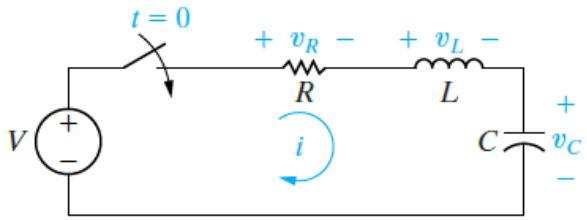


Figure 14: Step response of series RLC circuit

Differential equation that describes the capacitor voltage in series RLC circuit has the same form as the differential equation for the inductor current in parallel RLC circuit. Assume that zero energy is stored in the circuit at the instant the switch is closed. The current (i) is related to the capacitor voltage (v_C) by the expression:

$$i = C\frac{dv_C}{dt} \qquad (Eq.43)$$

From which:

$$\frac{di}{dt} = C\frac{d^2v_C}{dt^2} \qquad (Eq.44)$$

Substitute the derivatives into the circuit equation, the equation above becomes:

$$\frac{d^2v_C}{dt^2} + \left(\frac{R}{L}\right)\frac{dv_C}{dt} + \frac{v_C}{LC} = \frac{V}{LC} \qquad (Eq.45)$$

This equation has the same form as equation for the step response of parallel RLC circuit. Therefore, the procedure for finding v_C parallels that for finding i_L. Three possible solutions for v_C are:

$$v_C = V_f + A_1'e^{j\omega_1 t} + A_2'e^{j\omega_2 t} \qquad \text{(overdamped)} \qquad (Eq.46a)$$

$$v_C = V_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t \qquad \text{(underdamped)} \qquad (Eq.46b)$$

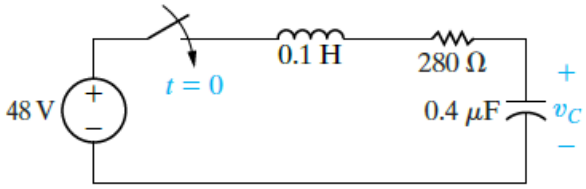
$$v_C = V_f + D_1'e^{-\alpha t} + D_2'e^{-\alpha t} \qquad \text{(critically damped)} \qquad (Eq.46c)$$

Where: Voltage V_f is the final value of v_C.

Hence the final value of v_C is the DC source voltage V.

Example 4: Step Response of Series RLC

No energy is stored in the 100 mH inductor or the 0.4 μF capacitor when the switch in the circuit shown in the figure below is closed. Find $v_C(t)$ for $t \geq 0$. [4 marks]



Solution

The roots of the characteristic equation are:

$$j\omega_1 = -\frac{280}{0.2} + \sqrt{\left(\frac{280}{0.2}\right)^2 - \frac{10^6}{(0.1)(0.4)}} = (-1400 + j4800)\ \text{rad/s}$$

And

$$j\omega_2 = (-1400 - j4800)\ \text{rad/s}$$

The roots are complex, so the voltage response is underdamped.

3. Response Characteristics of Second-Order RLC Circuits

The response characteristics of the second-order RLC circuit are compared according to the type or damping ratio in the tables given below.

Type of Circuit (Damping)	**When**	**Second-Order Circuit Response Characteristics**
Overdamped	$\alpha^2 > \omega_0^2$	The voltage or current approaches its final value without oscillation.
Underdamped	$\alpha^2 < \omega_0^2$	The voltage or current oscillates about its final value.
Critically damped	$\alpha^2 = \omega_0^2$	The voltage or current is on the verge of oscillating about its final value.

Table 1: Response of the second-order circuit