XMUT203 Analogue Circuits and Systems

Note 8a: Introduction to Electrical Power

1. Sine and Cosine

Alternating current (AC) utilises sines and cosines for moving large blocks of energy from one place to another. Considering a trigonometric function as shown below, we can analyse and design this transfer of energies.

$$f(t) = A\cos(\omega t + \theta) \qquad (Eq.1)$$

For a cosine as shown above, the function starts at t = 0 and the period of cosine is considered to be τ. Furthermore, we say that for this example, the phase shift is a leading phase angle θ. Thus, as illustrate in the figure below, the frequency of the sinusoidal waveform is:

$$f = \frac{1}{T} \qquad \text{or} \qquad \omega = 2\pi f \qquad (Eq.2)$$

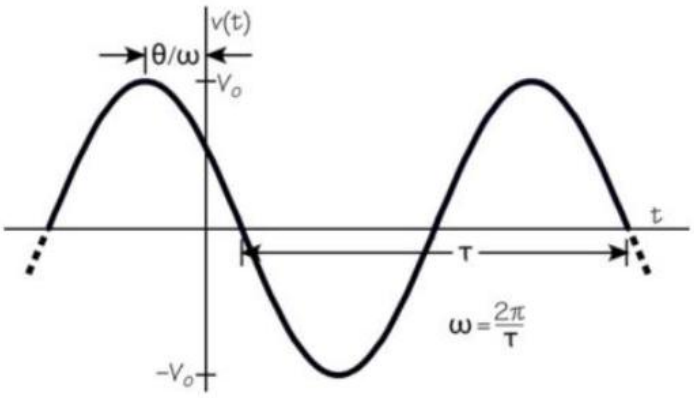


Figure 1: Sinusoidal signal waveform

1.1. Sinusoids Waveform

When the function is applied to represent the parameter of the circuit i.e. voltage, for a sinusoid voltage, it is represented as:

$$v(t) = V_m \sin \omega t \qquad (Eq.3)$$

Where:

V_m = Amplitude of sinusoid

ω = Angular frequency in radian/s

ωt = argument of the sinusoid

Now, if we include the phase shift of the signal into the equation i.e. relative position of the signal relative to another. Let's consider a sinusoidal signal as given below. The signal can be represented with the following equation:

$$v(t) = V_m \sin(\omega t + \phi) \qquad (Eq. 4)$$

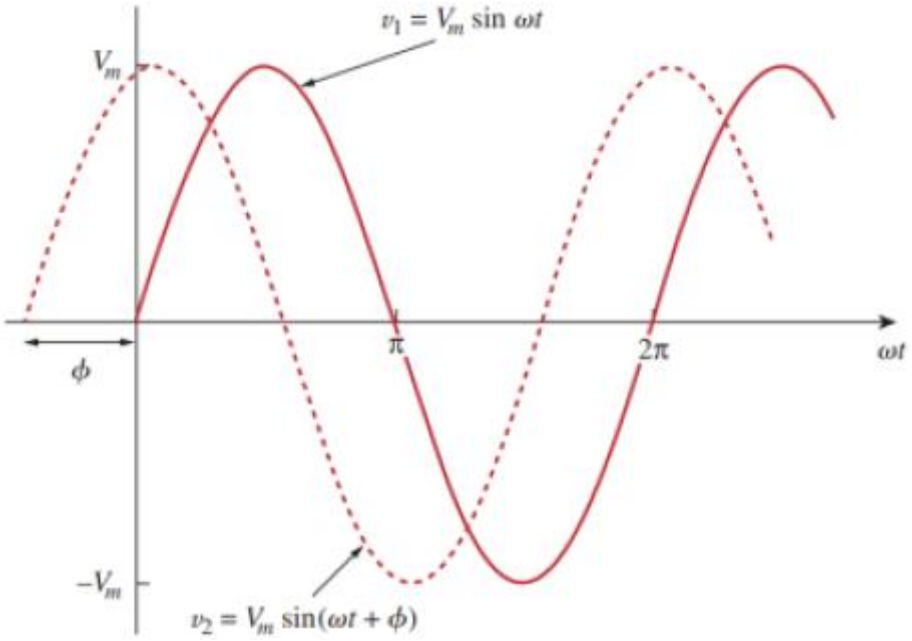


Figure 2: Sinusoidal signal with phase shift included

Example 1: Parameters of Sinusoidal Waveform

Find amplitude, phase, period, and frequency of the following signal: [10 marks]

$$v(t) = 12\cos(50t + 10°)$$

Solution

Referring to the equation given above, the amplitude of the signal is:

$$V_m = 12 \text{ V}$$

The phase shift is:

$$\phi = 10°$$

The angular frequency is:

$$\omega = 50 \text{ rad/s}$$

The period of the signal is:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257 \text{ s}$$

The frequency is:

$$f = \frac{1}{T} = 7.958 \text{ Hz}$$

Self-Exercise:

Now work backwards for 50 Hz keeping V_m and ϕ the same (i.e. $\omega = 2\pi f = 2\pi(50) = 314$ rad/s, thus $v(t) = 12\cos(314t + 10°)$).

2. AC Power

We can treat AC power as a function of time (in sine and cosine). We can represent the voltage and current of the circuit using these equations:

$$v(t) = V_m \cos(2\pi f t + \theta_v) \qquad (Eq. 5)$$

And

$$i(t) = I_m \cos(2\pi f t + \theta_i) \qquad (Eq. 6)$$

As a result, we can calculate the power using the following equation

$$p(t) = v(t)i(t) \qquad (Eq. 7)$$

Note that $p(t)$ is instantaneous power.

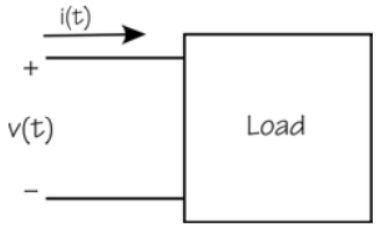


Figure 3: AC Power delivered to the load

But, this is not very useful, because we really need to know how much power is being delivered on average. More importantly, how much energy is being moved i.e. what power company want us to pay for the electricity that we consume.

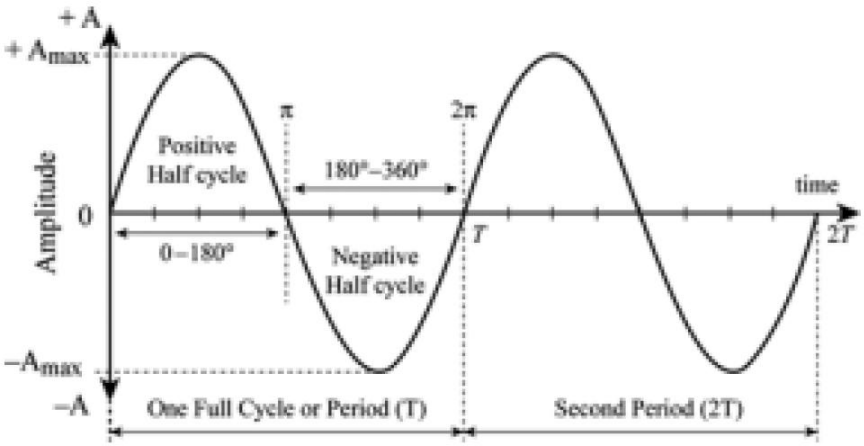


Figure 4: Sinusoidal signal

2.1. Instantaneous Power

Since the instantaneous power is the power at an instant of time represented with the following equation:

$$p(t) = v(t)i(t) \qquad (Eq. 8)$$

If $p(t)$ has positive value i.e. the current in the same direction as positive voltage polarity, it is considered as the source of power. On the other hand, if it is negative, then it is a sink (i.e. dissipate) of the power.

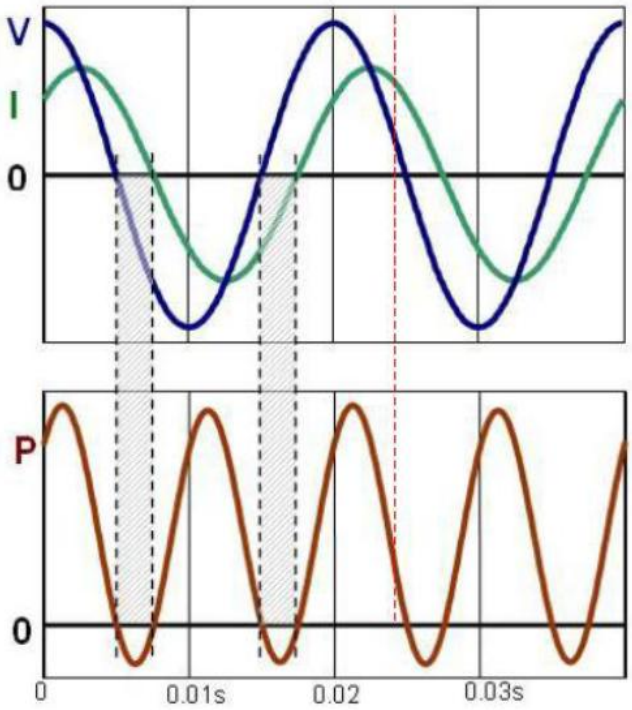


Figure 5: Voltage, current, and power waveforms

2.2. Energy

Energy is defined as the time integral of power and we often use symbol W (although its units are Joules). But, do not confuse this with unit for power Watts, W. Energy is calculated from the following equation:

$$W = \int_{t_1}^{t_2} p(t)\, dt \qquad (Eq. 9)$$

2.3. Average Power

The average power is the power averaged over entire cycle. It is also known as real power or active power and calculated from:

$$P_{ave} = \frac{1}{T}\int_0^t p(t)\, dt = \frac{1}{T}\int_0^t v(t)\, i(t) dt = \frac{W}{T} \qquad (Eq. 10)$$

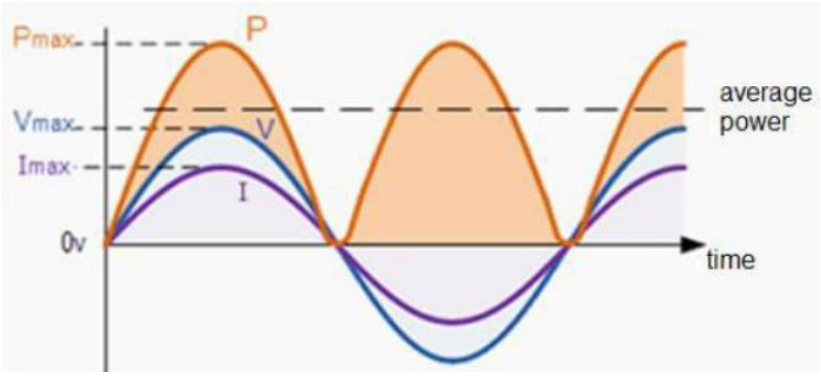
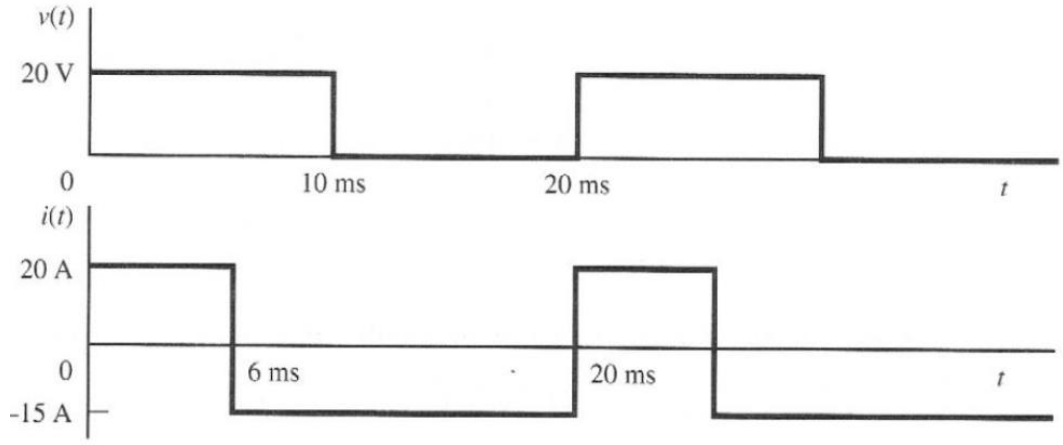


Figure 6: Determination of the average power

Example 2: Instantaneous Power

Given voltage and current waveforms as shown in the figures below.



Find:

a. Expressions for the voltage and current waveforms. [4 marks]

b. The instantaneous power. [2 marks]

c. Energy absorbed by the device in one period. [4 marks]

d. Average power absorbed by the device. [4 marks]

Solution

a. The equations for the voltage and current waveforms are:

Voltage is:

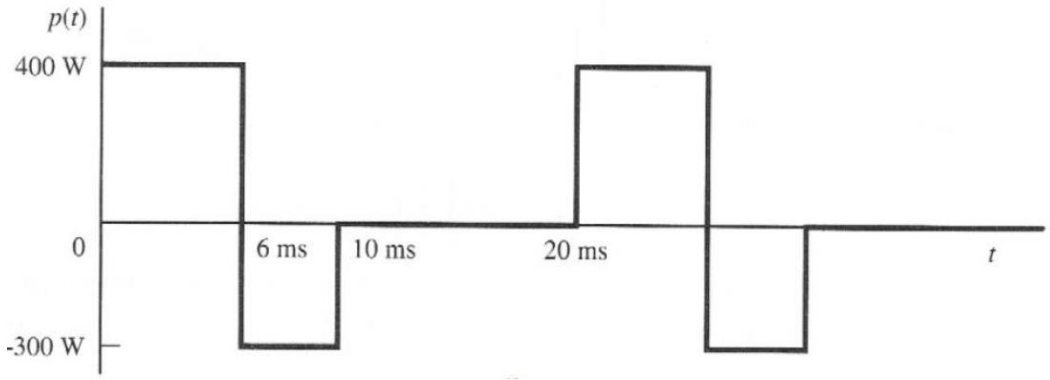
$$v(t) = \begin{cases} 20\ \text{V} & 0 < t < 10\ \text{ms} \\ 0\ \text{V} & 10\ \text{ms} < t < 20\ \text{ms} \end{cases}$$

Current is:

$$i(t) = \begin{cases} 20\ \text{A} & 0 < t < 6\ \text{ms} \\ -15\ \text{A} & 6\ \text{ms} < t < 20\ \text{ms} \end{cases}$$

b. The instantaneous power is simply:

$$p(t) = \begin{cases} 400\ \text{W} & 0 < t < 6\ \text{ms} \\ -300\ \text{W} & 6\ \text{ms} < t < 10\ \text{ms} \\ 0 & 10\ \text{ms} < t < 20\ \text{ms} \end{cases}$$



c. The energy absorbed by the device in one period is:

$$W = \int_0^t p(t)\,dt$$

$$= \int_0^{0.006} 400\,dt + \int_{0.006}^{0.01} (-300)\,dt + \int_{0.01}^{0.02} (0)\,dt$$

$$= 2.4 - 1.2 + 0 = 1.2\ \text{Joule}$$

d. The average power is:

$$P_{ave} = \frac{W}{T} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= \frac{1}{0.02}\left[\int_0^{0.006} 400\,dt + \int_{0.006}^{0.01} (-300)\,dt + \int_{0.01}^{0.02} (0)\,dt\right]$$

$$= 60 \text{ W}$$

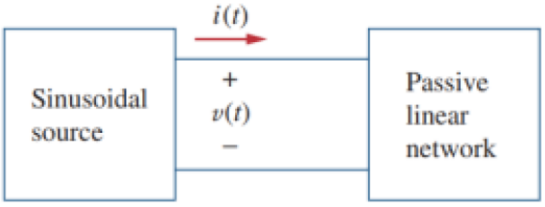
Example 3: AC Power

Given the following equations for voltage and current in the circuit:

$$v(t) = 120\cos(377t + 45°) \text{ V} \quad \text{and} \quad i(t) = 10\cos(377t - 10°) \text{ A}$$

Determine the instantaneous power and the average power absorbed by the passive linear network.

[10 marks]



Solution

Considering the two equations of the voltage and current given above, the instantaneous power is given by:

$$p(t) = v(t)i(t)$$

$$= 120\cos(377t + 45°)\,10\cos(377t - 10°)$$

$$= 1200\cos(377t + 45°)\cos(377t - 10°)$$

Applying trigonometry identity:

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

This gives:

$$p(t) = \frac{1200}{2}[\cos(377t + 45° + 377t - 10°)\cos(377t + 45° - (377t - 10°))]$$

$$= 600[\cos(754t + 35°) + \cos 55°]$$

Or

$$p(t) = 344.2 + 600\cos(754t + 35°) \text{ W}$$

Then, the average power is:

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2}(120)(10)\cos[45° - (-10°)]$$

$$= 600\cos 55° = 344.2\ \text{W}$$

2.4. DC Source

The DC power is obtained when DC voltage multiplied with average current, and it is calculated from:

$$P_{dc} = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\,dt = \frac{1}{T}\int_{t_0}^{t_0+T} V_{dc}\,i(t)dt \qquad (Eq.11)$$

If the circuit is purely resistive i.e. $v = Ri$, then the average power is calculated from:

$$P_{ave} = \frac{R}{T}\int_0^T i^2(t)\,dt \qquad (Eq.12)$$

2.5. Root Mean Square (RMS) Power

For the RMS current I, the average power is calculated from:

$$P_{ave} = RI_{RMS}^2 \qquad (Eq.13)$$

Hence, the term 'RMS':

$$P_{ave} = \frac{R}{T}\int_0^T i^2(t)\,dt \qquad (Eq.14)$$

Thus

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} \qquad (Eq.15)$$

2.6. AC Power Quantities - Averages

The average power comes from averaging the instantaneous power $p(t)$ over one period of the AC waveform.

$$p(t) = [V_m\cos(2\pi ft + \theta_v)][I_m\cos(2\pi ft + \theta_i)] \qquad (Eq.16)$$

Thus

$$p(t) = \frac{V_m I_m}{2}\left[\cos(\theta_v - \theta_i) + \cos\left(4\pi ft + (\theta_v - \theta_i)\right)\right] \qquad (Eq.17)$$

As a result, the average power is calculated from:

$$P_{ave} = \frac{V_m I_m}{2}[\cos(\theta_v - \theta_i)] \qquad \text{(i.e. good for mathematicians)} \qquad (Eq.18)$$

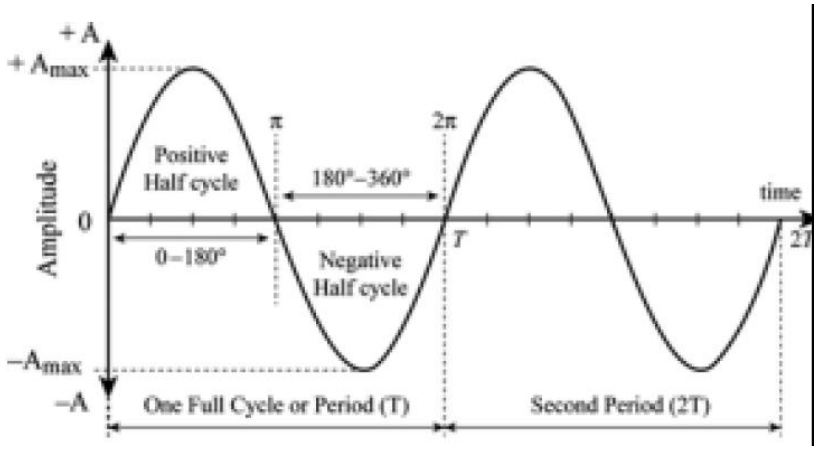


Figure 7:

For analysing the AC power, we do not use the peak values in AC systems, but we use RMS value instead. For sine wave.

$$V_{RMS} = \frac{V_m}{\sqrt{2}} = 0.707V_m \qquad (Eq.19)$$

Thus:

$$P_{ave} = V_{RMS}I_{RMS}\cos(\theta_v - \theta_i) \qquad (Eq.20)$$

2.7. RMS from Peak Values

The following table shows the conversion of the quantities from the peak value to the RMS value for a variety of common waveforms

Waveform	**Equation**
Rectangular pulse (duty cycle D)	$V_{RMS} = V_m\sqrt{D}$
Sinusoids	$V_{RMS} = V_m/\sqrt{2}$
Full-wave rectified sinusoids	$V_{RMS} = V_m/\sqrt{2}$
Half-wave rectified sinusoids	$V_{RMS} = V_m/2$
Sum of different frequency sinusoids	$V_{RMS} = \sqrt{V_{1,RMS}^2 + V_{2,RMS}^2 + \cdots}$
Triangular wave	$V_{RMS} = V_m/\sqrt{3}$

Note: Substitute I for V to determine currents.

Table 1: Conversions of peak value to RMS value for some waveforms

2.8. Sum of Periodic Waveforms

If the periodic voltage is the sum of two periodic voltage waveforms $v_1(t) + v_2(t)$, then its RMS value is determined from:

$$V_{RMS}^2 = \frac{1}{T}\int_0^T (v_1 + v_2)^2 dt$$

$$= \frac{1}{T}\int_0^T v_1^2\, dt + \frac{1}{T}\int_0^T 2v_1 v_2\, dt + \frac{1}{T}\int_0^T v_2^2\, dt \qquad (Eq.21)$$

If the voltages are of two different frequencies (orthogonal), then the term $v_1 v_2$ in the equation above is equal to zero.

3. Electrical Power

In this section, we cover topics related to several types of electrical power. In practice, we recognise that there are real power (P), reactive power (Q), and apparent power (S).

3.1. Real Electrical Power (P)

The P has the unit of Watts and the unit symbol W. The P is sometimes referred to as average power or real power, but much more commonly, it is just plain power. If someone says simply, "power," it means P.

$$p(t) = \frac{V_m I_m}{2}\left[\cos\theta_v - \theta_i + \cos\left(4\pi f t + (\theta_v - \theta_i)\right)\right] \qquad (Eq.22)$$

- This second term in the equation above is still power, even though its average is zero.
- From this second term, we define a term called the reactive power Q.
- As illustrated below, the real power is the power that flows one way for a quarter of a cycle, then the other way for a quarter of a cycle.

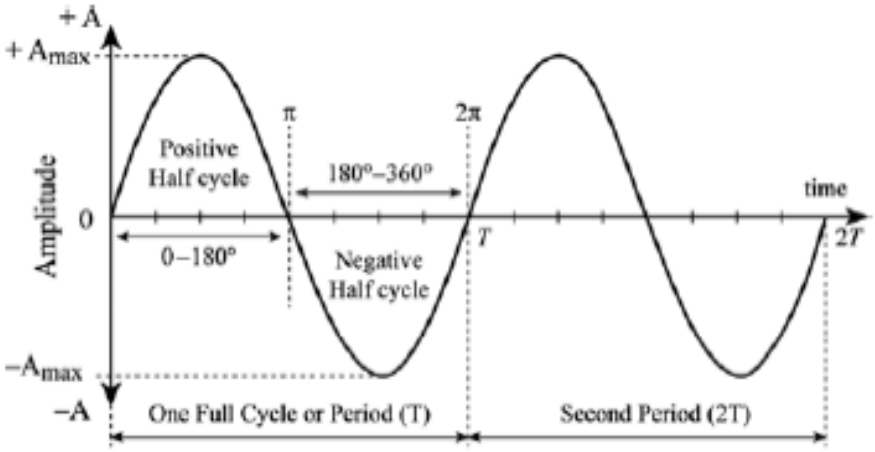


Figure 1: Sinusoidal waveform

3.2. Reactive Electrical Power (Q)

The reactive electrical power Q delivers no energy on average. It represents the energy that flows forwards and backwards. No net flow of energy, but it accounts for part of the current flowing in the

system – hence we need to include in our calculations. The Q is positive for loads that are primarily inductive and negative if the loads are primarily capacitive. Its units are VAR (volt-amp-reactive):

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) = V_{RMS} I_{RMS} \sin(\theta_v - \theta_i) \qquad (Eq. 23)$$

For more information about the reactive power, see further this YouTube video link: https://www.youtube.com/watch?v=VFPwgjGHqFQ

3.3. Why Q is important?

Take induction motor for example: we need current to supply the magnetic field that makes the motor work.



Figure 2: Induction motor

Just real power, P cannot supply this current i.e. hence we need reactive power, Q. The increase in current will increase losses. Hence, we need to quantify Q. If you include an inductor or a capacitor, your circuit is consuming Q (i.e. this is through the tracks/wires and coils).

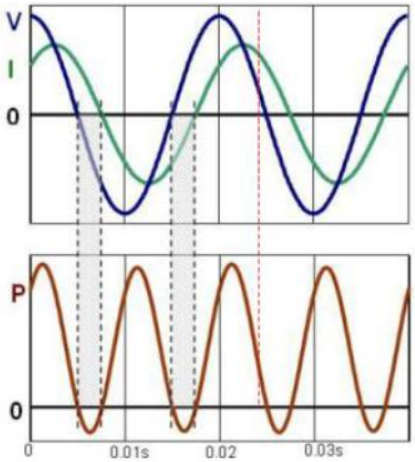


Figure 3: V, I, and P waveforms

There is more on the topic of the reactive power Q later in the subsequent courses in the programme.

3.4. Power Factor (PF)

Power factor is defined as the cosine of the angle between the voltage and the current. It is calculated from:

$$pf = \cos(\theta_v - \theta_i) \qquad (Eq. 24)$$

It is usually in the form of 'leading' or 'lagging' as the cosine value will always produce a positive value (i.e. 0 to 1).

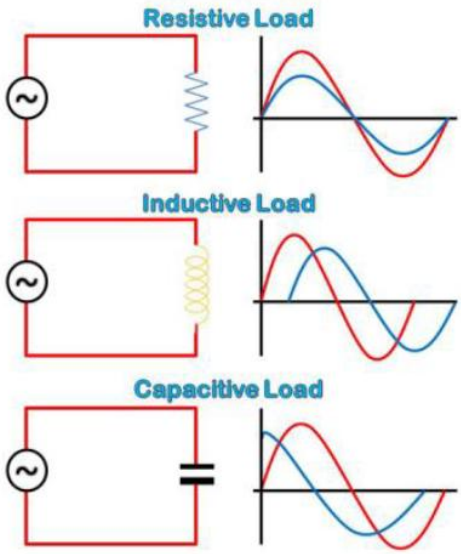


Figure 4: Types of loads in the power circuit

How do we know lead or lag? If the load is inductive, the current lags the voltage in time. As a result, the angle of current will be less than the voltage's angle. Then, the current lags voltage. As a result, power factor is lagging.

On other hand, if load is capacitive, the power factor is leading as the current leads.

3.5. Complex or Apparent Electrical Power (S)

Apparent power is another power quantity that is useful i.e. it is complex power S and calculated from:

$$S = P + jQ \qquad (Eq. 25)$$

The unit of complex power is volt-ampere (VA). The S is a complex number with a real part of P and an imaginary part of Q. The S is also the product of phasor voltage and phasor current:

$$S = VI^* \qquad (Eq. 26)$$

Note: Asterisk indicates the complex conjugate.

Furthermore, the power factor (pf) is defined as the ratio of average power to apparent power:

$$pf = \frac{P}{|S|} \quad \text{or} \quad pf = \frac{P}{S} = \cos(\theta_v - \theta_i) \qquad (Eq. 27)$$

3.6. Electrical Power Triangle

The figure showing below shows an easy way to visualise the relationship among P, Q, S, and power factor.

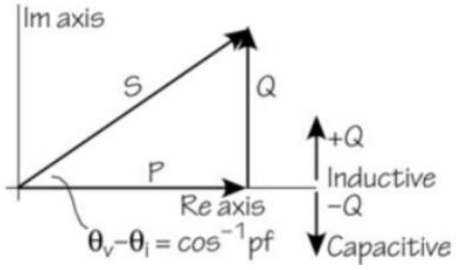
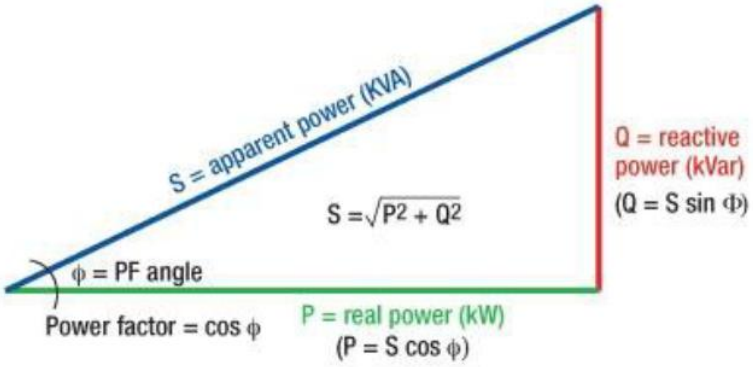


Figure 6: Power triangle

As shown below, the apparent power is calculated from real power and reactive power.



$$\phi = \theta_v - \theta_i$$

Figure 5: Power triangle

3.7. Apparent Electrical Power S - Summary

Apparent power S is magnitude of the complex power:

$$S = P + jQ \qquad (Eq.28)$$

Where:

$$S = |S| = \sqrt{P^2 + Q^2} \qquad (Eq.29)$$

Thus, we can calculate apparent power for RMS values.

$$S = V_{RMS} I_{RMS} \qquad (Eq.30)$$

And

$$\angle S = \tan^{-1}\left(\frac{Q}{P}\right) = pf \qquad (Eq.31)$$

The unit of apparent power S is VA, reactive power Q is Var, and real power P is Watt.

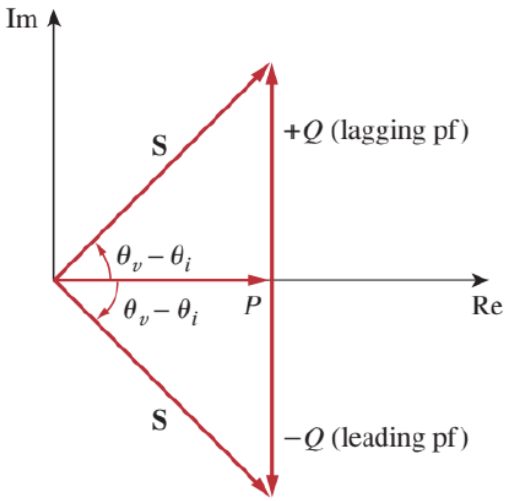
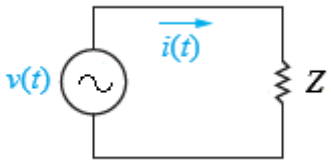


Figure 6: Apparent power, real power, and reactive power

Example 4: Apparent Power and Power Factor

A series-connected load draws a current $i(t) = 4\cos(100\pi t + 10°)$ A when the applied voltage is $v(t) = 120\cos(100\pi t - 20°)$ V.



a. Find the apparent power and the power factor of the load. [4 marks]

b. Determine the power factor using the load impedance. [4 marks]

c. Determine the element values that form the series-connected load. [4 marks]

Solution

a. The apparent power is:

$$S = V_{RMS}I_{RMS} = \left(\frac{120}{\sqrt{2}}\right)\left(\frac{4}{\sqrt{2}}\right) = 240 \text{ VA}$$

The power factor is:

$$pf = \cos(\theta_v - \theta_i) = \cos(-20° - 10°) = 0.866 \text{ (leading)}$$

The pf is leading because the current leads the voltage.

b. The pf may also be obtained from the load impedance:

$$Z = \frac{V}{I} = \frac{120\angle -20°}{4\angle 10°} = 30\angle -30° = 25.98 - j15\ \Omega$$

Thus

$$pf = \cos(-30°) = 0.866\ (\text{leading})$$

c. The load impedance Z can be modelled by a 25.98 Ω resistor in series with a capacitor with:

$$X_C = -\frac{1}{\omega C} = -15$$

Assuming the line frequency is 50 Hz, thus:

$$C = \frac{1}{15(2\pi f)} = \frac{1}{15(100\pi)} = 212.2\ \mu\text{F}$$

3.8. Reactive Electrical Power

For a given electrical system, real and reactive powers are calculated from:

$$P = V_{RMS} I_{RMS} \cos(\theta_v - \theta_i) \qquad (Eq.32)$$

And

$$Q = V_{RMS} I_{RMS} \sin(\theta_v - \theta_i) \qquad (Eq.32)$$

The P is the actual power dissipated by the load i.e. the useful power. The Q is energy exchange between source and reactive part of the load.

- The $Q = 0$ for resistive loads (unity pf).
- The $Q < 0$ for capacitive loads (leading pf).
- The $Q > 0$ for inductive loads (lagging pf).

3.8. Summary

Complex power:

$$S = P + jQ = V_{RMS}(I_{RMS})^* = |V_{RMS}||I_{RMS}|\angle(\theta_v - \theta_i) \qquad (Eq.33)$$

Apparent power:

$$S = |S| = |V_{RMS}||I_{RMS}| = \sqrt{P^2 + Q^2} \qquad (Eq.34)$$

Real power:

$$P = \text{Re}(S) = S\cos(\theta_v - \theta_i) \qquad (Eq.35)$$

Reactive power:

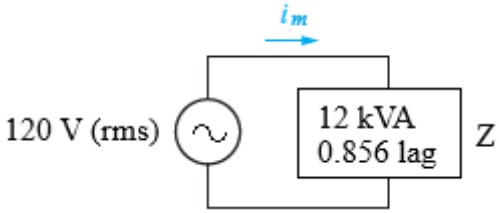
$$Q = \text{Im}(S) = S\sin(\theta_v - \theta_i) \qquad (Eq.36)$$

Power factor:

$$\frac{P}{S} = \cos(\theta_v - \theta_i) \qquad (Eq.37)$$

Example 5: Load Power and Power Factor

A load Z draws 12 kVA at a power factor of 0.856 lagging from a 120-V rms sinusoidal source.



Calculate:

a. the average and reactive powers delivered to the load, [6 marks]

b. the peak current, and [6 marks]

c. the load impedance. [2 marks]

Solution

a. Given that $pf = \cos\theta = 0.856$, we obtain the power angle as:

$$\theta = \cos^{-1} 0.856 = 31.13°$$

If the apparent power is S = 12,000 VA, then the average or real power is:

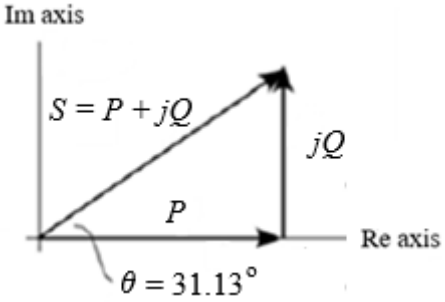
$$P = S\cos\theta = (12 \times 10^3)(0.856) = 10,272 \text{ kW}$$

While the reactive power is:

$$Q = S\sin\theta = (12 \times 10^3)(0.517) = 6.204 \text{ kVA}$$

b. Since the pf is lagging, the complex power is:

$$S = P + jQ = 10.272 + j6.204 \text{ kVA}$$



From $S = V_{RMS}I^*_{RMS}$, we obtain:

$$I_{RMS}^{*} = \frac{S}{V_{RMS}} = \frac{10{,}272 + j6{,}204}{120\angle 0°} = 85.6 + j51.7 \text{ A} = 100\angle 31.13° \text{ A}$$

Thus, $I_{RMS} = 100\angle -31.13°$ and the peak current is:

$$I_m = \sqrt{2} I_{RMS} = \sqrt{2}(100) = 141.4 \text{ A}$$

c. The load impedance is:

$$Z = \frac{V_{RMS}}{I_{RMS}} = \frac{120\angle 0°}{100\angle -31.13°} = 1.2\angle 31.13° \ \Omega$$

Thus, this is an inductive impedance.

4. Principle of Power Conservation

Just like DC system, the principle of conservation of power applies to AC systems as well.

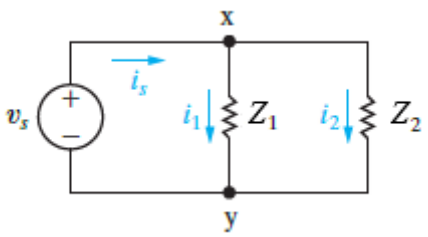


Figure 7: Parallel circuit configuration

Given a parallel circuit as shown above.

$$I = I_1 + I_2 \qquad (Eq.38)$$

Thus

$$S = VI^{*} = V(I_1^{*} + I_2^{*}) = VI_1^{*} + VI_2^{*} = S_1 + S_2 \qquad (Eq.39)$$

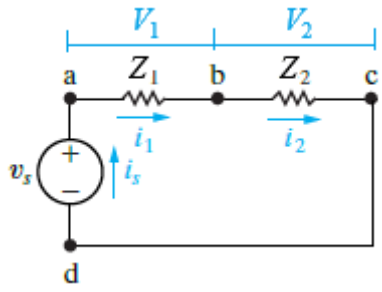


Figure 8: Series circuit configuration

For series configuration as shown above, the voltage is calculated as:

$$V = V_1 + V_2 \qquad (Eq.40)$$

Thus

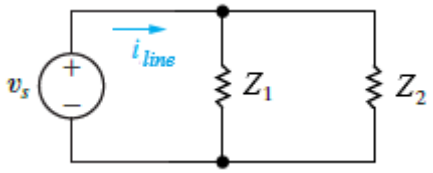
$$S = VI^* = (V_1 + V_2)I^* = V_1I^* + V_2I^* = S_1 + S_2 \qquad (Eq.41)$$

Whether the loads are connected in series or in parallel, the total power supplied by the source equals the total power delivered to the load, so:

$$S = S_1 + S_2 + \cdots + S_N \qquad (Eq.42)$$

Example 6: Electrical Distribution Load

As shown in the figures below, the electricity system has two loads. Find total power, reactive power, apparent power, and the line current. [20 marks]



Solution

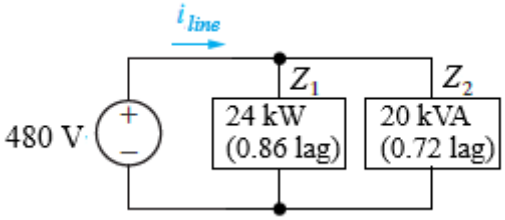
Load 1:

The power in the first load is:

P_1 = 24 kW (with a power factor of 0.86 lagging)

If we look at the power triangle and remember that the tangent of the angle is Q/P, then the reactive power is:

$$Q_1 = P_1 \tan(\cos^{-1} pf) = 24 \tan(\cos^{-1} 0.86) = 14.24 \text{ kVAR}$$



Load 2:

The power in the second load is:

$$P_2 = |S_2| pf_2 = (20)(0.72) = 14.4 \text{ kW}$$

And

$$Q_2 = |S_2| \sin(\cos^{-1} pf_2) = (20)(0.694) = 13.88 \text{ kVar}$$

The power triangle tells us that Q and S are related by the sine of the power-factor angle.

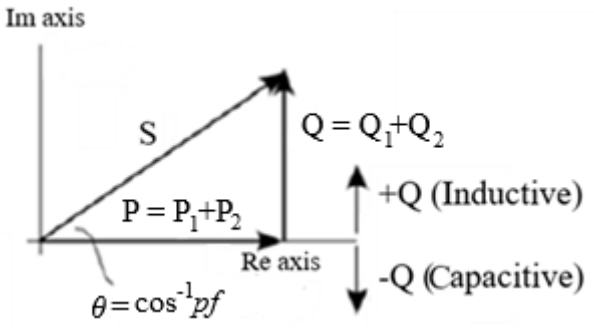
Conservation principle for P, S and Q:

$$P_{total} = P_1 + P_2 = 24 \text{ kW} + 14.4 \text{ kW} = 38.4 \text{ kW}$$

$$Q_{total} = Q_1 + Q_2 = 14.24\ \text{kW} + 13.88\ \text{kW} = 28.12\ \text{kVar}$$

$$S_{total} = P_{total} + jQ_{total} = 38.4\ \text{kW} + \text{j}28.12\ \text{kVAR} = 47.6\angle 36.2°\ \text{kVA}$$

$$pf_{total} = \frac{P_{total}}{|S_{total}|} = \frac{38.4\ \text{kW}}{47.6\ \text{kVA}} = 0.807\ \text{(lagging)}$$



The current is calculated from $S = VI^*$:

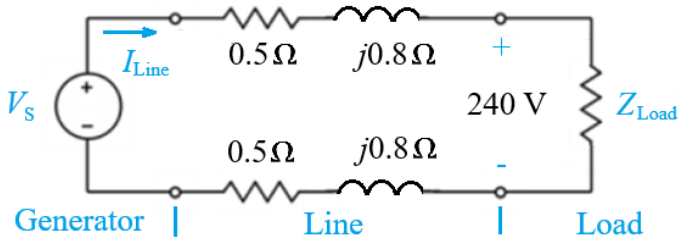
$$I^*_{line} = \frac{S_{total}}{V_{line}} = \frac{47.6 \times 10^3 \angle 36.2°}{480} = 99.2\angle 36.2°\ \text{A}$$

Note the sign, thus

$$I_{line} = 99.2\angle -36.2°\ \text{A}$$

Example 7: Electrical Motor Load

For a 2 HP motor load with 0.78 lagging power factor and 85% efficiency, find the power delivered to the motor, the line current, the power lost in the line, the total P, Q, and S, the power factor at the generator, the percent voltage regulation, and the percent efficiency. [24 marks]



Solution

If 1 HP is 746 W and the motor is 85% efficient, so the motor power input is:

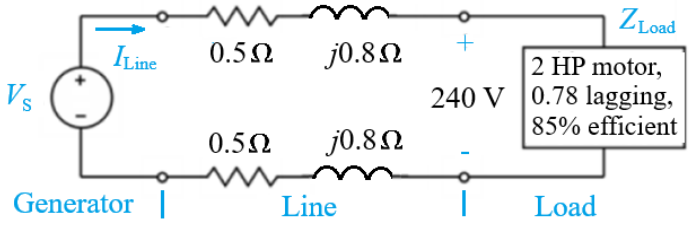
$$P_{load} = \frac{P}{\%\eta} = \frac{2(746)}{0.85} = 1.755\ \text{kW}$$

The power factor angle is:

$$\theta = \cos^{-1} 0.78 = 38.74°$$

The reactive power input to the motor is:

$$Q_{load} = 1.755 \tan(38.74°) = 1.408\ \text{kVar}$$



The line current is calculated from:

$$P_{load} = |V||I_{line}|(pf_{load})$$

So

$$I_{line} = \frac{P_{Load}}{V_{load}(pf_{load})} \angle -\cos^{-1} pf$$

$$= \frac{1.755 \times 10^3}{(240)(0.78)} \angle -\cos^{-1} 0.78$$

$$= 9.375 \angle -38.7° \text{ A}$$

The line power loss is mainly resistive part:

$$P_{line} = R_{line}|I_{line}|^2$$

$$= (2 \times 0.5\ \Omega)(9.375\ \text{A})^2$$

$$= 87.9\ \text{W}$$

The generator's voltage using KVL:

$$V_s = V_{load} + V_{line}$$

$$= V_{load} + (R_{line} + jX_{line})I_{line}$$

$$= 240 + (0.5 + j0.8 + 0.5 + j0.8)(9.375 \angle -38.74°)$$

$$= 240 + (1 + j1.6)(9.375 \angle -38.74°)$$

$$= 240 + (1.887 \angle 40°)(9.375 \angle -38.74°)$$

$$= 256.8 \angle 1.26° \text{ V}$$

We need to get Q and S. Beginning with apparent power:

$$S = VI^*$$

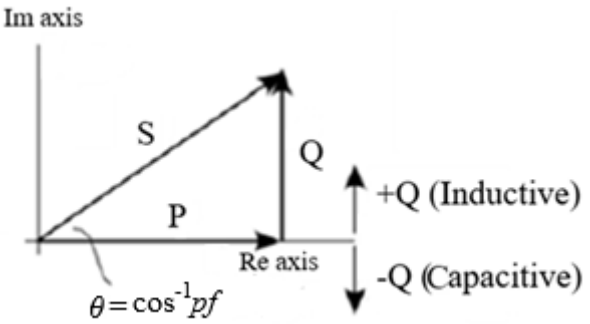
Thus

$$S_s = V_s I_s = (256.8 \angle 1.26°)(9.375 \angle 38.74°) = 2.408 \text{ kVA} \angle 40.0°$$

$$P_s = \text{Re}[S_s] = 2.408 \text{ kVA} \times \cos 40° = 1.844 \text{ kW}$$

$$Q_s = \text{Im}[S_s] = 2.408 \text{ kVA} \times \sin 40° = 1.548 \text{ kVar}$$

$$pf_s = \cos(40.0°) = 0.766 \text{ (lagging)}$$



Now, the voltage regulation (i.e. voltage raise due to sudden loss of load):

$$\%VR = \left(\frac{|V_s| - |V_{load}|}{|V_{load}|}\right) \times 100\%$$

$$= \left(\frac{256.8 - 240}{240}\right) \times 100\%$$

$$= 7.0\%$$

The percent efficiency is:

$$\eta = \left(\frac{P_{load}}{P_s}\right) \times 100\%$$

$$= \left(\frac{1.755}{1.844}\right) \times 100\%$$

$$= 95.2\%$$

5. Electricity Consumption Cost

Costing of electricity depends on generation, transmission and distribution costs and retail costs. The cost is also heavily influenced by fluctuating fuel prices (coal, gas), supply/demand imbalances, and weather.

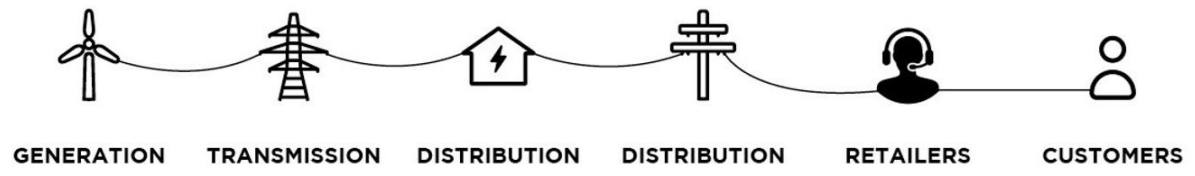


Figure 9: Electricity generation, transmission and distribution

5.1. Factors in Calculating Power Bills

In general, key items considered in calculating the power bills include infrastructure investment, maintenance, regulatory policies, and the mix of energy sources, such as hydropower or fossil fuels.

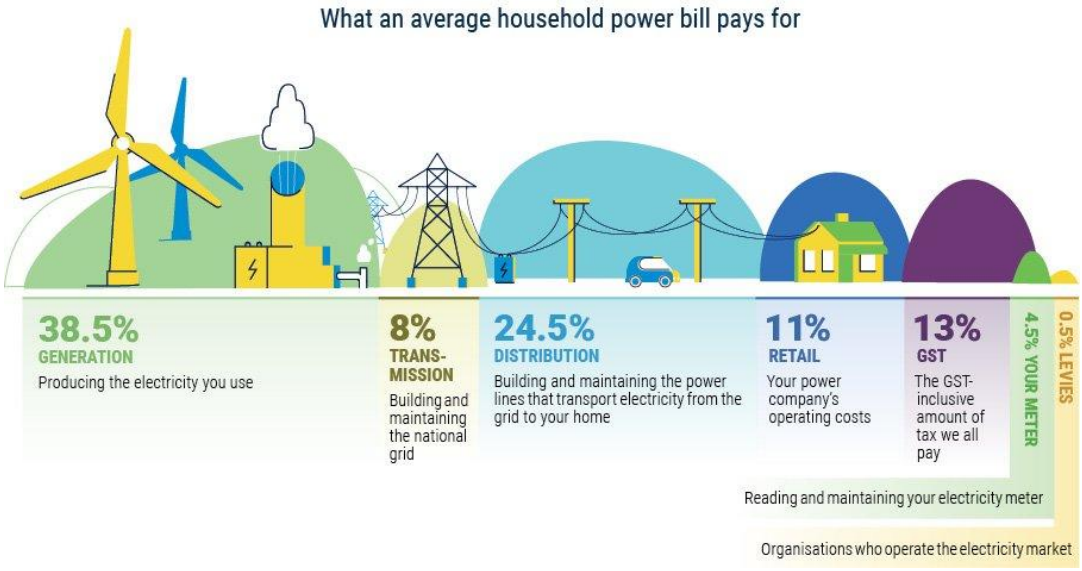


Figure 10: Breakdown of the cost of electricity in New Zealand

Factors considered for costing the electricity:

- Investment in the electricity generation and transmission.
- Total energy consumed.
- Efficiency of the use and power factor (leads to penalties when lacking).

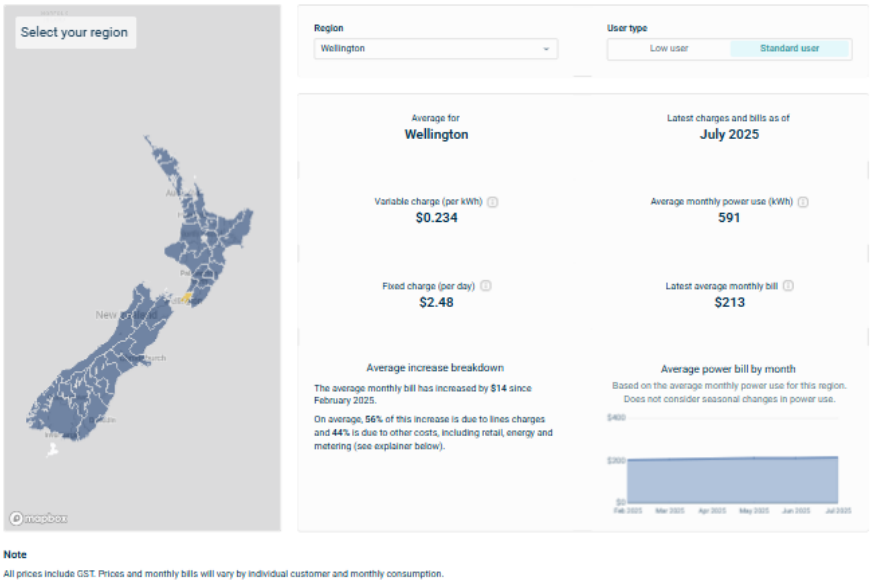


Figure 11: Average power bill for a standard user in Wellington region (https://www.ea.govt.nz/data-and-insights/charts-and-dashboards/regional-power-prices/)

Example 8: Electricity Consumption

A manufacturing industry consumes 200 MWh in one month. If the maximum demand is 1,600 kW, calculate the electricity bill based on the following two-part rate: [10 marks]

- Demand charge: $5.00 per month per kW of billing demand.
- Energy charge: 8 cents per kWh for the first 50,000 kWh, 5 cents per kWh for the remaining energy.

Solution

The demand charge is:

$$\text{Max. Demand Cost} = (\$\ 5.00/\text{kW})(1{,}600\ \text{kW}) = \$\ 8{,}000$$

The energy charge for the first 50,000 kWh is:

$$\text{Initial Charge Cost} = (\$\ 0.08/\text{kWh})(50{,}000\ \text{kWh}) = \$\ 4{,}000$$

The remaining energy is:

$$W = 200{,}000\ \text{kWh} - 50{,}000\ \text{kWh} = 150{,}000\ \text{kWh}$$

The corresponding energy charge is:

$$\text{Energy Cost} = \$\ \text{Charge} \times W = (\$\ 0.05)(150{,}000\ \text{kWh}) = \$\ 7{,}500$$

Total bill of the month:

$$\text{Total Cost} = \text{Demand Cost} + \text{Initial Charge Cost} + \text{Energy Cost}$$

$$= \$\ 8{,}000 + \$\ 4{,}000 + \$\ 7{,}500 = \$\ 19{,}500$$

5.2. Role of Loads in Electricity Pricing

Loads with low power factors are costly to serve because they require large currents. A load with nonzero reactive power Q means that energy flows back and forth between the load and the source, giving rise to additional power losses. Hence, power companies penalise customers with power factor issues.

Example 9: Energy Costing

A 300-kW load supplied at 13 kV (RMS) operates 520 hours a month at 80 percent power factor. Calculate the average cost per month based on this simplified tariff: [8 marks]

- Energy charge: 6 cents per kWh
- Power-factor penalty: 0.1 percent of energy charge for every 0.01 that pf falls below 0.85.
- Power-factor credit: 0.1 percent of energy charge for every 0.01 that pf exceeds 0.85.

Solution

The energy consumed is:

$$W = P \times h = (300\ \text{kW})(520\ \text{h}) = 156{,}000\ \text{kWh}$$

The operating power factor, pf = 80% = 0.8, is 5(0.01) below the prescribed power factor of 0.85. Since there is 0.1 percent energy charge for every 0.01, there is a power-factor penalty charge of 0.5 percent. This amounts to an energy charge of:

$$\Delta W = W \times \%\text{charge} = 156{,}000\left(\frac{0.85 - 0.80}{0.01}\right)\left(\frac{0.1}{100}\right) = 780\ \text{kWh}$$

The total energy is:

$$W_T = W + \Delta W = 156{,}000\ \text{kWh} + 780\ \text{kWh} = 156{,}780\ \text{kWh}$$

The cost per month is given by:

$$\text{Cost/Month} = \text{UnitPrice} \times W_T = (\$\ 0.06)(156{,}780) = \$\ 9{,}406.80$$