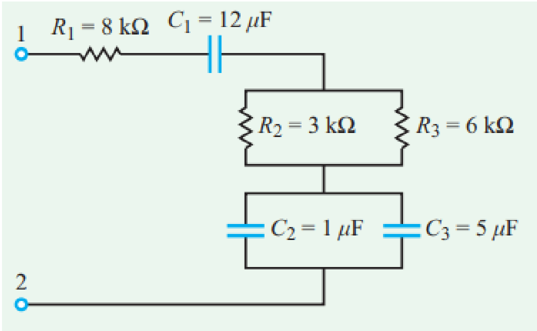
ECEN203 Tutorial 5: RC and RL First Order Circuits (with answers)

Answer all 19 questions. Answers are provided for your self-checking.

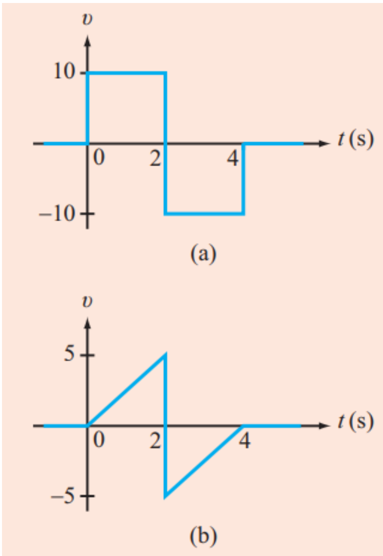
Q1. Reduce the circuit below into an equivalent configuration.



(Answer: 

)

Q2. Express the waveforms shown in (a) and (b) below in terms of unit step functions.

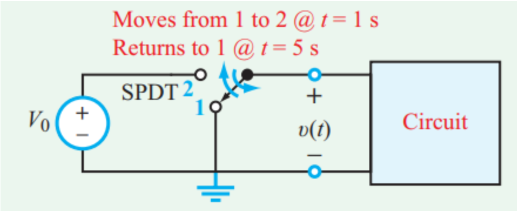


(Answer: (a) υ(t) = 10 u(t) − 20 u(t − 2) + 10 u(t − 4), (b) υ(t) = 2.5 r(t) − 10 u(t − 2) − 2.5 r(t − 4).)

Q3. How is u(t) related to u(−t)?

(Answer: They are mirror images of one another (with respect to the y-axis).)

Q4. Consider the SPDT switch below. Assume that it started out at position 2, was moved to position 1 at t = 1 s, and then moved back to position 2 at t = 5 s. <u>This is the reverse of the sequence shown in the figure</u>. Express υ(t) in terms of (a) units step functions and (b) the rectangle function.



(Answer: (a) $v(t) = V_0[u(1-t) + u(t-5)]$, (b) $v(t) = V_0\left[1 - \text{rect}\left(\frac{t-3}{4}\right)\right]$.)

Q5. The radioactive decay equation for a certain material is given by $n(t) = n_0 e^{-t/\tau}$, where n_0 is the initial count at t = 0. If τ = 2 × 108 s, how long is its half-life? [Half-life $t_{1/2}$ is the time it takes a material to decay to 50 percent of its initial value.]

(Answer: $t_{1/2}$ = 1.386 × 108 s = 4 years, 144 days, 12 hours, 10 minutes, 36 s.)

Q6. If the current i(t) through a resistor R decays exponentially with a time constant τ , what is the value of the power dissipated in the resistor at t = τ , compared with its value at t = 0?

(Answer: $p(t) = i^2R = I_0^2 R(e^{-t/\tau})^2 = I_0^2 Re^{-2t/\tau}$, $p(\tau)/p(0) = e^{-2} = 0.135$ or 13.5 percent.)

Q7. It is desired to build a parallel-plate capacitor capable of storing 1 mJ of energy when the voltage across it is 1 V. If the capacitor plates are 2 cm × 2 cm each and its insulating material is Teflon, what should the separation *d* be? Is such a capacitor practical?

Table 5-2: Relative electrical permittivity of common insulators: $\varepsilon_r = \varepsilon/\varepsilon_0$ and $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

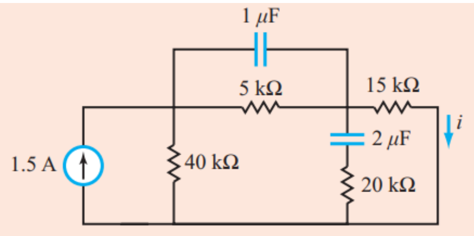
Material	Relative Permittivity ε_r
Air (at sea level)	1.0006
Teflon	2.1
Polystyrene	2.6
Paper	2–4
Glass	4.5–10
Quartz	3.8–5
Bakelite	5
Mica	5.4–6
Porcelain	5.7

(Answer: $d = 3.72 \times 10^{-12}$ m. No, it is not practical to build a capacitor with such a small d, because it is about two orders of magnitude smaller than the typical spacing between two adjacent atoms in a solid material.)

Q8. Instead of specifying *A* and calculating the spacing *d* needed to meet the 1 mJ requirement in Q7, suppose we specify *d* as 1 μm and then calculate *A*. How large would *A* have to be?

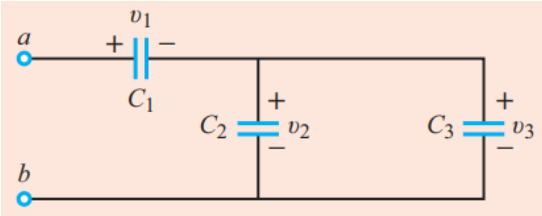
(Answer: A = 10.4 m × 10.4 m, equally impractical!)

Q9. Determine the current *i* in the circuit below, under dc conditions.



(Answer: i = 1 A)

Q10. Determine C_{eq} and $\upsilon_{eq}(0)$ at terminals (a, b) for the circuit below given that $C_1 = 6\ \mu F$, $C_2 = 4\ \mu F$, $C_3 = 8\ \mu F$, and the initial voltages on the three capacitors are $\upsilon_1(0) = 5$ V and $\upsilon_2(0) = \upsilon_3(0) = 10$ V, respectively.



(Answer: $C_{eq} = 4\ \mu F$, $\upsilon_{eq}(0) = 15$ V.)

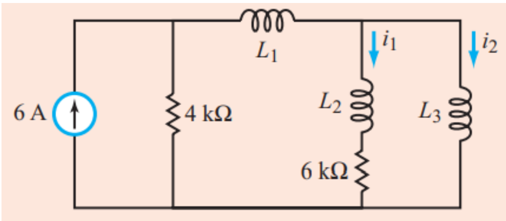
Q11. Suppose the circuit in Q10 is connected to a dc voltage source $V_0 = 12$ V. Assuming that the capacitors had no charge before they were connected to the voltage source, determine υ_1 and υ_2 given that $C_1 = 6\ \mu F$, $C_2 = 4\ \mu F$, and $C_3 = 8\ \mu F$.

(Answer: $\upsilon_1 = 8$ V, $\upsilon_2 = 4$ V.)

Q12. Calculate the inductance of a 20-turn aircore solenoid if its length is 4 cm and the radius of its circular cross section is 0.5 cm.

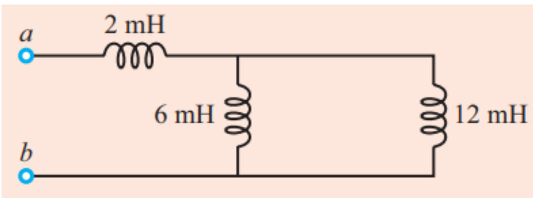
(Answer: $L = 9.87 \times 10^{-7}$ H $= 0.987\ \mu$H.)

Q13. Determine currents i_1 and i_2 in the circuit below, under dc conditions.



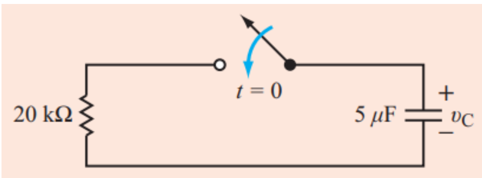
(Answer: $i_1 = 0$, $i_2 = 6$ A.)

Q14. Determine L_{eq} at terminals (a, b) in the circuit below.



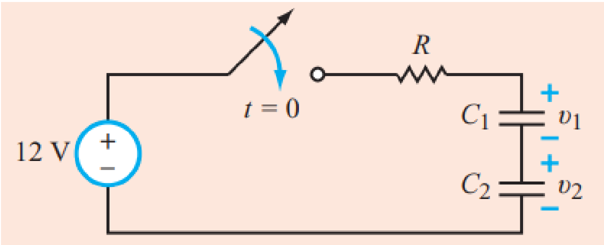
(Answer: $L_{eq} = 6$ mH.)

Q15. If in the circuit below $\upsilon_C(0^-) = 24$ V, determine $\upsilon_C(t)$ for $t \geq 0$.



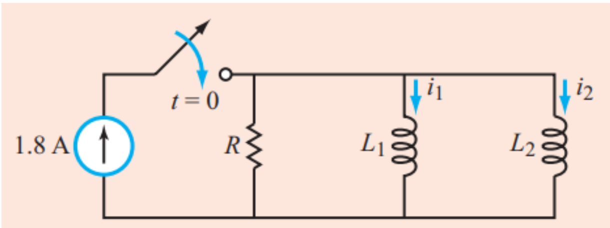
(Answer: $\upsilon_C(t) = 24e^{-10t}$ V for $t \geq 0$.)

Q16. Determine $\upsilon_1(t)$ and $\upsilon_2(t)$ for $t \geq 0$, given that in the circuit below $C_1 = 6\ \mu F$, $C_2 = 3\ \mu F$, R = 100 k , and neither capacitor had any charge prior to t = 0.



(Answer: $\upsilon_1(t) = 4(1 - e^{-5t})$ V, for $t \geq 0$, $\upsilon_2(t) = 8(1 - e^{-5t})$ V, for $t \geq 0$.)

Q17. Determine $i_1(t)$ and $i_2(t)$ for $t \geq 0$ given that, in the circuit below, $L_1 = 6$ mH, $L_2 = 12$ mH, and R = 2 . Assume $i_1(0^-) = i_2(0^-) = 0$.



(Answer: $i_1(t) = 1.2(1 - e^{-500t})\,u(t)$ A, $i_2(t) = 0.6(1 - e^{-500t})\,u(t)$ A.)

Q18. The input signal to an ideal integrator circuit with $RC = 2 \times 10^{-3}$ s and $V_{cc} = 15$ V is given by $\upsilon_s(t) = 2 \sin 100t$ V. What is $\upsilon_{out}(t)$?

(Answer: $\upsilon_{out}(t) = 10[\cos(100t) - 1]$ V.)

Q19. Repeat Q18 for a differentiator instead of an integrator.

(Answer: $\upsilon_{out}(t) = -0.4 \cos 100t$ V.)