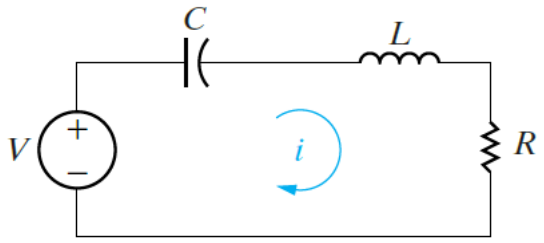
XMUT203 Analogue Circuits and Systems

Tutorial 6: Second-Order RLC Circuits (Answer)

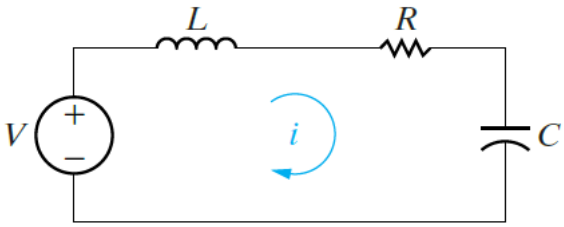
A. Introduction to Second-Order RLC Circuits

1. Find the transfer functions $G(j\omega) = \tilde{V}_o(\omega)/\tilde{V}_i(\omega)$ for each of the circuits shown in the figures given below.

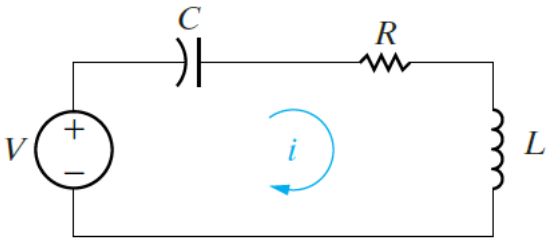
   a. Circuit 1 (e.g. V_o is taken across the resistor). [Answer: $G(\omega) = j\omega RC/(1 + j\omega RC - \omega^2 LC)$ or $G(\omega) = -[-j\omega(R/L)]/[\omega^2 - j\omega(R/L) - 1/LC]$] [10 marks]



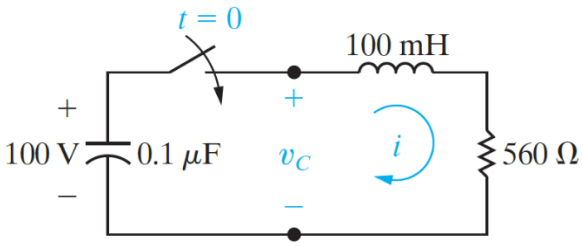
   b. Circuit 2 (e.g. V_o is taken across the capacitor). [Answer: $G(\omega) = 1/[1 + j\omega RC - \omega^2 LC]$ or $G(\omega) = [-j\omega(1/LC)]/\omega^2 - j\omega(R/L) - 1/LC]$] [10 marks]



   c. Circuit 3 (e.g. V_o is taken across the inductor). [Answer: $G(\omega) = [-\omega^2 LC]/[1 + j\omega RC - \omega^2 LC]$ or $G(\omega) = [\omega^2]/[\omega^2 - j\omega(R/L) - 1/LC]$] [10 marks]



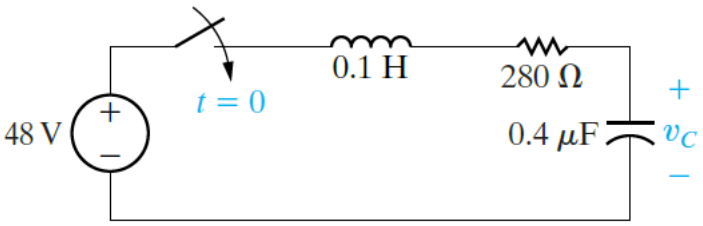
2. The 0.1 μF capacitor in the natural response series RLC circuit shown in figure below is charged to 100 V. At t = 0 the capacitor is discharged through a series combination of a 100 mH inductor and a 560 Ω resistor.



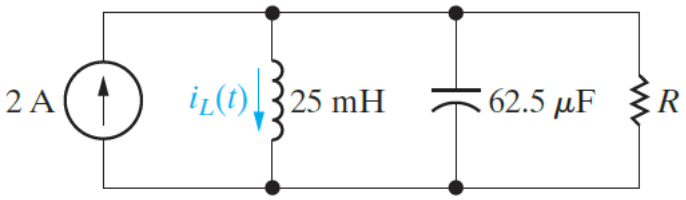
a. Find $i(t)$ for t > 0. [Answer: $i(t) = 0.1402e^{-2,800t}\sin 9,600t \quad t \geq 0$] [28 marks]

b. Find $v_c(t)$ for t > 0. [Answer: $v_c(t) = (100\cos 9,600t + 29.17\sin 9,600t)e^{-2,800t} \quad t \geq 0$] [8 marks]

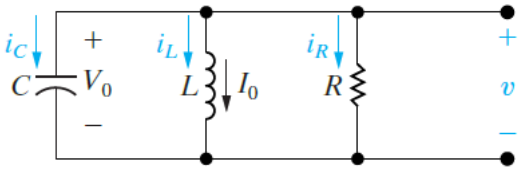
3. No energy is stored in the 100 mH inductor or the 0.4 μF capacitor of a step response series RLC circuit when the switch in the circuit shown in the figure below is closed. Find $v_c(t)$ for $t > 0$. [Answer: $v_c(t) = 48 - 48e^{-1,400t}\cos 4,800t - 14e^{-1,400t}\sin 4,800t \quad t \geq 0$] [24 marks]



4. Assume that at the instant the 2 A DC current source is applied to the circuit in the figure below, the initial current in the 25 mH inductor is 1 A, and the initial voltage on the capacitor is 50 V (i.e. positive at the upper terminal). Find the expression for $i_L(t)$ for $t \geq 0$ if R equals 12.5 Ω. [Answer: $i_L(t) = 2 - e^{-640t}\cos 480t + 2.83e^{-640t}\sin 480t$ A, $t \geq 0$] [26 marks]



5. The voltage $v(t)$ of a natural response parallel RLC circuit as shown in the figure below is $v(t) = 120e^{-400t}\cos 300t + 80e^{-400t}\sin 300t$ V.



When the capacitor is 250 F, find the values and expression for the following:

a. L. [Answer: 16 mH] [8 marks]

b. R. [Answer: 5 Ω] [2 marks]

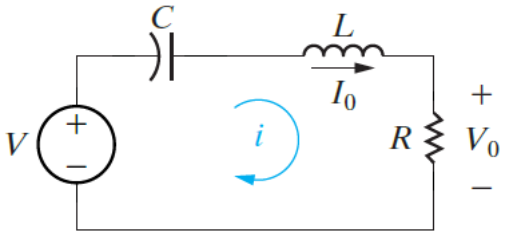
c. V_0. [Answer: 120 V] [2 marks]

d. I_0. [Answer: -18 A] [12 marks]

e. $i_L(t)$. [Answer: $i_L(t) = e^{-400t}(-18\cos 300t + \sin 300t)$ A, $t \geq 0$]. [10 marks]

B. Transient and Frequency Response of Second-Order RLC Circuits

6. For the series resonant RLC circuit given below.

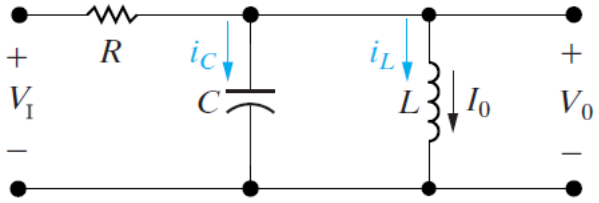


a. Let's calculate the current flowing in the series resonant circuit as a function of the frequency of the input voltage. In this case, we have drawn the circuit directly in the phasor representation. [Answer: $\tilde{I}(\omega)/\tilde{V}_i(\omega) = 1/[R + j\omega L + 1/j\omega C]$] [6 marks]

b. Simulate the gain response of the system with $R = C = L = 1$. [8 marks]

c. Simulate the frequency response of the system in MATLAB. [8 marks]

7. The following figure shows a series-parallel RLC circuit with the resistor R is connected in series with a parallel of inductor L and capacitor C. The input and output of the circuit are as indicated in the diagram.



The transfer function of the output voltage over the input voltage $G(\omega) = V_o(\omega)/V_i(\omega)$ of the circuit is:

$$G(\omega) = \frac{j\omega/(RC)}{(j\omega)^2 + j\omega/(RC) + 1/(LC)}$$

In this type of circuit, the product LC controls the resonant frequency while RC controls the bandwidth. To tune the resonant frequency of the circuit to the frequency of 1 rad/s, set $L = C = 1$ and vary the value of the R to tune the bandwidth of the frequency response. In the filter circuit applications, this type of RLC circuit is commonly termed as bandpass filter circuit.

a. Simulate the frequency response of the circuit for R = L = C = 1. Comment on the result of the simulation. Note: use `tf()` function in MATLAB to specify the circuit's transfer and use bode function to plot the frequency response of the circuit. [8 marks]

b. To get a narrower bandwidth, try increasing values of R. Simulate the circuit for additional values of R = 2 and 20 Ω. [8 marks]

8. For the series-parallel RLC circuit that you have in the previous circuit, simulate its transient response. Confirm the attenuation properties of the circuit with R = 20 by simulating how this circuit transforms sine waves with frequency 0.9, 1, and 1.1 rad/s. [10 marks]

C. Design of Second-Order RLC Circuits

9. Design both series and parallel resonant RLC circuits that have a resonant frequency of 876 kHz, and a Q-factor of 10. Use a fixed capacitor value of 1 nF. What values of inductance and resistance are required? [Answer: L = 33 µH in both cases. R_{series} = 18.2 Ω, R_{parallel} = 1.82 kΩ] [6 marks]

10. A series-resonant RLC circuit has L = 1 mH and C = 10 µF. If the circuit is to have a bandwidth (B) of 15.9 Hz, what value of R will be needed, and what will be the resulting Q-factor of the circuit? [Answer: Q = 100, R = 0.1 Ω] [6 marks]

11. Design a series-resonant RLC circuit that passes signals in the range of 1.75 MHz to 1.85 MHz (that is, the bandwidth must extend to these frequencies). Use a 10 pF capacitor. [Answer: L = 783 µH , R = 492 Ω] [6 marks]

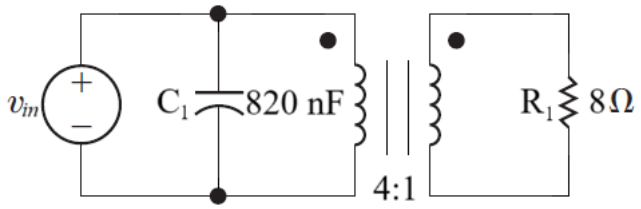
12. You are working on a receiver that is a RLC circuit required to collect a signal that is being transmitted at 100 kHz. The circuit must use a fixed inductor of 10 µH.

    a. What value capacitor should be used to build a series-resonant circuit with a resonant frequency at 100 kHz? [Answer: C = 253 nF] [2 marks]

    b. You are now restricted to using a single capacitor with an E12 series value. What is the resonant frequency closest to the desired 100 kHz that you could achieve? [Answer: f_0 = 96.8 kHz] [4 marks]

    c. Choose a value of R that ensures that your modified resonant circuit contains the original 100 kHz within its bandwidth. [Answer: $R \geq 0.403\ \Omega$] [6 marks]

13. An 8 Ω speaker is connected as part of a resonant circuit with a transformer as shown in the figure below.



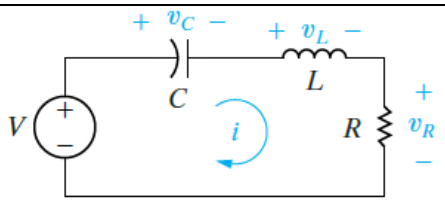
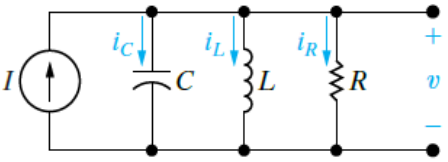
    a. If the circuit is intended to sound a single tone at 6 kHz, what should the equivalent inductance of the transformer be as seen from the primary? [Answer: $L = 860 \times 10^{-6}$ H] [2 marks]

    b. What will the Q-factor of the resulting circuit be? [Answer: Q = 3.95] [2 marks]

    c. You find that the circuit has been incorrectly configured, with the capacitor placed in the secondary side of the transformer, rather than the primary. What would the resonant frequency and Q-factor of the circuit then be? [Answer: f_0 = 24 kHz, $Q \approx 1$] [8 marks]

Appendix A – RLC Resonant Circuit

The parameters and equations of series RLC circuit are compared with parallel RLC circuit in the table below.

Parameter	Series	Parallel
Circuit diagram		
Transfer function	$\frac{V_R(\omega)}{V(\omega)} = \frac{j\omega RC}{(1-\omega^2 LC)+j\omega RC}$	$\frac{I_R(\omega)}{I(\omega)} = \frac{j\omega L}{(R-\omega^2 LCR)+j\omega L}$
Resonant frequency	$\omega_0 = \frac{1}{\sqrt{LC}}$	$\omega_0 = \frac{1}{\sqrt{LC}}$
Cut-off frequencies	$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{\omega_0^2}}$ $\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{\omega_0^2}}$	$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{\omega_0^2}}$ $\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{\omega_0^2}}$
Bandwidth	$B_s = \omega_2 - \omega_1 = \frac{R}{L}$	$B_p = \omega_2 - \omega_1 = \frac{1}{RC}$
Q-factor	$Q = \frac{\omega_0}{B_s} = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$	$Q = \frac{\omega_0}{B_p} = \omega_o RC = \frac{R}{\omega_o L}$