XMUT204 Electronic Design

Note 2b: Diode Models and Circuit Analysis

1. Introduction

In the first part of XMUT203 course, we have spent considerable time analysing circuits consisting of voltage sources and so-called linear elements such as resistors in order to be able to calculate the voltage at a particular point in the circuit, or to calculate the current through a branch of the circuit.

We would now like to use the same techniques as previous to analyse circuits that may also contain diodes. However, from the diode I-V curve (see Figure 4 in the previous chapter), we realise that this will be considerably more complicated, as the diode is a highly non-linear circuit element, with current through the diode highly dependent on the applied bias voltage and the polarity.

In order to make it possible to analyse a circuit containing one or more diodes, we now model the behaviour of a real diode. These models will only approximate the behaviour of a real diode, but will make it easier to analyse a diode containing circuit.

We can refine these diode models by increasing the complexity of the model, which will also increase the precision with which we can analyse the circuit. Note that these models will not be used to describe the reverse bias breakthrough region of operation.

Before we look at diode models, we will first use the technique of load line analysis to calculate the currents and voltages in a diode circuit.

2. Load Line Analysis

Load line analysis is a graphical technique to find the current and voltages in a circuit. It consists of simultaneously plotting two graphs and finding their intersection, which is equivalent to finding the simultaneous solution of two equations. In the case of a diode the two equations will consist of:

(a) I-V curve for the specific diode in the circuit (non-linear).
(b) Line defined by KVL applied around the circuit loop, taking into account voltage sources and other components (resistors) in the circuit (linear) – this is the actual load line.

The intersection of these two points is the operating point of the circuit, from which the relevant current and voltage and be determined.

Example 1: The I-V curve for a diode is given below (or can be sketched from the diode equation or experimental measurements). This diode is now connected in a circuit with a power supply and resistor as shown. Determine the current that will flow through the diode with an applied voltage of E = 5 V.

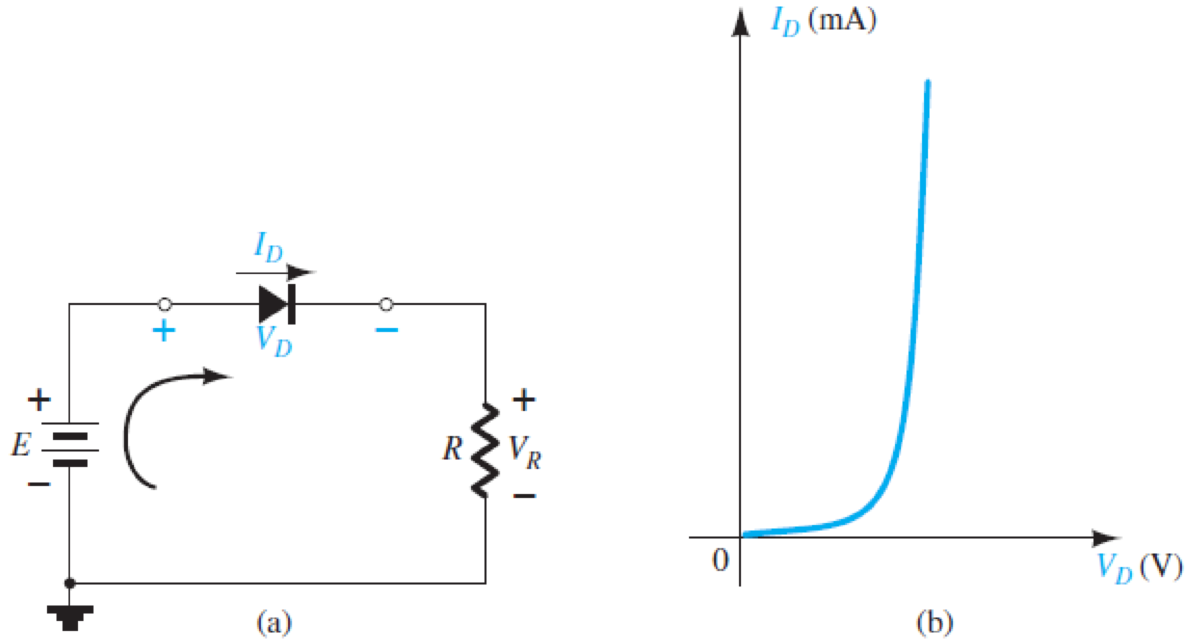


Figure 1: (a). The circuit (network) in which the diode is used. (b). The diode curve in the forward bias region. The intersection of these two curves will define the operating point of the device.

The diode is then forward biased and we can write the KVL equation around the loop as:

$$E - V_D - V_R = 0 \qquad \Rightarrow \qquad E = V_D + I_D R$$

The two variables that define the diode curve (I_D and V_D) thus also appear in the KVL loop, so that we can plot these two equations on the same graph. The load line resulting from the KVL equation can easily be plotted if we calculate: (1) the value of the current in the circuit when there is no diode in the circuit (V_D = 0 V) and (2) the value of the voltage drop over the diode (V_D) when there is no current flowing in the circuit (I_D = 0).

(1) From the KVL equation when $V_D = 0$ V we have $E = I_D R \Rightarrow I_D = E/R$. This will define the value of I_D on the graph where $V_D = 0$

(2) Similarly, when $I_D = 0$ we have $V_D = E$ which defines the value of V_D on the graph when $I_D = 0$.

A straight line between these points will then define the load line of the circuit, and a change in the load (the resistor) will change the intersection on the vertical axis. If we sketch both the load line and the diode curve on the same graph, it will look as in Figure 2.

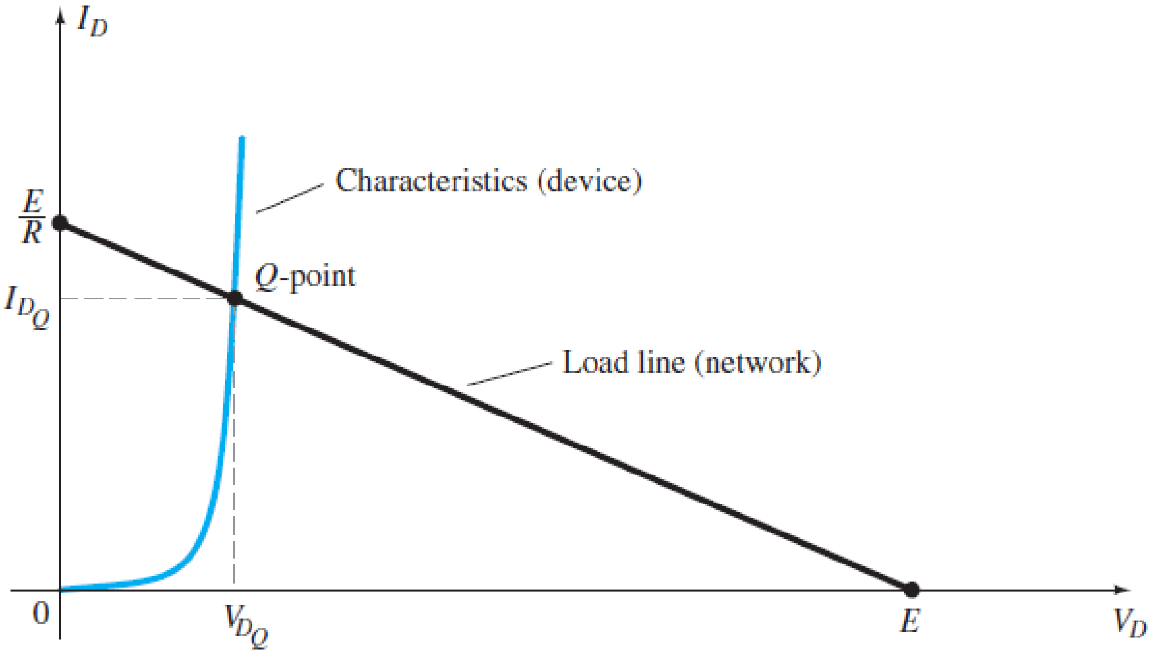
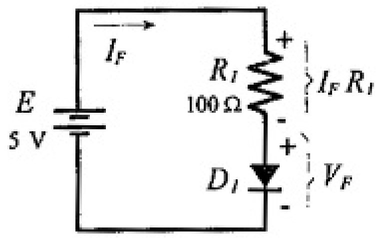


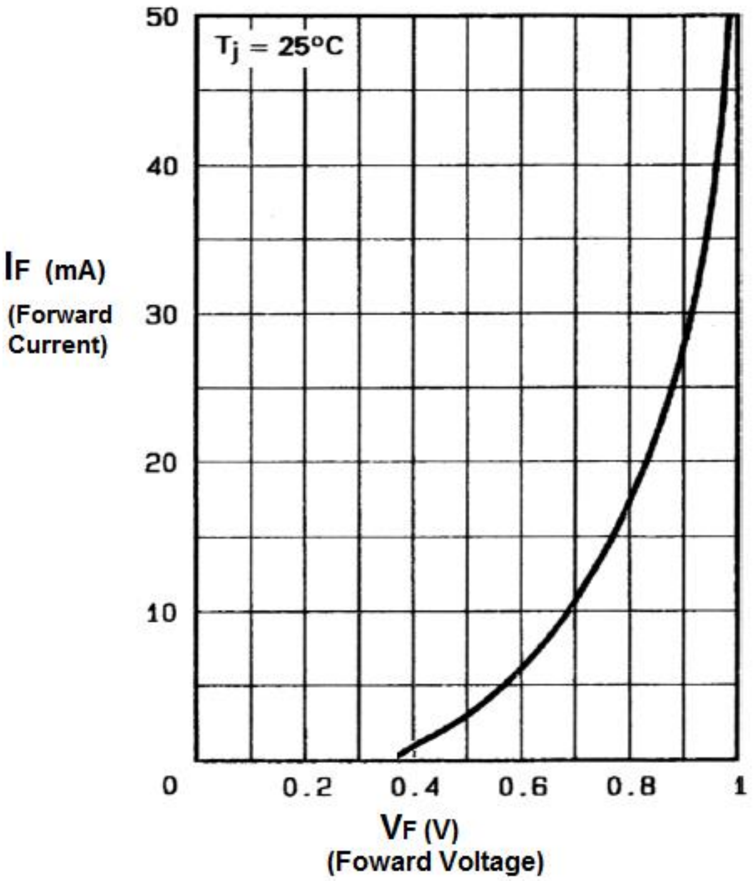
Figure 2: Construction of a load line to get the Q point.

The intersection between these two lines now defines the operating point of the device – the so-called quiescent point or Q-point. The values of I_D and V_D at this Q-point now define the voltage drop across the diode as well as the current through the diode.

Example for Tutorial 1 – Diode Load Line

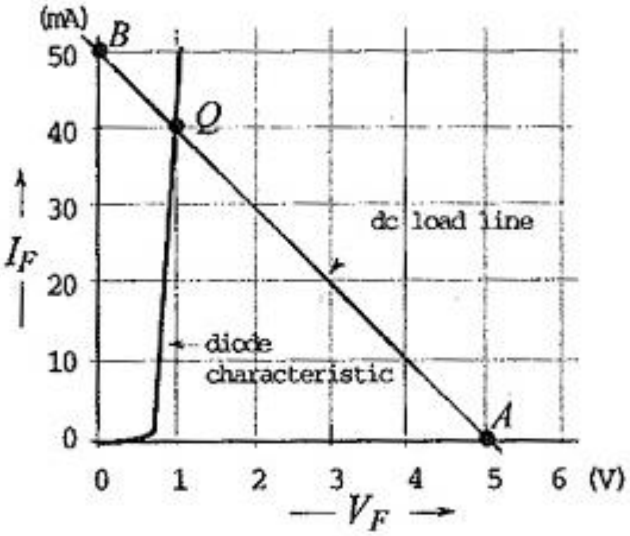
1. Sketch the load line if the diode is forward biased by a 10 V power supply and a 1 kΩ current limiting resistor is placed in series. Determine in your sketch the operating condition of the diode. [5 marks]





Answer

The graph for the load line analysis is given in the figure below



Extreme points in the graph for the biasing of the diode are:

$$I_F(\max) = \frac{V_F}{R_D} = \frac{5}{100} = 50 \text{ mA}$$

And

$$V_F(\max) = I_F \times R_D = (50\ \mathrm{mA})(100\ \Omega) = 5\ \mathrm{V}$$

Notice that the maximum forward voltage of the diode is 5 V at point A in the graph and the maximum forward current of the diode is 50 mA at point B.

The recommended operating condition of the diode is supposed to be designed at point Q in the graph.

3. The Ideal Diode Model

In the simplest model for the diode, it is viewed as an element that has zero resistance (short circuit) in forward bias and infinite resistance (open circuit) in reverse bias. This model then allows current to flow in only one direction and is only dependant on the polarity of the bias voltage. The operation of the diode can then also be described as being in one of two states:

(1) The ON state when $i_D > 0$
(2) The OFF state when $V_D < 0$ and thus $i_D = 0$.

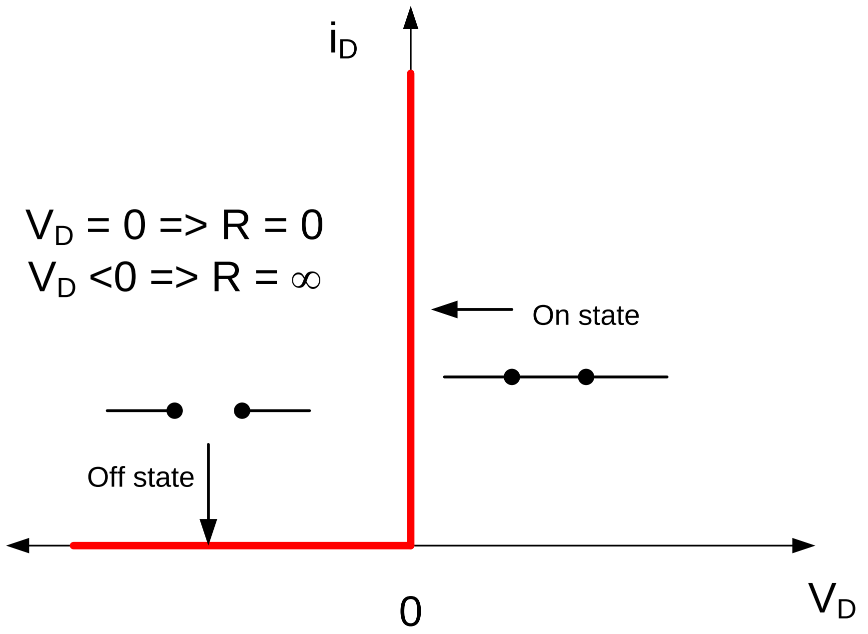


Figure 3: The ideal diode model for a real diode.

Although it may not be evident, this is generally an acceptable model for the diode as long as we are using large enough forward bias voltages so that we are past the cut-on point of the diode.

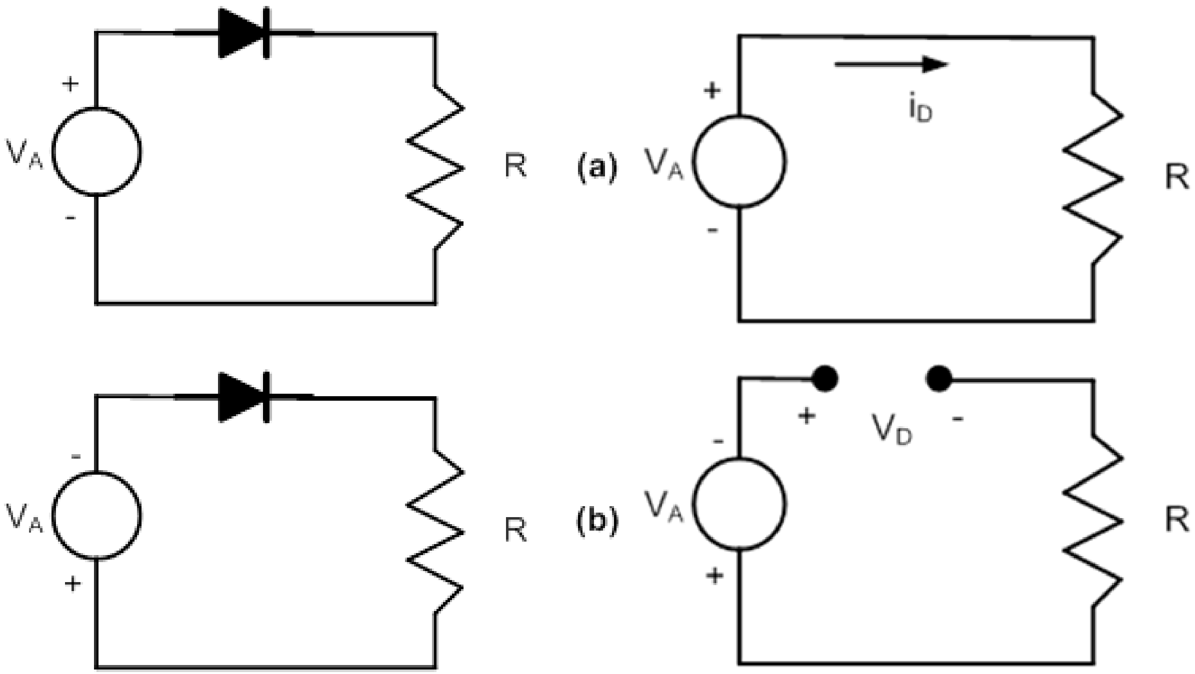


Figure 4: The ideal diode models on the right for a diode in forwards bias (a) and reverse bias (b).

The two situations of forward and reverse bias for a diode with resistor in series can be seen in Figure 4. Using the ideal diode model for the forward bias situation (a), the diode can be modelled as a short (no voltage drop over it) with a current i_D flowing through it. In the reverse bias case, the model will produce an open circuit with voltage drop V_D over it and no current flowing.

Using these two models now allows us to use our normal voltage and current laws (Ohm, Kirchhoff) to calculate voltages and currents in the circuit. For example, assuming that we have an applied voltage V_A = 10 V and R = 1 kΩ, we can calculate that we will have a current of 10 mA flowing in the forward bias case and will have a voltage drop of V_D = -10 V over the diode in the reverse bias case.

4. The Assumed States Method

If we need to analyse a circuit that contains multiple diodes or multiple voltage sources, we can use a systematic method called the **assumed states method** to obtain the state modelled ideal diode in each case. This method is based on the two possible states that a diode can have. If we then have two diodes in the circuit, we will have four possible circuit configurations, while 3 diodes in the circuit will produce eight possible circuit configurations. The method then uses the following procedure:

(1) Identify all possible state combinations.
(2) Analyse each state by replacing the diode with the corresponding short or open circuit.
(3) Determine which state is consistent for each diode: ON with $i_D > 0$ or OFF with $V_D < 0$.

Example 2: Use the assumed states method to determine if the diode in Figure 5 below is on or off and calculate the associated current through the diode (if forward biased) or voltage drop across the diode (if reverse biased).

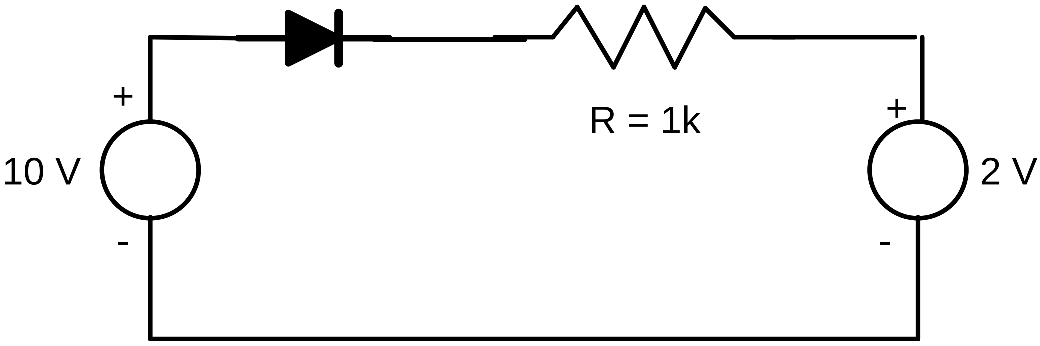


Figure 5: Diode circuit for Example 2

The diode (and the circuit) can be in one of two possible states:

State 1 - Diode ON:

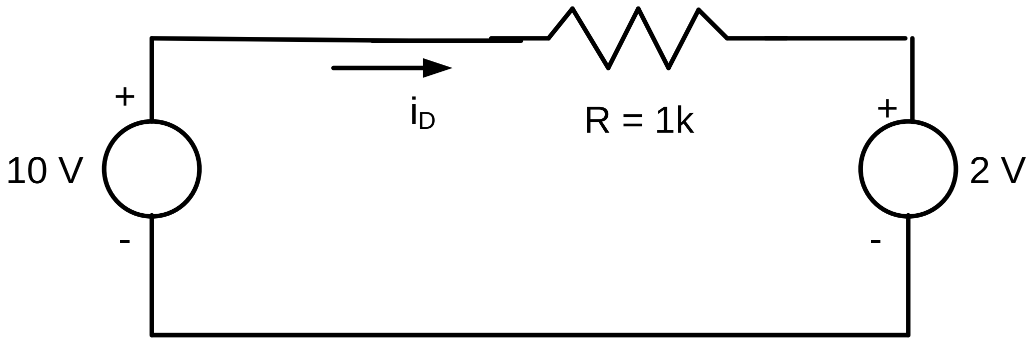


Figure 6: Diode circuit for Example 2 (Diode = ON)

The diode is now simply a short and a KVL around the circuit will give: 10 V – $i_D R$ – 2 V = 0, so that i_D = 8 mA. As this value of i_D is consistent with the forward biased case, it would appear that this is the correct state of the diode. However, also test for the alternative state.

State 2 - Diode OFF:

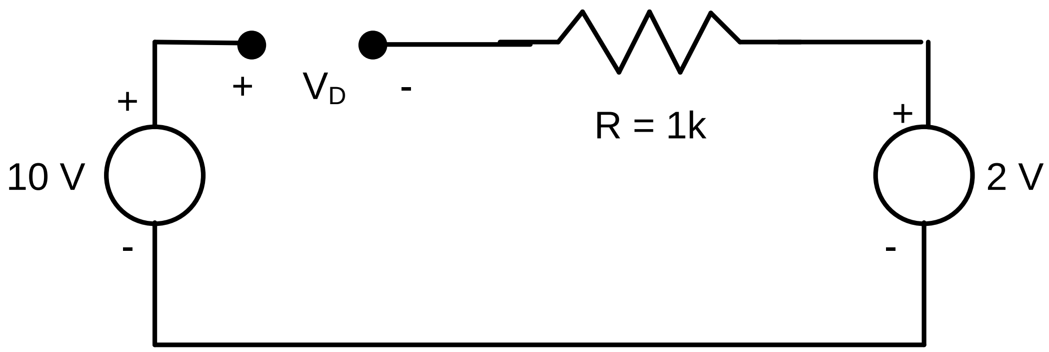


Figure 7: Diode circuit for Example 2 (Diode = OFF)

The diode is now an open circuit and a KVL around the circuit will now give:

$$10\text{ V} - V_D - i_D R - 2\text{ V} = 0 \qquad (\text{with } i_D = 0)$$

We can then calculate V_D to yield V_D = 8 V. This is not consistent with the assumed state of the diode OFF ($V_D < 0$) so we can accept that this is not the state of the diode.

If we have more than one diode in the circuit, the number of circuit states will scale as 2^n, where n is the number of diodes in the circuit. Thus, for two diodes in the circuit, we will have four possible circuit states. For three diodes, we will have eight possible circuit states, etc. A slightly more complex example follows below.

Example 3: Use the assumed states method in the circuit below (Figure 6) to analyse the circuit states of the two diodes. Also, calculate the current flowing through each of the resistors.

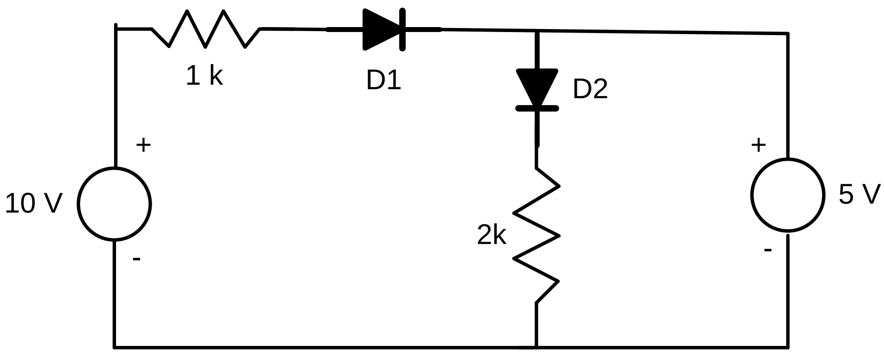


Figure 8: Diode circuit for Example 3

We can thus draw up a states table for the diode as follows:

State	Diode 1	Diode 2	Consistent?
1	OFF	OFF	
2	OFF	ON	
3	ON	OFF	

4	ON	ON	

Table 1: States of the diode circuit for Example 3

If we consider the first possible circuit state with both diodes OFF, we will get a circuit that looks as below where the two diodes will both be open circuit and we will then have zero current flowing:

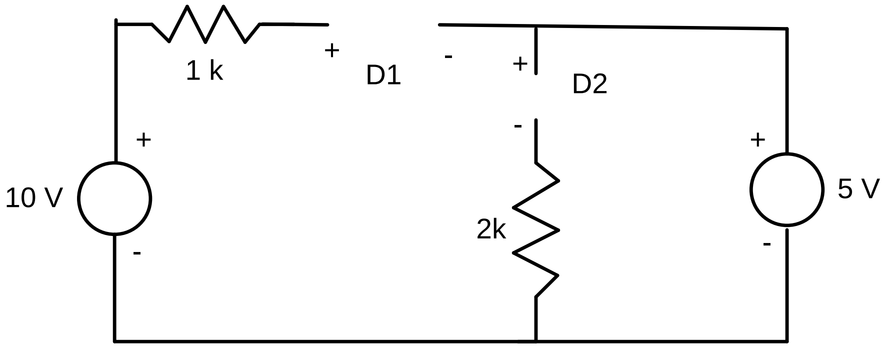


Figure 9: Diode circuit for Example 3 (D1 = OFF, D2 = OFF)

To check if this state is consistent, we perform a KVL around any of the loops: For the right-hand side loop, we will get:

$$5\ V - V_{D2} - V_{R2k} = 0$$

But, the voltage over the 2 kΩ resistor is zero, so that the voltage over diode D2 will be:

$$V_{D2} = 5\ V$$

This results that the voltage over D2 is positive is not consistent with our model that the voltage over a diode that is OFF must be < 0 V, so that the first assumed state will not be consistent.

State	Diode 1	Diode 2	Consistent?
1	OFF	OFF	No

Table 2: Diode circuit for Example 3 (State 1)

We can then look at the second possible circuit state, i.e. D1 = OFF and D2 = ON, producing the circuit below:

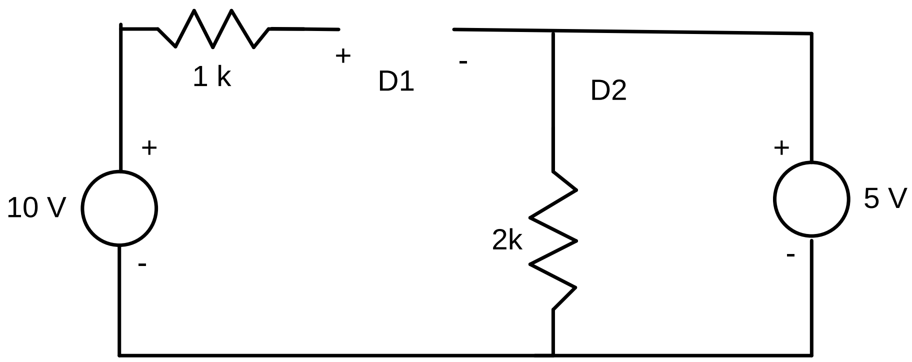


Figure 10: Diode circuit for Example 3 (D1 = OFF, D2 = OFF)

We can do a KVL by going around the outer loop:

$$10\text{ V} - V_{R1k} - V_{D1} - 5\text{ V} = 0$$

But, the current through the 1k resistor will be zero, so $V_{R1k} = 0$ and we have:

$$V_{D1} = -5\text{ V} + 10\text{ V} = 5\text{ V}$$

However, this calculation that $V_{D1} = 5$ V which is > 0 V is not consistent with the second assumed state which was that D1 is OFF (meaning V_{D1} must be < 0 V), so that this state is not consistent.

State	Diode 1	Diode 2	Consistent?
2	OFF	ON	No

Table 3: Diode circuit for Example 3 (State 2)

State 3 will be D1 = ON and D2 = OFF, but we have already for State 1 shown that the case of D2 = OFF is not consistent, so without having to do any calculations we can say:

States	Diode 1	Diode 2	Consistent?
3	ON	OFF	No

Table 4: Diode circuit for Example 3 (State 3)

The last remaining state will be both the diodes on, yielding a circuit containing two shorts as follows:

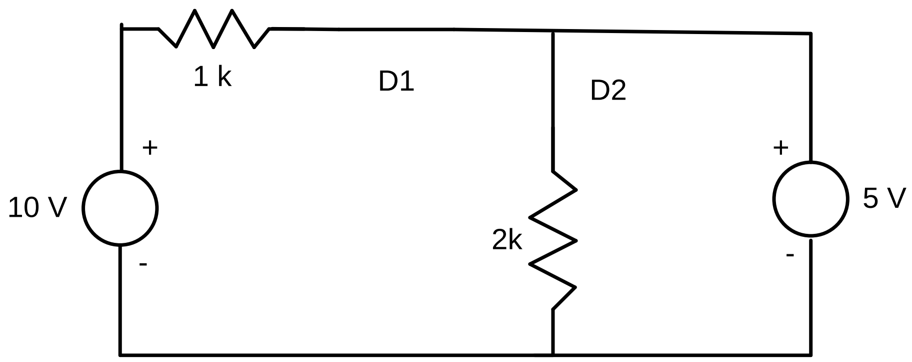


Figure 11: Diode circuit for Example 3 (D1 = ON, D2 = ON)

A KVL can be written for the outer loop as:

$$10\text{ V} - I_{D1} - 5\text{ V} = 0 \text{ so that } 10\text{ V} - 1000.I_{D1} - 5\text{ V} = 0$$

Solving for I_{D1} we get I_{D1} = 5 V/1000 = 5 mA

This value of $I_{D1} > 0$ is then consistent with a diode that is on and so far, this state seems consistent. We can also evaluate the right hand loop by KVL:

$$5\text{ V} - ID2 = 0 \text{ so that } ID2 = 5/2000 = 2.5\text{ mA}$$

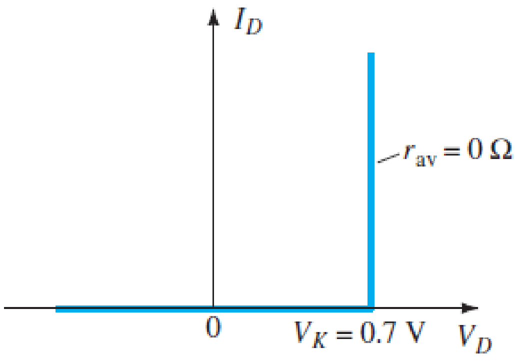
This value is again consistent with a diode that is ON and we can say that state 4 with both diodes ON is the only consistent state of the circuit and the currents through the two diodes will be ~ 5 mA through D1 and ~ 2.5 mA through D2. We can also calculate the voltage drop over the two resistors and should obtain a value of 5 V over each of the two resistors.

States	Diode 1	Diode 2	Consistent?
4	ON	ON	Yes

Table 5: Diode circuit for Example 3 (State 4)

5. The Ideal Diode + Voltage Source Model

This model is a refinement of the ideal diode model we used previously by recognising that there is a (nearly) constant voltage drop across the diode during forward bias operation (~ 0.7 V for a Si diode). A voltage source (reversed in polarity from the diode polarity) is then placed in series with an ideal diode as shown in Figure 7. This should give an improved circuit analysis compared to the ideal diode model.



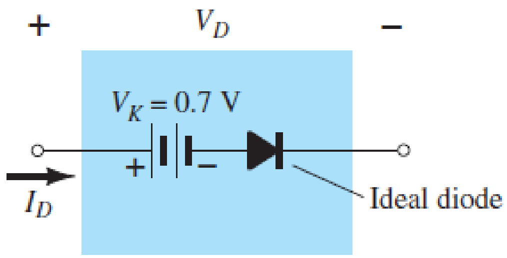


Figure 12: The ideal diode + voltage source model

Example 4: Repeat the previous example (Example 3, Figure 8) using the assumed states method but analyse the circuit using the ideal diode plus voltage source model.

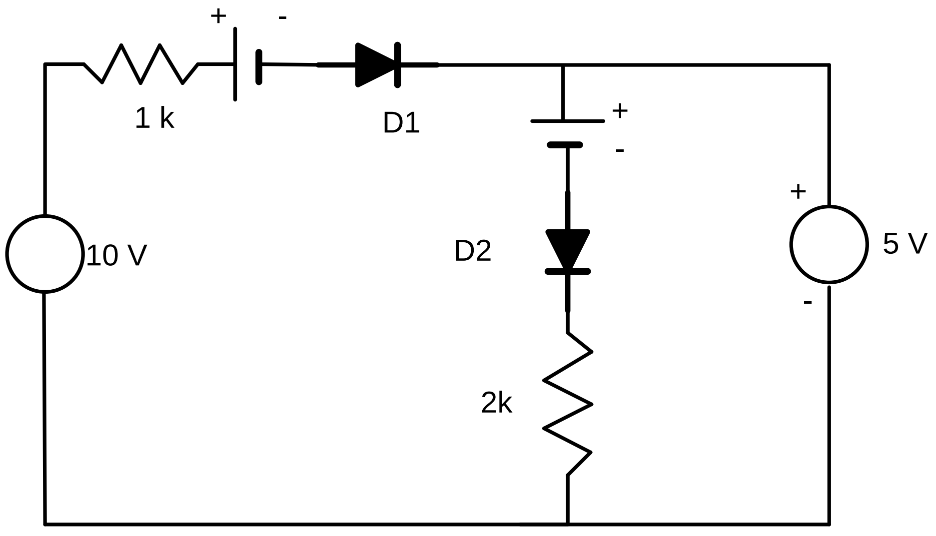


Figure 13: Circuit modelled using ideal diodes + voltage source.

In the previous example, we used an ideal diode model and the assumed states method and showed that both the diodes were ON. We will again have to check the state of each of the diodes for the new model, but can start by assuming that they are both ON and check if this is consistent. This state can then be modelled as below (both diodes as shorts):

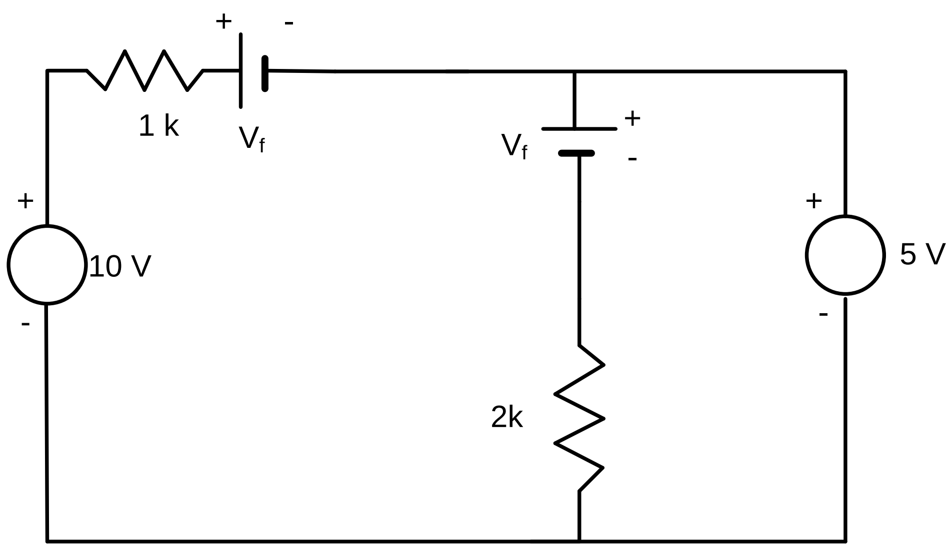


Figure 14: Circuit modelled using ideal diodes + voltage source (D1, D2 = ON)

Writing a KVL around the outer loop:

$$10\text{ V} - I_{D1}R_1 - V_f - 5\text{ V} = 0 \text{ thus } I_{D1} = (5\text{ V} - V_f)/1000$$

I_{D1} will then be > 0 when $V_f < 5$ V and we can expect this to be the case, as Vf ~0.7 V for a Si diode. The assumption that diode D1 is on is then correct. If we assume the value of $V_f = 0.7$ V for these diodes, the value of Id1 will be 4.3 mA.

Similarly, we can write a KVL around the right hand loop as:

$$5\text{V} - V_f - I_{D2}R_2 = 0 \text{ and } I_{D2} = (5-V_f)/2000$$

which will produce positive values for the current as long as $V_f < 5$ V which is to be expected. Diode D2 can then again assumed to be on and a diode current $I_{D2} = 2.15$ mA will flow for a voltage drop of $V_f = 0.7$ V over the diode.

We can also calculate the voltage drop over both the resistors as 4.3 V. These values differ slightly from our previous calculations, but we can expect the later values to be closer to the values of the real circuit as we are now using a better (more accurate) model!

6. More Sophisticated Models

We can further refine our diode models, for example by taking into account that a diode that is ON has a small resistance and thus introducing a further resistor in the model. Such a piecewise linear model will further improve the accuracy with which circuits can be simulated. However, the gains are relatively small and the ideal diode + voltage source model is normally accurate enough for most applications.

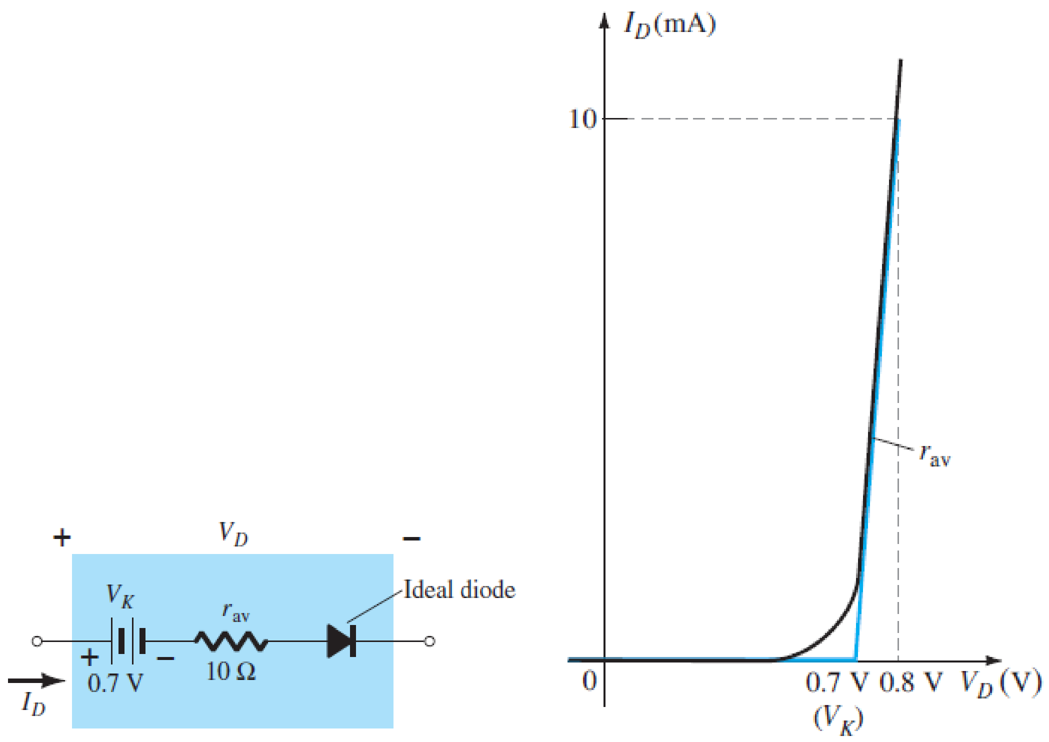


Figure 15: Piecewise linear diode model

In practice, diodes tend to have resistance across it and the amount of resistance depends on the type and biasing current.

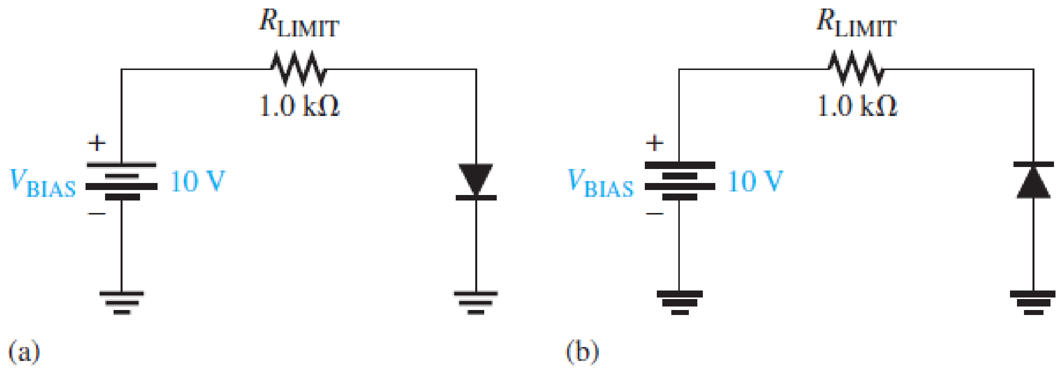
The slope of the curve in the diagram above is actually the resistance of the diode at a given biasing condition.

Example for Tutorial 2 – Diode Circuits

1. Given two diode circuits in the figure below, for each of the diode models e.g. ideal, voltage source, and voltage source + resistance models, answer the following questions.

   a. Determine the forward voltage and forward current for the diode in part (a) of the figure below for each of the diode models. Also, find the voltage across the limiting resistor in each case. Assume $r'_d = 10\ \Omega$ at the determined value of forward current. [9 marks]

   b. Determine the reverse voltage and reverse current for the diode in part (b) of the figure below for each of the diode models. Also, find the voltage across the limiting resistor in each case. Assume $I_R = 1\ \mu A$. [9 marks]



Answer

a. The analysis for the first circuit is given below.

For ideal diode model:

- Forward biased current of the diode is:

$$V_F = 0 \text{ V}$$

- Forward biased current of the diode is:

$$I_F = \frac{V_{BIAS}}{R_{LIMIT}} = \frac{10 \text{ V}}{1.0 \text{ k}\Omega} = 10 \text{ mA}$$

- The voltage across the limiting resistor is:

$$V_{R_{LIMIT}} = I_F R_{LIMIT} = (10 \text{ mA})\ (1.0 \text{ k}\Omega) \ = \ 10 \text{ V}$$

For voltage source model:

- Forward biased voltage of the diode is:

$$V_F = 0.7 \text{ V}$$

- Forward biased current of the diode is:

$$I_F = \frac{V_{BIAS} - V_F}{R_{LIMIT}} = \frac{10 \text{ V} - 0.7 \text{ V}}{1.0\text{k}\Omega} = 9.3 \text{ mA}$$

- The voltage across the limiting resistor is:

$$V_{R_{LIMIT}} = I_F R_{LIMIT} = (9.3 \text{ mA})\ (1.0 \text{ k}\Omega) = 9.3 \text{ V}$$

For voltage source + resistance model:

- Forward biased current of the diode is:

$$I_F = \frac{V_{BIAS} - V_F}{R_{LIMIT} + r_d'} = \frac{10 \text{ V} - 0.7 \text{ V}}{1.0\text{k}\Omega + 10\Omega} = 9.21 \text{ mA}$$

- Forward biased voltage of the diode is:

$$V_F = 0.7\ \text{V} + I_F r'_d\ =\ 0.7\ \text{V} + (9.21\ \text{mA})\ (10\ \text{A}) = 792\ \text{mV}$$

- The voltage across the limiting resistor is:

$$V_{R_{LIMIT}} = I_F R_{LIMIT} = (9.21\ \text{mA})\ (1.0\ \text{k}\Omega) = 9.21\ \text{V}$$

b. The analysis for the second circuit is given below.

For ideal diode model:

- Reverse biased current of the diode is:

$$I_R = 0\ \text{A}$$

- Reverse biased voltage of the diode is:

$$V_R = V_{BIAS} = 10\ \text{V}$$

- The voltage across the limiting resistor is:

$$V_{R_{LIMIT}} = 0\ \text{V}$$

For voltage source model:

- Reverse biased current of the diode is:

$$I_R = 0\ \text{A}$$

- Reverse biased voltage of the diode is:

$$V_R = V_{BIAS} = -10\ \text{V}$$

- The voltage across the limiting resistor is:

$$V_{R_{LIMIT}} = 0\ \text{V}$$

For voltage source + resistance model:

- Reverse biased current of the diode is:

$$I_R = 1\ \mu\text{A}$$

- Reverse biased voltage of the diode is:

$$V_{R_{LIMIT}} = I_R R_{LIMIT} = (1\ \mu\text{A})(1.0\ \text{k}\Omega) = 1\ \text{mV}$$

- The voltage across the limiting resistor is:

$$V_R = V_{BIAS} - V_{R_{LIMIT}} = 10\ \text{V} - 1\ \text{mV} = 9.999\ \text{V}$$

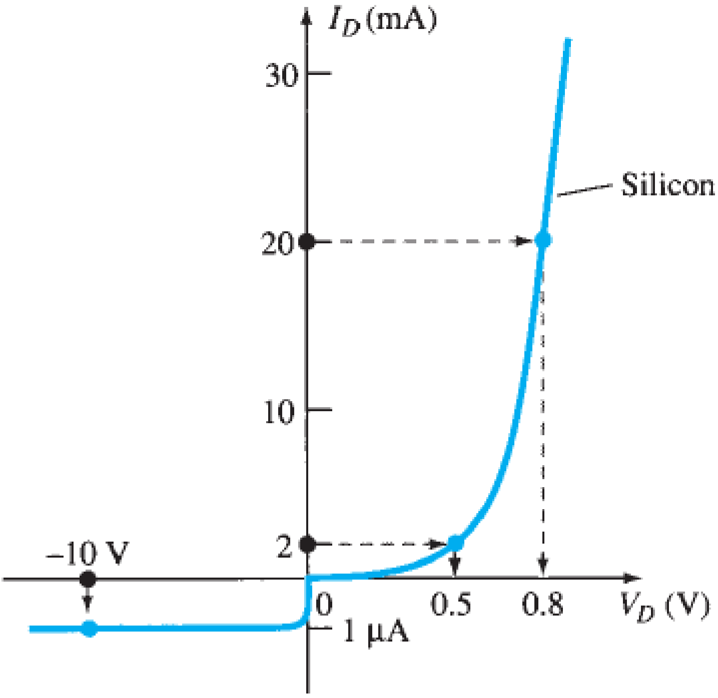
2. Determine the dc resistance levels for the diode of the figure given below at:

[8 marks]

a. Current I_D = 2 mA (low level).
b. Current I_D = 20 mA (high level).

c. Voltage V_D = -10 V (reverse-biased).

d. Comment on the results obtained in parts (a)-(c).



Answer

a. At I_D = 2 mA, V_D = 0.5 V (from the curve) and diode resistance is:

$$R_D = \frac{V_D}{I_D} = \frac{0.5\ \mathrm{V}}{2\ \mathrm{mA}} = 250\ \Omega$$

b. At I_D = 20 mA, V_D = 0.8 V (from the curve) and diode resistance is:

$$R_D = \frac{V_D}{I_D} = \frac{0.8\ \mathrm{V}}{20\ \mathrm{mA}} = 40\ \Omega$$

c. At V_D = -10 V, $I_D = -I_s$ = -1 uA (from the curve) and diode resistance is:

$$R_D = \frac{V_D}{I_D} = \frac{10\ \mathrm{V}}{1\ \mathrm{mA}} = 10\ \mathrm{M}\Omega$$

d. The results clearly are supporting some of the earlier comments regarding the dc resistance levels of a diode.