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# XMUT204 Electronic Design

# Note 3f: BJT Frequency Response Analysis

## 1. Small-Signal (AC) Analysis of Common Emitter Feedback Amplifier Circuit

We will start the design with the common-emitter amplifier circuit as shown in the figure given below.

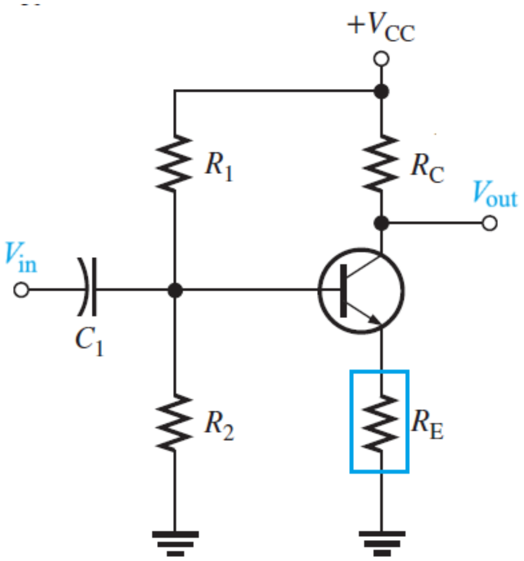


**Figure 1**: Common-emitter BJT amplifier with feedback resistor

Assumptions:

- Small-signal gain $\beta_{ac}$ is large so the small-signal base current $i_b$ may be neglected.
- The DC value of $V_{BE}$ remains constant ~ 0.7 V, so that small signal $v_{be}$ is very small.

As $v_{be}$ is very small, we will have the following potential differences to be the same:

$$v_e = v_b = v_i$$

So, the AC collector current is:

$$i_e = \frac{v_e}{R_E}$$

As $i_c \approx i_e$, we have:

$$v_o = -i_c R_C \approx -i_e R_C = -\left(\frac{v_e R_C}{R_E}\right)$$

As $v_e \approx v_i$, we will have:

$$v_o \approx -\left(\frac{v_i R_C}{R_E}\right)$$

And the voltage gain will be given by $v_o/v_i \approx -R_C/R_E$ which for components in the above circuit will be:

$$\frac{v_o}{v_i} \approx -\left(\frac{R_C}{R_E}\right) = -\frac{2.2\ \mathrm{k\Omega}}{1\ \mathrm{k\Omega}} = -2.2$$

The voltage gain of the amplifier is now only controlled by the value of passive components and is independent of DC gain, $\beta$!

## 2. A Comment on the Voltage Gain

We have an expression for the voltage gain given by:

$$\frac{v_o}{v_i} \approx -\left(\frac{R_C}{R_E}\right)$$

This considerably smaller than the gain without feedback in the common-emitter configuration.

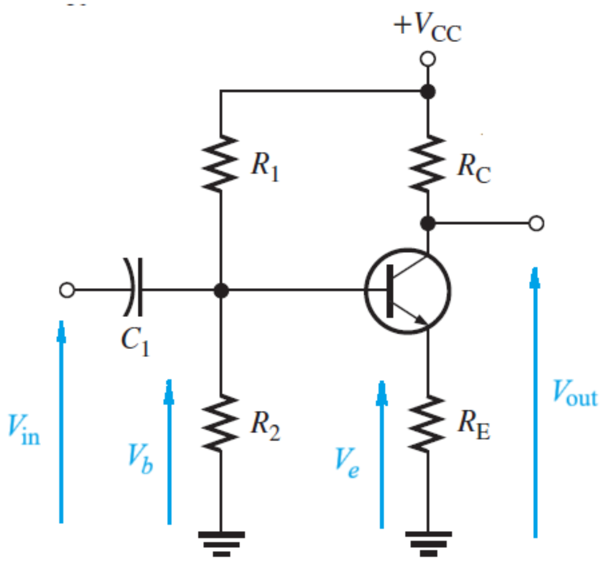


**Figure 2**: Voltage gain ($V_o/V_i$) of the common-emitter BJT amplifier

In the common-emitter configuration without feedback, the gain was given by:

$$A_v = -\left(\frac{R_E}{r_e}\right)$$

Where: $r_e = 1/g_m$ with $g_m$ is the transconductance (ratio of $I_C/V_{BE}$).

Thus, it would appear that we can increase the gain simply by increasing this ratio of $R_C/R_E$.

This can only be done to a limited extent in practice:

- $R_E$ cannot be too small as we need a significant amount of feedback to make circuit work.
- $R_C$ cannot be too large to limit the current that flows.

We have thus traded off increased stability for a loss of high gain.

### 3. Use of an Emitter Bypass Capacitor

Previously stated that the use of an emitter bypass capacitor leads to increased gain as the small passes through this capacitor and to ground.

In this way, the DC feedback is maintained, but AC feedback is removed.

At low frequencies, the effect of the bypass capacitor will fall, and small signal gain will fall.

This point is determined by the frequency at which the impedance of the bypass capacitor becomes appreciable compared to the parallel combination of $R_E$ and $r_e$, where $r_e$ resistance looking into the emitter.

We have: $r_e = 1/g_m$ and it can be shown that:

$$r_e \approx \frac{1}{40 I_E} = \frac{25\ \mathrm{mV}}{I_E}$$

Where: the transconductance of the amplifier, $g_m = I_C/V_{BE}$ and $I_E \approx I_C$.

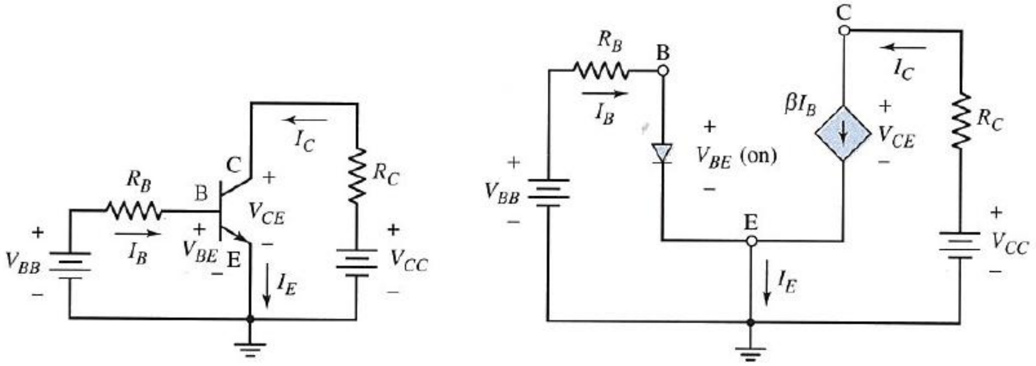


**Figure 3**: Model of BJT amplifier

The value of $r_e$ is then typically only a few Ohms and is a lot smaller than $R_E$.

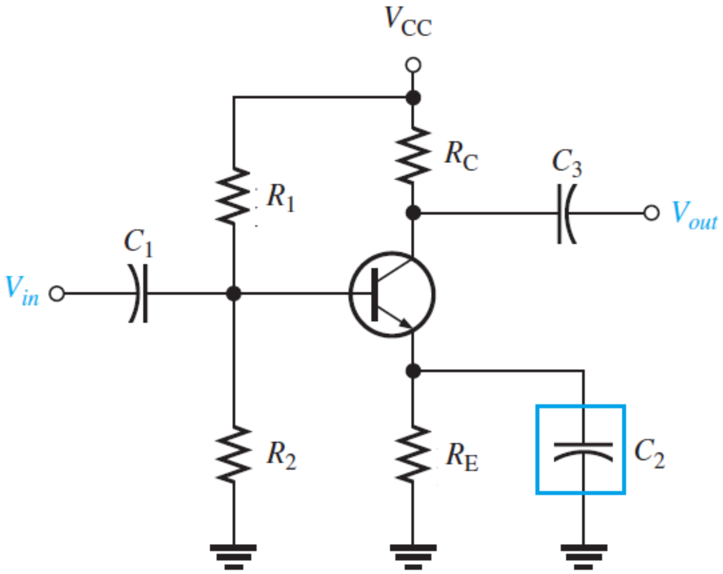


**Figure 4**: Design of BJT amplifier with emitter-bypass capacitor

### 3.1. Cut-Off Frequency of Bypass Capacitor

The equation for the cut-off frequency due the bypass capacitor is then:

$$f_{co} = \frac{1}{2\pi C_E r_e}$$

We can use the above expression to calculate the size of the bypass capacitor that will be required to provide a sufficient low cut-off for the desired application.

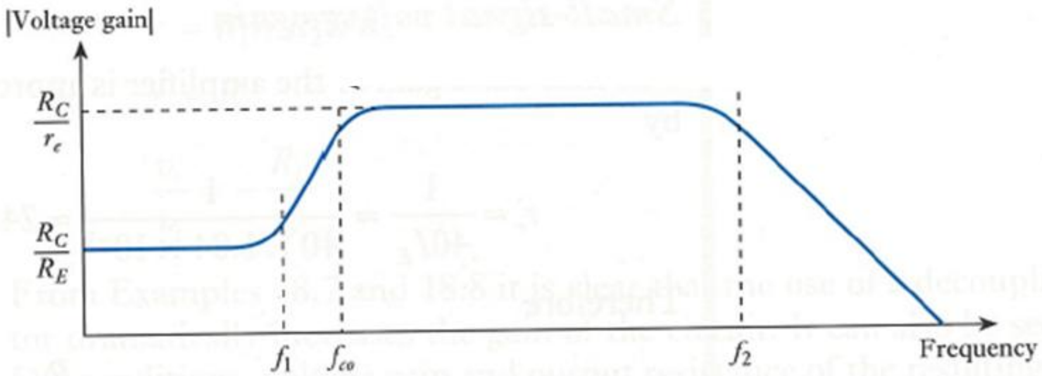


**Figure 5**: Cut-off frequencies of common-emitter amplifier with emitter resistor and decoupling capacitor

Below the cut-off frequency, the gain will drop by -6 dB per octave until the impedance of common emitter becomes comparable to $R_E$ e.g. further below this frequency, the gain of the amplifier will level out as $R_E$ now dominates the response. The frequency at which this occurs is given by:

$$f_1 = \frac{1}{2\pi C_E R_E}$$

Notice in the frequency response graph of the amplifier that there are cut-off frequencies due to coupling input capacitor ($f_1$) and coupling output capacitor ($f_2$).

### 3.2. Bandwidth of BJT Amplifier

The frequency response of the BJT amplifier depends on the values of lower cut-off frequency ($f_1$), upper cut-off frequency ($f_2$), and frequency due to addition of the bypass capacitor ($f_{co}$).

The frequency response of the given amplifier shown in the diagram below is for common-emitter amplifier with emitter resistor and decoupling capacitors.

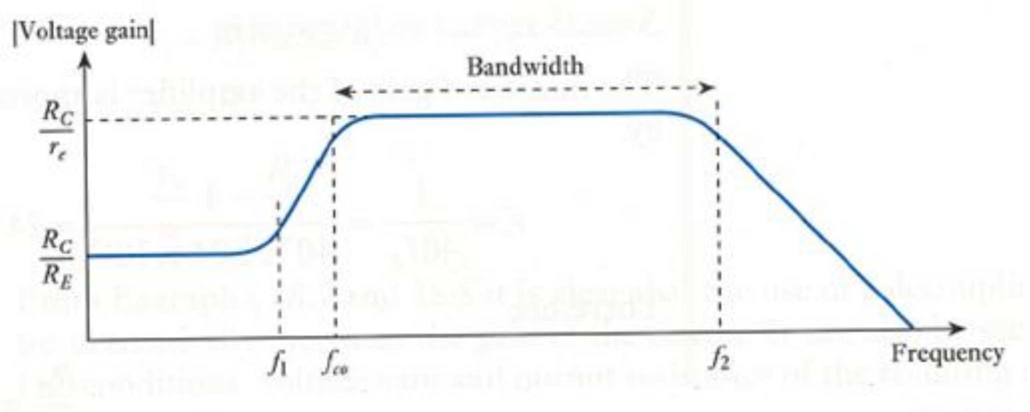


**Figure 6**: Frequency response and bandwidth of common-emitter amplifier with emitter resistor and decoupling capacitor

With the bypass capacitor in place, the small-signal gain is then given by:

$$A_v = -\frac{R_C}{r_e}$$

In our case, we have:

$$r_e \approx \frac{1}{40 I_E} = \frac{1}{40(1 \times 10^{-3}\,\mathrm{A})} = 24\,\Omega$$

Thus, small-signal gain of the amplifier is:

$$A_v = -\frac{5.6\,\mathrm{k\Omega}}{24\,\Omega} = -233$$

Which is considerably larger than $A_v = R_E/R_C$ that we calculated earlier.

## 4. Swamped Resistor

A swamped amplifier uses a partially bypassed emitter resistance to minimize the effect of $r_e'$ on the gain to achieve gain stability.

The total external emitter resistance, $R_E$, is formed with two separate emitter resistors, $R_{E1}$ and $R_{E2}$.

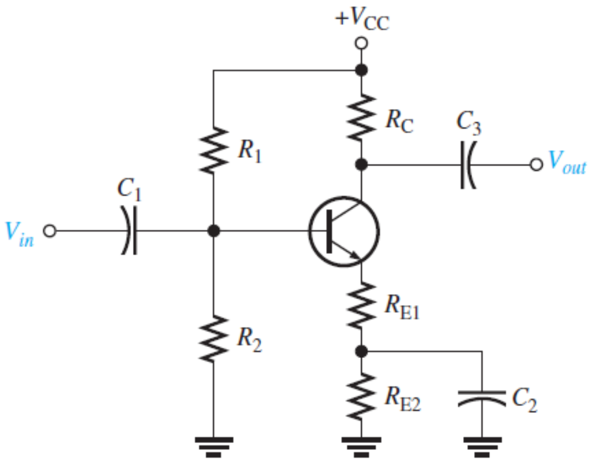


**Figure 7**: Swamped resistor in the common-emitter BJT amplifier

Both $R_{E1}$ and $R_{E2}$ affect the DC bias while only $R_{E1}$ affects the AC voltage gain:

$$A_v = \frac{R_C}{r_e' + R_{E1}}$$

If $R_{E1}$ is at least 10 times larger than $r_e'$, then the effect of $r_e'$ is minimised and the approximate voltage gain for the swamped amplifier is:

$$A_v \cong \frac{R_C}{R_{E1}}$$

## 5. Frequency Response of Common Emitter BJT Amplifier

The frequency response of a common-emitter BJT amplifier depends on a number breakpoint or critical frequencies:

- Lower-critical frequencies ($f_{cl1}$, $f_{cl2}$, and $f_{cl3}$) are three low-frequency $RC$ circuits of the coupling and bypass capacitors.
- Upper-critical frequencies ($f_{cu1}$ and $f_{cu2}$) are two high-frequency $RC$ circuits of the transistor's internal capacitances.

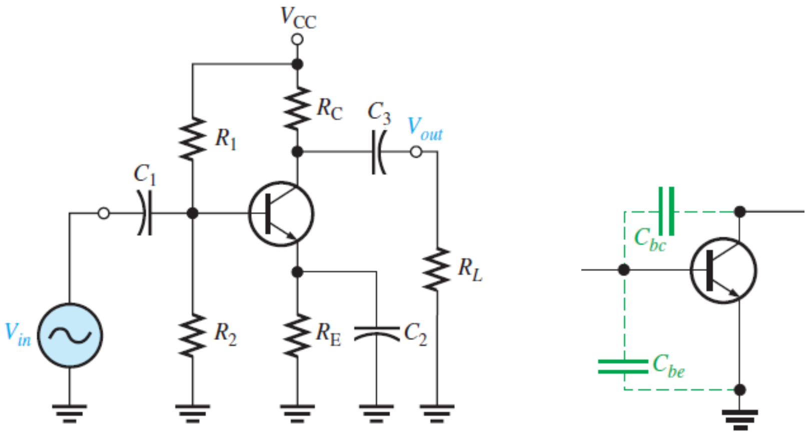


**Figure 8**: A common-emitter BJT amplifier with coupling and bypass capacitors (left) and BJT internal capacitances (right)

The lower and upper frequency responses of the BJT amplifier as shown above are illustrated in the graph below.

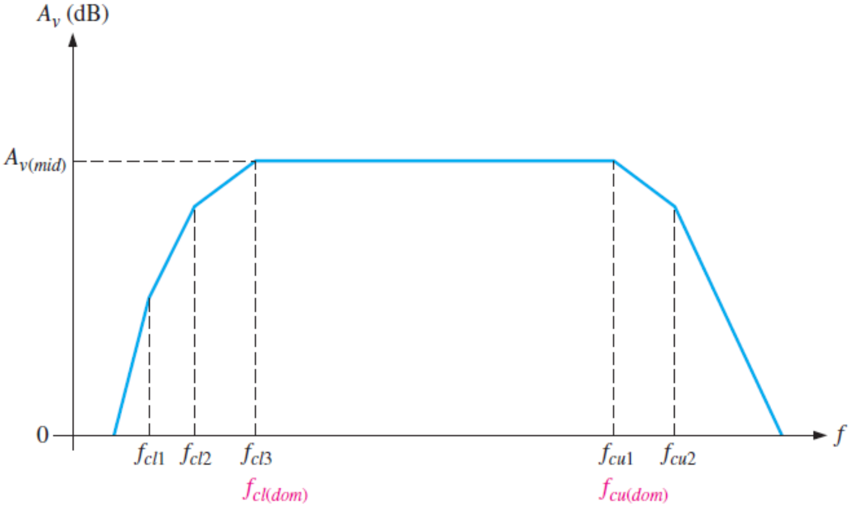


**Figure 9**: Frequency Response of BJT Amplifier

As illustrated in the diagram, there are cut-off frequencies of the amplifier in the lower frequency region of the frequency response of the amplifier e.g. $f_{cl1}$, $f_{cl2}$, and $f_{cl3}$ due to these coupling and bypass capacitors e.g. $C_1$, $C_2$, and $C_3$ respectively.

There are also cut-off frequencies at higher frequency region of the frequency response of the amplifier e.g. $f_{cu1}$ and $f_{cu1}$ due to present of the parasitic capacitances in the BJT observed at high frequency.

### 5.1. Determine the Lower Cut-Off Frequency

The low-frequency ac equivalent circuit of the common-emitter BJT amplifier consists of three high-pass $RC$ circuits.

- Input coupling capacitor $C_1$ and the resistance looking in at the base of the amplifier ($R_{in}$).
- Output coupling capacitor $C_3$, the resistance looking in at the collector of the amplifier ($R_{out}$), and the load resistance ($R_L$).
- Emitter-bypass capacitor $C_2$ and the resistance looking in at the emitter of the amplifier ($R_{in(emitter)}$).

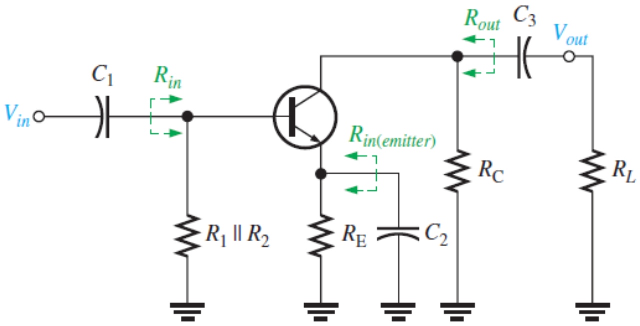


**Figure 10**: Lower cut-off frequencies RC circuit

### 5.1.1. Input Coupling Capacitor

The lower cut-off frequency of the $RC$ circuit at the input of the amplifier is determined from:

$$X_{C(\text{input})} = \frac{1}{2\pi f_{cl(\text{input})} C_1}$$

The equivalent resistance at the input of the amplifier is:

$$R_{in} = R_1 \parallel R_2 \parallel R_{in(\text{base})}$$

Where:

$$R_{in(\text{base})} = \beta_{ac} r_e'$$

Neglecting the reactance at the input of the amplifier, the impedance at the input of the amplifier is:

$$X_{C(\text{input})} = R_{in}$$

So, the lower cut-off frequency due to capacitance at the input of the amplifier is:

$$f_{cl(\text{input})} = \frac{1}{2\pi R_{in} C_1}$$

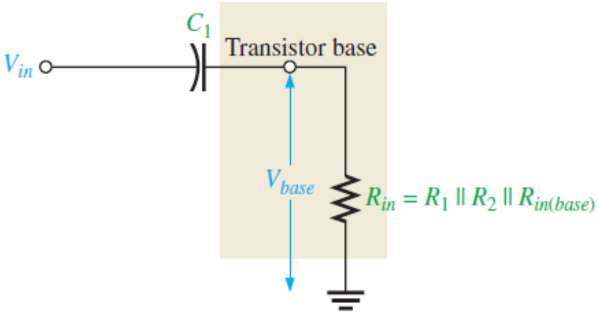


**Figure 11**: Equivalent circuit of input capacitor equivalent RC circuit

If the resistance of the input source is considered, the lower cut-off frequency of input $RC$ circuit is:

$$f_{cl(\text{input})} = \frac{1}{2\pi(R_s + R_{in})C_1}$$

### 5.1.2. Output Coupling Capacitor

The collector and load resistors make up the equivalent impedance at the output of amplifier.

$$X_{C(\text{output})} = R_C + R_L$$

The lower cut-off frequency of the $RC$ circuit at the output of the amplifier is:

$$f_{cl(\text{output})} = \frac{1}{2\pi(R_C + R_L)C_3}$$

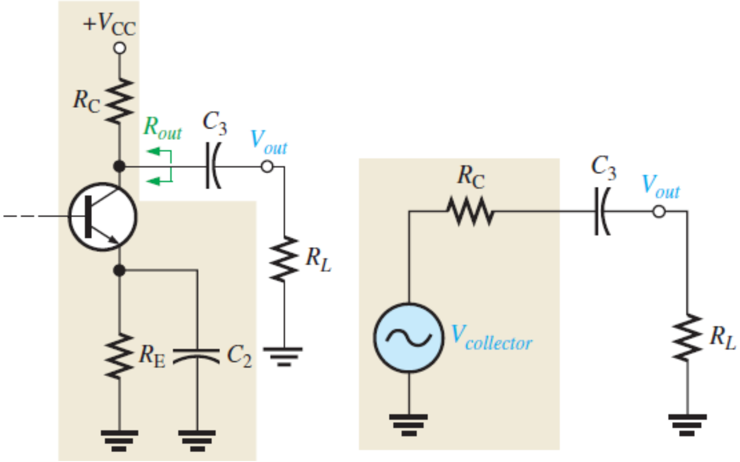


**Figure 12**: Low-frequency output capacitor equivalent RC circuit

### 5.1.3. Emitter Bypass Capacitor

Equivalent (bypass) input resistance at the emitter is determined from:

$$R_{in(\text{emitter})} = r_e' + \frac{V_e}{I_e}$$

Since $V_b = V_e$, $I_e \cong I_c$, and $I_c = \beta_{ac} I_b$, thus:

$$R_{in(\text{emitter})} \cong r_e' + \frac{V_b}{\beta_{ac} I_b}$$

Knowing $V_b = I_b R_{th}$, the above equation becomes:

$$R_{in(\text{emitter})} = r_e' + \frac{I_b R_{th}}{\beta_{ac} I_b}$$

Rearranging the above equation, the input resistance at the emitter is:

$$R_{in(emitter)} = r_e' + \frac{R_{th}}{\beta_{ac}}$$

The total impedance at the emitter is a parallel combination of this equivalent input resistance at the emitter ($R_{in(emitter)}$) and emitter resistor ($R_E$).

$$X_{C(\text{bypass})} = R_{in(\text{emitter})} \parallel R_E$$

The lower cut-off frequency of the $RC$ circuit at the (bypass) emitter of the amplifier is:

$$f_{cl(\text{bypass})} = \frac{1}{2\pi\left[\left(r_e' + \frac{R_{th}}{\beta_{ac}}\right) \parallel R_E\right] C_2}$$

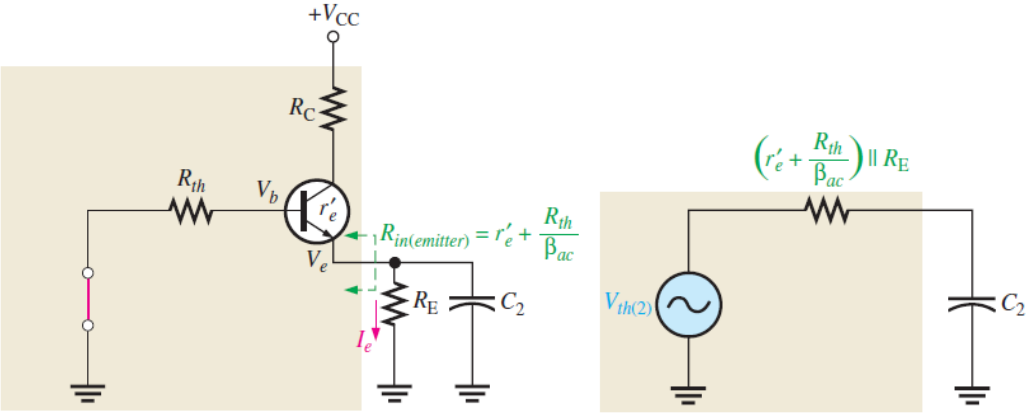


**Figure 13**: Low-frequency bypass capacitor equivalent RC circuit

The following diagram shows the frequency response of the BJT amplifier with lower cut-off frequencies. Notice the three lower cut-off frequencies $f_{ci(bypass)}$, $f_{ci(output)}$, and $f_{ci(input)}$.

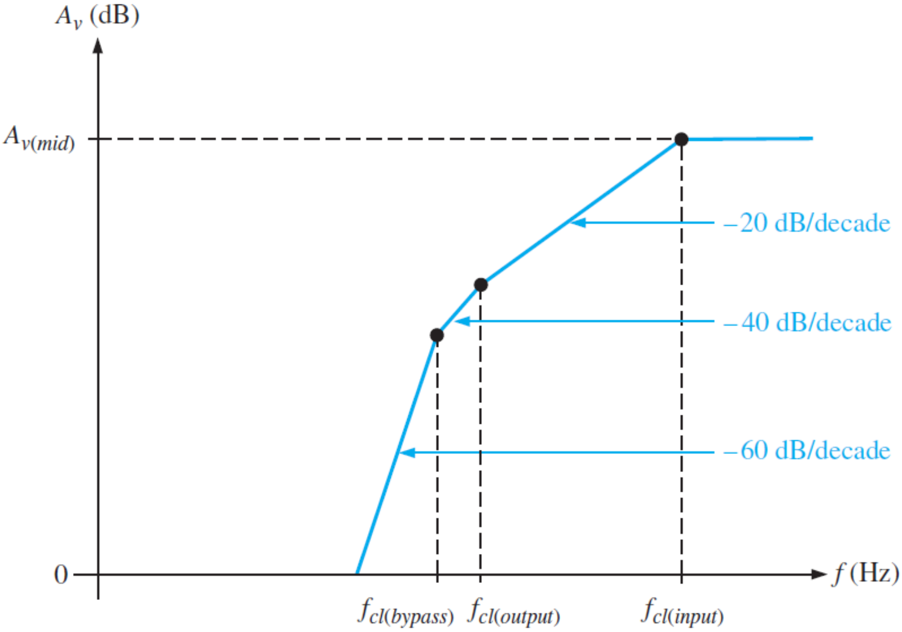


**Figure 14**: Lower cut-off frequency response of BJT amplifier

### 5.2. Determining the Upper Cut-Off Frequency

At higher frequency, there are several parasitic capacitances inside the BJT amplifier that determine the cut-off frequencies in the higher frequency range of the frequency response of the amplifier.

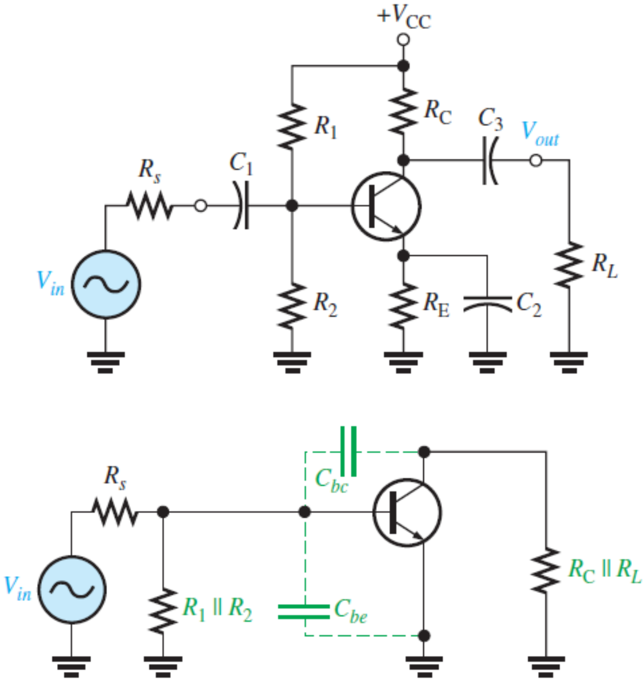


**Figure 15**: Parasitic capacitances inside the common-emitter BJT amplifier

Internal capacitances, $C_{be}$ and $C_{bc}$, which are significant only at high frequencies. $C_{bc}$ appears as the input capacitance from base to ground and $C_{be}$ simply appears as a capacitance to ac ground.

According to John M. Miller, these parasitic capacitances decrease the gain of a common emitter circuit at higher frequencies. The amount of negative gain-reducing feedback is related to both current gain, and amount of collector-base capacitance.

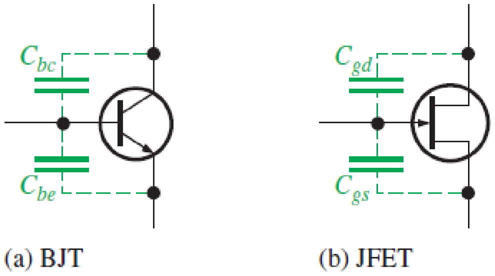


**Figure 16**: Parasitic capacitances between the terminals of BJT and JFET

The following diagram shows general case of Miller input and output capacitances. In this case, $C$ represents $C_{bc}$ or $C_{gd}$.

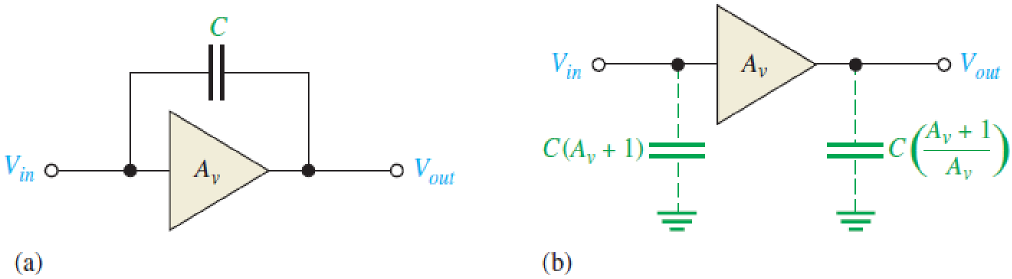


**Figure 17**: general case of Miller input and output capacitances and its equivalent circuit

### 5.1.1. Effects of Parasitic Capacitances

At lower frequencies, the internal capacitances have a very high reactance because of their low capacitance value (usually only a few picofarads) and the low frequency value. Therefore, they look like opens and have no effect on the transistor's performance.

As the frequency goes up, the internal capacitive reactance goes down, and at some point, they begin to have a significant effect on the transistor's gain. When the reactance of $C_{be}$ (or $C_{gs}$) becomes small enough, a significant amount of the signal voltage is lost due to a voltage-divider effect of the signal source resistance and the reactance of $C_{be}$, as illustrated in the figure below.

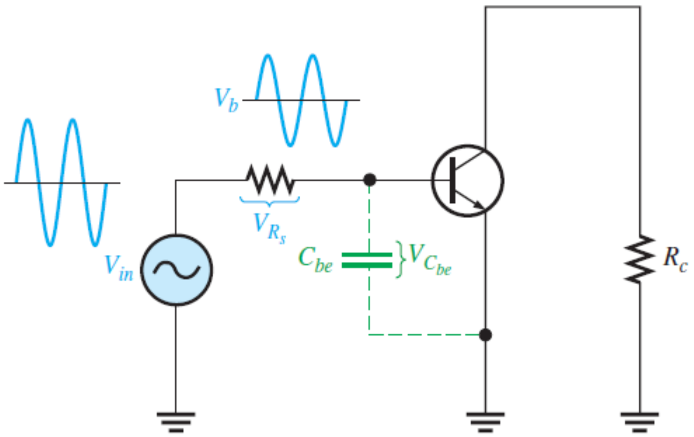


**Figure 18**: Effects of parasitic $C_{be}$ capacitance in BJT amplifier circuit

When the reactance of $C_{bc}$ (or $C_{gd}$) becomes small enough, a significant amount of output signal voltage is fed back out of phase with the input (negative feedback), thus effectively reducing the voltage gain. This is illustrated in the figure below.

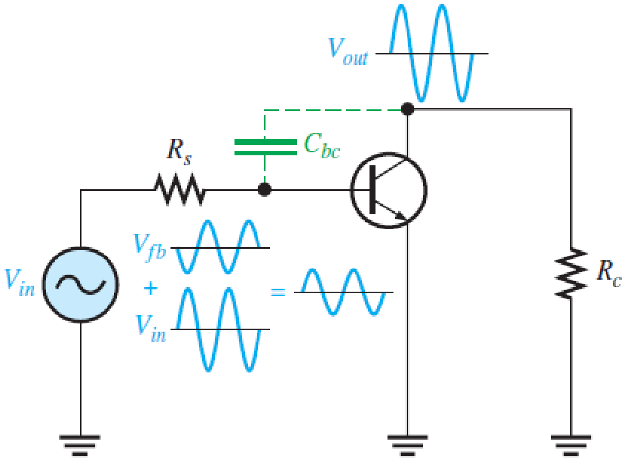


**Figure 19**: Effects of parasitic $C_{bc}$ capacitance in BJT amplifier circuit

Following Miller's theorem, these parasitic capacitances of the BJT will form equivalent higher-frequency RC circuit:

- $C_{be}$ simply appears as a capacitance to ac ground in parallel with Miller input capacitance.

- $C_{bc}$ appears in the Miller input capacitance from base to ground:

$$C_{in(\text{Miller})} = C_{bc}(A_v + 1)$$

- $C_{bc}$ simply appears in the Miller output capacitance (that is also in series with collector resistor):

$$C_{out(\text{Miller})} = C_{bc}\left(\frac{A_v + 1}{A_v}\right)$$

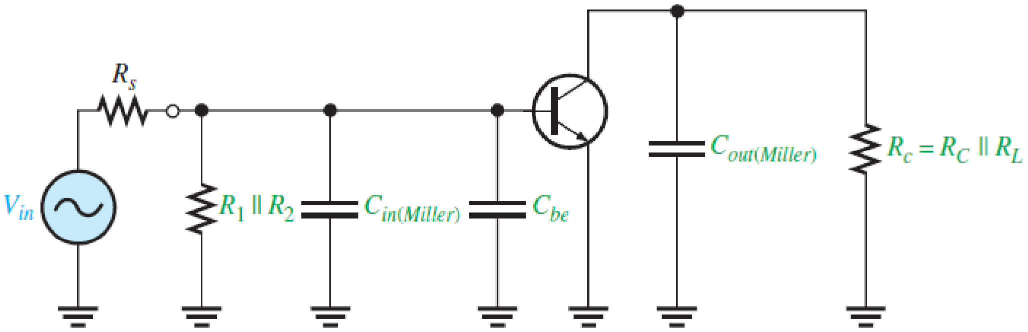


**Figure 20**: High-frequency equivalent RC circuit (note that parasitic capacitances included at the input and output of the amplifier)

### 5.1.2. Input Parasitic Capacitance

High-frequency input $RC$ circuit is parallel of Miller input capacitance with base-emitter capacitance and the total impedance at the base.

The equivalent impedance at the input of the amplifier is a parallel combination of internal voltage source resistance, the parallel of biasing input resistors, and reflected input emitter resistance of the BJT at the input.

$$X_{C(\text{input})} = R_s \parallel R_1 \parallel R_2 \parallel \beta_{ac} r_e'$$

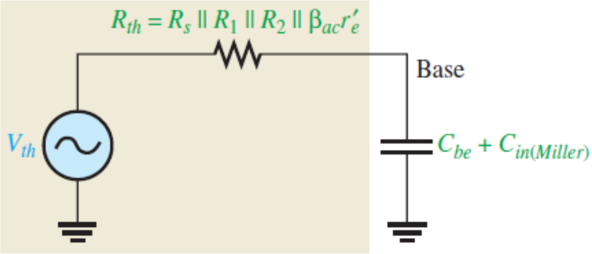


**Figure 21**: High-frequency input equivalent RC circuit

Therefore, upper cut-off frequency due to parasitic capacitances at the input of the amplifier circuit is:

$$f_{cu(\text{input})} = \frac{1}{2\pi(R_s \parallel R_1 \parallel R_2 \parallel \beta_{ac} r_e')C_{tot}}$$

Where: $C_{tot} = C_{be} + C_{in(\text{Miller})}$

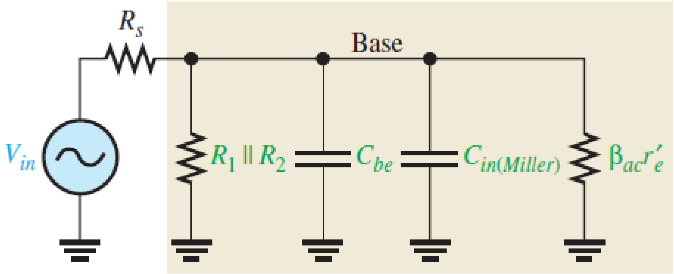


**Figure 22**: High-frequency input equivalent RC circuit (with source resistance included)

### 5.1.3. Output Parasitic Capacitance

High-frequency output $RC$ circuit is formed by the Miller output capacitance and the resistance looking in at the collector. The equivalent impedance at the output of the amplifier is the parallel combination of collector resistor and load resistor.

$$X_{C(\text{output})} = R_C \parallel R_L$$

As a result, the upper cut-off frequency due to parasitic capacitances at output of the amplifier is:

$$f_{cu(\text{output})} = \frac{1}{2\pi(R_C \parallel R_L)C_{out(\text{Miller})}}$$

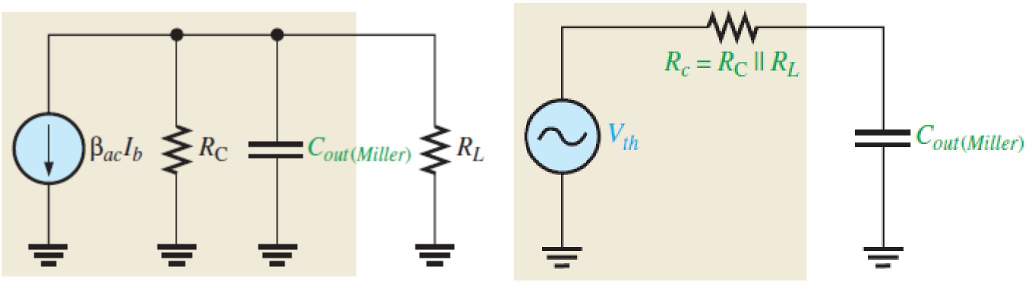


**Figure 23**: High-frequency output equivalent $RC$ circuit

The following diagram shows the frequency response of the BJT amplifier with upper cut-off frequencies. Notice the two upper cut-off frequencies $f_{cu(input)}$ and $f_{cu(output)}$.

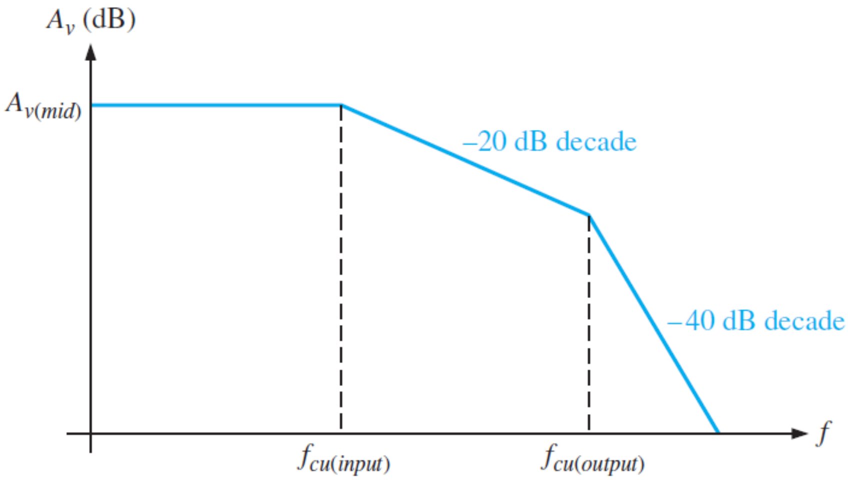


**Figure 24**: Upper cut-off frequency response of BJT amplifier

## 6. Design Procedures

Given the following common emitter BJT amplifier as described in the following figure.

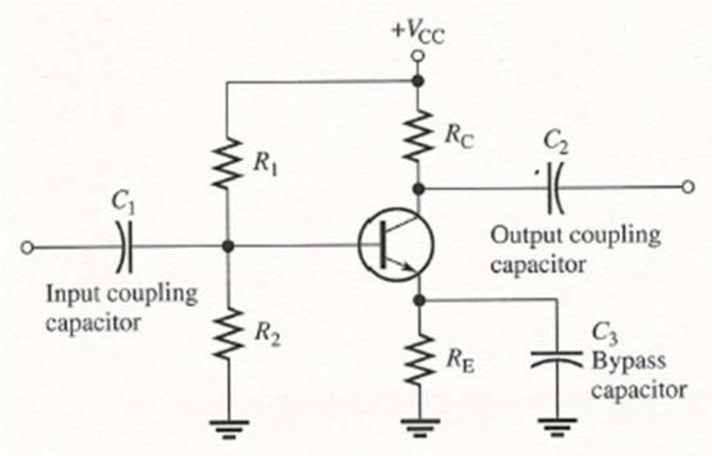


**Figure 25**: An example BJT amplifier circuit for design procedure

Need to select components for the following functions:

1. $C_1$ is to capacitively couple the input signal to the base. Blocks DC component of this signal to the base.
2. $C_2$ is to block any DC component of the output signal. Only amplified AC component will then appear at $V_{out}$.
3. $R_1$ and $R_2$ provide a voltage dividing network on the input to ensure that BE junction is forward biased at a stable voltage.
4. $R_C$ and $R_E$ provide a voltage dividing network on the output side to ensure the transistor is operating in the active region with the correct Q-point. $R_E$ acts as the negative feedback to stabilise the gain to variations in $\beta$ and $R$.
5. $C_E$ is a bypass capacitor to short the ac signal around the emitter resistor and increase the voltage gain.

**Design Overview**

Design example of a basic transistor-based amplifier and use of an emitter bypass capacitor.

The heuristic processes that are involved in the BJT amplifier design are as outlined as follows:

1. Work on overall design – gain of amplifier, peak-to-peak output voltage swing.

2. Perform Q point determination – determine the quotient collector current and collector to emitter voltage drop.
3. Design the biasing circuit design – calculate the values of the output resistors (collector and emitter resistor) and input resistors (voltage divider resistors).
4. Calculate the values of capacitors – work on the input and output coupling capacitors and emitter bypass capacitor.

**Example 1 – Basic BJT Amplifier Design with Input Coupling Capacitor**

Design a single stage amplifier using an NPN transistor to give a small signal voltage gain of -4 and a maximum peak to peak voltage swing of 10 V when used with a high impedance load. The amplifier is expected to have a rating output current of 1 mA.

It should operate from a 15 V supply and be AC coupled with a gain of the amplifier that its bandwidth is approximately constant at frequency 100 Hz - 5 kHz.

a. What the basic amplifier circuit look like? [4 marks]

b. Sketch the load line and determine the Q-point for DC bias. [12.5 marks]

c. Calculate values of collector resistor ($R_C$), actual voltage gain ($A_v$), and emitter resistor ($R_E$). [7.5 marks]

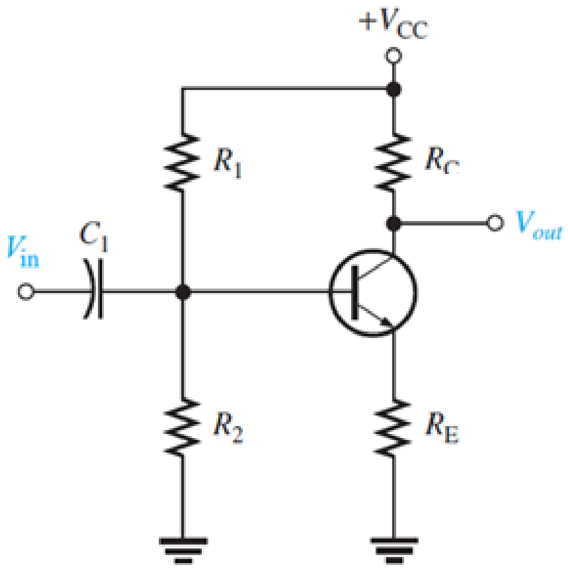
d. Calculate values of base bias resistors ($R_1$ and $R_2$). [12.5 marks]

e. Considering the effect of input resistance, determine the input coupling capacitor. [7.5 marks]

f. Finally, add an emitter bypass capacitor of 22 µF and calculate how this will influence your gain. [6 marks]

**Answer**

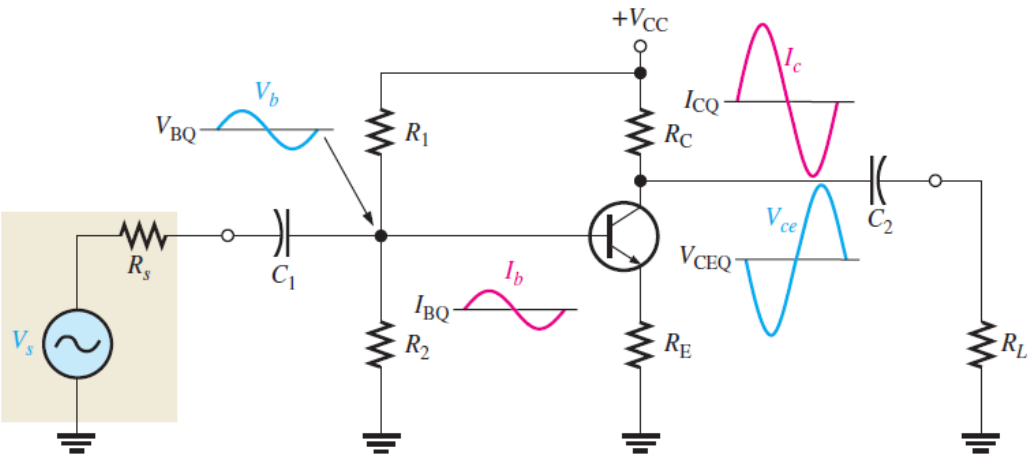
a. What should the basic BJT amplifier circuit look like? The following figure shows the BJT amplifier circuit to be designed in this example exercise.



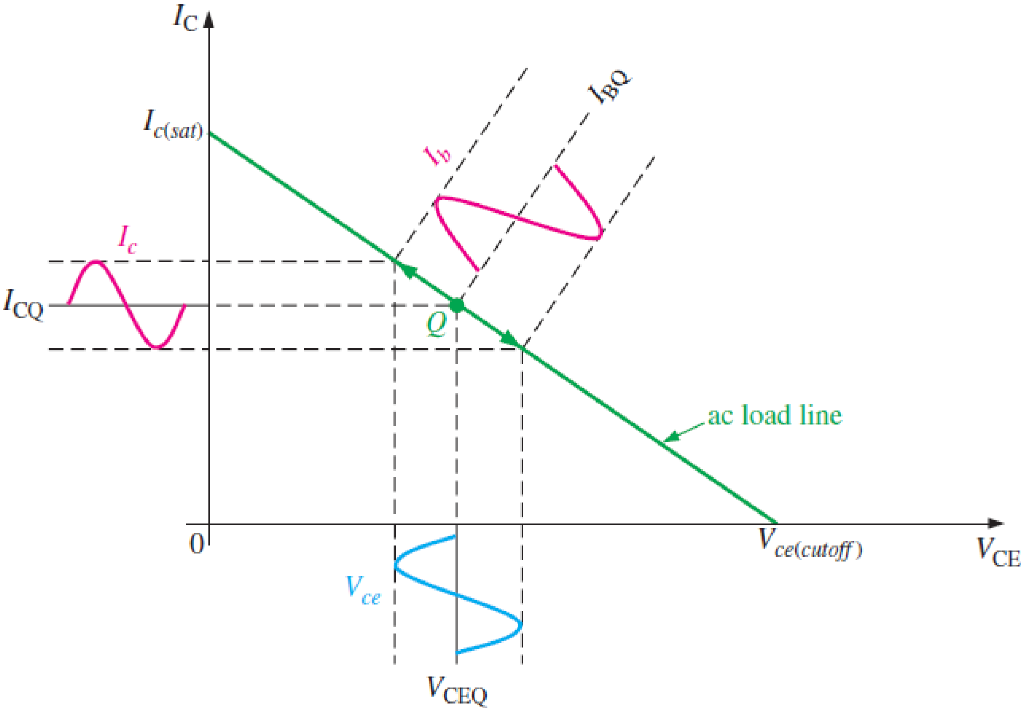
We will design a common emitter configuration with a feedback resistor as discussed before.

Important to realise that there will not be any single correct answer – your circuit will depend on the assumptions, design choices and trade-offs you make along the way.

b. Decide on a Q-point of operation for DC bias:



The current and voltage swings of the DC biasing are as shown in the figure below.



We need to accommodate a large voltage swing of 10 V peak-to-peak, so the operating point needs to be at least 5 V above ground and at least 5 V below the positive voltage supply.

Choose $V_{CQ}$ to be ≈ 9.5 V, i.e. 5.5 V below $V_{CC}$ = 15 V

c. Must now decide on a suitable collector current e.g. it should match the rating output current of the amplifier (the required current to flow to the load). As the load impedance is high – decide on $I_C \approx 1$ mA.

Calculate a value for $R_C$:

$$R_C = \frac{(V_{CC} - V_{CQ})}{I_C}$$

$$= \frac{15\text{ V} - 9.5\text{ V}}{1\text{ mA}} = 5.5\text{ k}\Omega$$

As 5.5 kΩ is not a standard resistor value, we choose the closest value from the E12 standard, i.e. 5.6 kΩ.

Then, we choose $R_E$ to meet the small signal voltage gain.

The voltage gain of the amplifier is:

$$A_v = -\left(\frac{R_C}{R_E}\right) = -4 = -\frac{5.6\ \mathrm{k\Omega}}{R_E}$$

So that $R_E$ = 1.4 kΩ which is again not a standard transistor value.

We can use combinations of resistors to obtain the desired value of $R_E$ and thus $A_v$, or we can choose the nearest E12 standard resistor value and accept that the gain will be slightly off.

Choose $R_E$ = 1.3 kΩ so that gain should be:

$$A_v = -\left(\frac{R_C}{R_E}\right) = -\frac{5.6\ \mathrm{k\Omega}}{1.3\ \mathrm{k\Omega}} = -4.2$$

d. Calculation of base bias resistors:

The Q-point voltage at the emitter can now be calculated by assuming $I_E \approx I_C \approx 1\ \mathrm{mA}$. Thus:

$$V_{EQ} = I_{EQ}R_E = (1\ \mathrm{mA})\ (1.3\ \mathrm{k\Omega}) \ = \ 1.3\ \mathrm{V}$$

The base voltage must then be at:

$$V_{EQ} + V_{BE} = 1.3\ \mathrm{V} + 0.7\ \mathrm{V} = 2.0\ \mathrm{V}$$

To achieve this voltage at the base, we can calculate the voltage divider as:

$$\left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = 2\ \mathrm{V}$$

The choice of values for $R_1$ and $R_2$ is a trade-off between high input resistance and a too high current through the base (a basic assumption).

A good rule of thumb is to make $R_2$ approximately 10 times $R_E$. Thus $R_2$ = 13 kΩ and we can calculate $R_1$ as:

$$R_1 = R_2\left(\frac{V_{CC} - V_B}{V_B}\right)$$

$$= 13\ \text{k}\Omega \left(\frac{15\ \text{V} - 2\ \text{V}}{2\ \text{V}}\right) = 84.5\ \text{k}\Omega$$

Choose $R_1$ = 82 kΩ as the real E12 standard resistor value. This practice makes small differences in values of $V_B$, $V_{CQ}$ and $I_E$.

e. We need to consider the effect of input and output resistances and the choice of coupling capacitors.

The input resistance into the amplifier is given approximately be the parallel combination of $R_1$ and $R_2$ (neglecting any loading from the transistor) so that:

$$R_{in} \approx R_1 \parallel R_2 = 11.2\ \text{k}\Omega$$

The presence of the input coupling capacitor will then produce a low-frequency cut-off of the signal given by:

$$f_{C1} = \frac{1}{2\pi C_1 R_{in}}$$

Design requirements needed to gain to be approximately constant down to 100 Hz – choose the low frequency cut-off ~ 10 times lower than that i.e. 10 Hz:

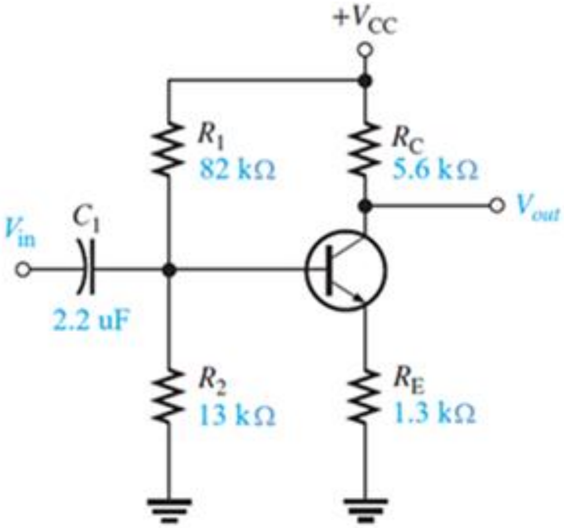
$$C_1 = \frac{1}{2\pi f_C R_{in}} = \frac{1}{2\pi(10)(11.2 \times 10^3\Omega)} = 1.4\ \mu\text{F}$$

Use a non-polarised capacitor of value bigger than 1.4 µF, in this case, it is 2.2 µF.

The final circuit design of the common-emitter BJT amplifier circuit with the calculated values of the components:

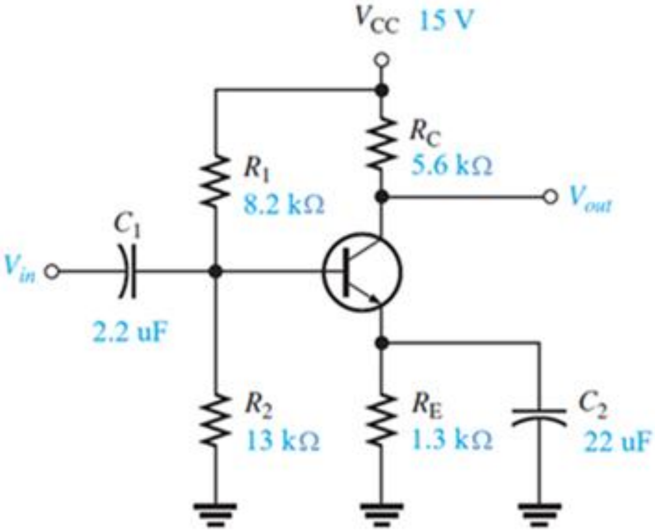
- $R_1 = 82\ \text{k}\Omega.$
- $R_2 = 13\ \text{k}\Omega.$
- $R_C = 5.6\ \text{k}\Omega.$
- $R_E = 1.3\ \text{k}\Omega.$
- $C_1 = 2.2\ \mu\text{F}.$

The amplifier circuit given in the figure below is the final circuit design of BJT amplifier.



f. With the emitter bypass capacitor of 22 µF in place, the voltage gain of the amplifier is calculated from:

$$A_v = \frac{R_C}{r_e'}$$



Knowing that $I_E$ = 1 mA, the value of AC emitter resistance is determined from

$$r_e' = \frac{25\text{ mV}}{I_E} = \frac{25\text{ mV}}{1\text{ mA}} = 25\ \Omega$$

So, the voltage gain of the amplifier is now:

$$A_v = \frac{5.6\ \text{k}\Omega}{25\ \Omega} = 224$$

**Further Example for Tutorial**

There are two exercises that you could attempt by yourself to extend your BJT amplifier design skills and knowledge.

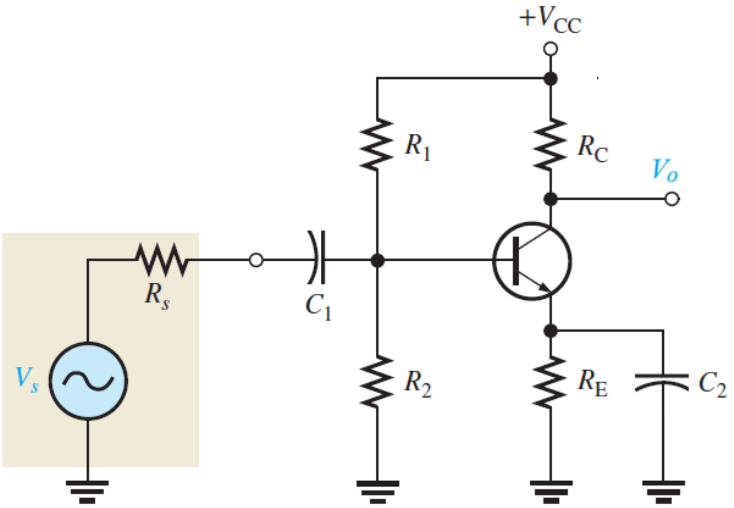
It is important that through these exercises you could determine the more precise values of the components used in the amplifier circuit.

The BJT amplifier is then enhanced further with bypass capacitor for more voltage gain of the amplifier and swamping resistor to stabilise the operating condition of the amplifier.

We need to also determine the other components that will be required for realising common-emitter BJT amplifier with feedback resistor and bypass emitter capacitor.

**Exercise 2 – Design of BJT Amplifier with Emitter Bypass Capacitor and Input Voltage Source**

Extend further the example design of BJT amplifier including an emitter bypass capacitor and a input voltage source with an internal resistance $R_s$.



a. Calculate the value of the emitter bypass capacitor if the frequency of the operation of the BJT amplifier is constant down to 100 Hz. [10 marks]

b. Calculate the actual voltage gain of the amplifier with the emitter-bypass capacitor included. [4 marks]

c. If we consider the input loading of the transistor and the input terminal of the amplifier is connected to a voltage source with internal resistance of 10 $\Omega$, calculate the value of input-coupling capacitor. Assume that the gains of the BJT, $\beta_{dc} = \beta_{ac} = 100$. [6 marks]

**Answer**

a. The value of the emitter-bypass capacitor ($C_2$) is calculated using the following equation:

$$f_{cl(\text{bypass})} = \frac{1}{2\pi\left[\left(r_e' + \frac{R_{Th}}{\beta_{ac}}\right) \parallel R_E\right]C_2}$$

First, calculate the AC emitter resistance of the BJT, $r_e'$:

$$r_e' = \frac{1}{40I_E} = \frac{1}{40(1 \times 10^{-3})} = 25\ \Omega$$

The Thevenin resistor of the potential divider biasing resistors at the input is:

$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(82\ \text{k}\Omega)(13\ \text{k}\Omega)}{82\ \text{k}\Omega + 13\ \text{k}\Omega} = 11.22\ \text{k}\Omega$$

As the lower frequency range of the amplifier is 100 Hz, choose the cut-off frequency of the emitter-bypass capacitor at 10 times less than 100 Hz e.g. 10 Hz.

Knowing $r_e' = 25\ \Omega$ and $R_{Th} = 11.22\ \text{k}\Omega$, the value of the emitter-bypass capacitor is:

$$C_2 = \frac{1}{2\pi[(25\ \Omega\ + 11.22\ \text{k}\Omega) \parallel 1.3\ \text{k}\Omega](10\ \text{Hz})}$$

But

$$R_{\text{emitter}} = (25\ \Omega\ + 11.22\ \text{k}\Omega) \parallel 1.3\ \text{k}\Omega = \frac{(11.245\ \text{k}\Omega)(1.3\ \text{k}\Omega)}{11.245\ \text{k}\Omega + 1.3\ \text{k}\Omega} = 1.165\ \text{k}\Omega$$

So

$$C_2 = \frac{1}{2\pi R_{\text{emitter}} C_2} = \frac{1}{2\pi(1.165\ \text{k}\Omega)(10\ \text{Hz})} = 13.67\ \mu\text{F}$$

b. Actual voltage gain of the amplifier with the emitter-bypass capacitor included is calculated from the following equation.

$$A_v = -\frac{R_C}{r_e' + r_c}$$

Note that $r_c$ is the equivalent series resistance (ESR) of the capacitor. It is assumed to be very small ($r_c \ll r_e$), so:

$$A_v = -\frac{R_C}{r_e'}$$

Knowing that $R_C = 5.6\ \mathrm{k\Omega}$ and $r_e' = 25\ \Omega$, with emitter-bypass capacitor is in parallel with the emitter-feedback resistor, the voltage gain of the amplifier is:

$$A_v = -\frac{5.6\ \mathrm{k\Omega}}{25\ \Omega} = -224$$

As a result, this gain of the amplifier with the emitter-bypass capacitor is a lot more than without the emitter-bypass capacitor (e.g. 4.2).

c. With loadings from the transistor ($R_{in(base)}$) and internal resistance of the voltage source ($R_s = 10\ \Omega$) are considered, the cut-off frequency of the input-coupling capacitor is determined from the following equation:

$$f_{cl(\mathrm{input})} = \frac{1}{2\pi(R_s + R_{in})C_1}$$

Where:

$$R_{in} = R_1 \parallel R_2 \parallel R_{in(\mathrm{base})} \quad \text{and} \quad R_{in(\mathrm{base})} = \beta_{ac} r_e'$$

If the AC gain of the BJT, $\beta_{ac} = 100$, to determine the value of the input-coupling capacitor, we calculate the equivalent resistance at the input of the amplifier.

$$R_{in(\mathrm{base})} = \beta_{ac} r_e' = (100)(25) = 2.5\ \mathrm{k\Omega}$$

And

$$\begin{aligned} R_{in} &= R_1 \parallel R_2 \parallel R_{in(\mathrm{base})} \\ &= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in(\mathrm{base})}}} \\ &= \frac{1}{\frac{1}{82\ \mathrm{k\Omega}} + \frac{1}{13\ \mathrm{k\Omega}} + \frac{1}{2.5\ \mathrm{k\Omega}}} = 2.04\ \mathrm{k\Omega} \end{aligned}$$

So, with loadings from the transistor and internal resistance of the voltage source are considered, the value of the input-coupling capacitor is now:

$$C_1 = \frac{1}{2\pi(R_s + R_{in})f_{cl(\text{input})}}$$

$$= \frac{1}{2\pi(10\ \Omega + 2.04\ \text{k}\Omega)(10)} = 7.8\ \mu\text{F}$$

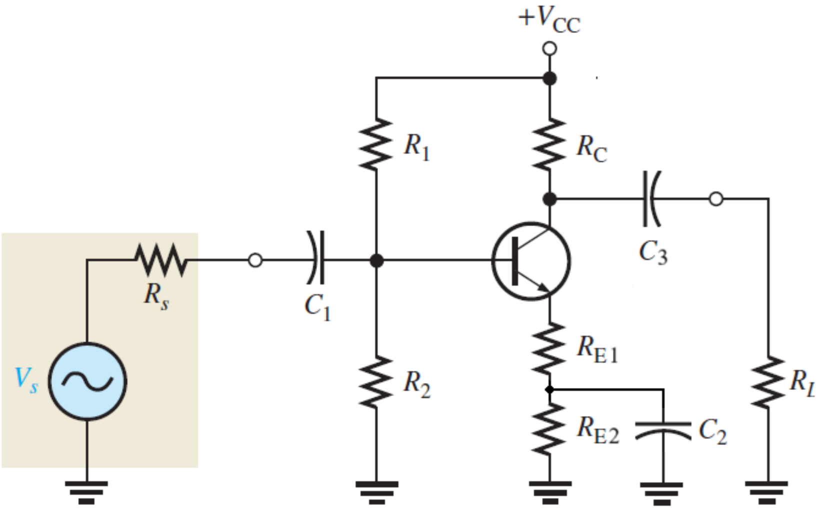
Referring to E12 standard value, choose capacitor value of 8.2 μF. If the value of input coupling capacitor was 2.2 μF before, then it is now 8.2 μF.

## Exercise 3 – Design of BJT Amplifier with Swamping Resistor, Load Resistor, and Output Coupling Capacitor

Extend the given amplifier design in the Exercise 2 by adding a swamping resistor at the emitter leg of the transistor. This resistor is included to improve the stability of the amplifier gain.

- The AC voltage gain is dependent on $r_e'$ since $A_v = R_C/r_e'$.
- Also, $r_e'$ depends on $I_E$ and on temperature. This causes the gain to be unstable over changes in temperature.

For the BJT amplifier circuit with the swamping resistor as shown in the figure below:



a. Calculate the value of the swamping resistor ($R_{E1}$) at the emitter. Determine the voltage gain of the amplifier after the inclusion of the swamping resistor. [4 marks]

b. If the output of the amplifier is connected to a load resistor of 8 $\Omega$, calculate the value of the coupling capacitor at the output of the amplifier. [4 marks]

c. With the voltage gain of the amplifier as calculated in part (a) and knowing that the values of parasitic capacitances $C_{bc} = 2.4$ pF and $C_{be} = 20$ pF from the datasheet of the BJT, estimate the cut-off frequencies in the input and output terminals of the amplifier at high frequency. [16 marks]

**Answer**

a. The swamping emitter resistor $R_{E1}$ is determined as follows:

Knowing that AC emitter resistance of the BJT ($r_e'$) is 25 $\Omega$ , make $R_{E1}$ to be at least 10 times of the $r_e'$.

$$R_{E1} = 10r_e' = 10(25\ \Omega) = 250\ \Omega$$

The voltage gain of the amplifier with the swamping resistor included is:

$$A_v = -\frac{R_C}{R_{E1}} = -\frac{5.6\ \text{k}\Omega}{250\ \Omega} = -22.4$$

Although the voltage gain now is less than the voltage gain of the amplifier without the swamping resistor, but the stability of the amplifier is better, and the amplifier is less dependent on operating temperature of the amplifier circuit.

b. The coupling capacitor at the output of the amplifier is calculated as follows.

$$f_{cl(\text{output})} = \frac{1}{2\pi(R_C + R_L)C_3}$$

So, knowing that $R_L = 8\ \Omega$, the value of coupling capacitor at the output of the amplifier is:

$$C_3 = \frac{1}{2\pi(R_C + R_L)f_{cl(\text{output})}}$$

$$= \frac{1}{2\pi(5.6\ \text{k}\Omega + 8\ \Omega)(10)} = 2.84\ \mu\text{F}$$

By considering E12 standard value, choose 3.3 μF for the output coupling capacitor.

c. Knowing that the values of parasitic capacitances $C_{bc} = 2.4$ pF and $C_{be} = 20$ pF from the datasheet of the BJT, the cut-off frequencies in the input and output terminals of the amplifier at high frequency are estimated as follows.

So, the cut-off frequency in the input of the amplifier at high frequency is:

$$f_{cu(\text{input})} = \frac{1}{2\pi(R_s \parallel R_1 \parallel R_2 \parallel \beta_{ac}r_e')C_{tot}}$$

Where: $C_{tot} = C_{be} + C_{in(\text{Miller})}$

The equivalent Miller capacitance at the input of the BJT is:

$$C_{in(\text{Miller})} = C_{bc}(A_v + 1)$$

$$= (2.4 \times 10^{-12})(22.4 + 1) = 56.16\ \text{pF}$$

The total capacitance at the input of the amplifier is:

$$C_{tot} = C_{be} + C_{in(\text{Miller})}$$

$$= (20\ \times 10^{-12})\ + (56.16 \times 10^{-12}\ )\ =\ 76.16\ \text{pF}$$

The equivalent resistance at the input of the amplifier is:

$$R_{in} = R_s \parallel R_1 \parallel R_2 \parallel \beta_{ac}r_e' = \frac{1}{\frac{1}{R_s} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{\beta_{ac}r_e'}}$$

$$= \frac{1}{\frac{1}{10\ \Omega} + \frac{1}{82\ \text{k}\Omega} + \frac{1}{13\ \text{k}\Omega} + \frac{1}{(100)(25\ \Omega)}} = 9.95\ \Omega$$

So, the cut-off frequency in the input of the amplifier at high frequency is:

$$f_{cu(\text{input})} = \frac{1}{2\pi R_{in} C_{tot}}$$

$$= \frac{1}{2\pi(9.95)(76.16\ \times 10^{-12})} = 210\ \text{MHz}$$

The cut-off frequency in the output of the amplifier at high frequency is calculated from:

$$f_{cu(\text{output})} = \frac{1}{2\pi(R_C \parallel R_L)C_{out(\text{Miller})}}$$

The equivalent Miller capacitance at the output of the BJT is:

$$C_{out(\text{Miller})} = C_{bc}\left(\frac{A_v + 1}{A_v}\right)$$

$$= (2.4 \times 10^{-12})\left(\frac{22.4 + 1}{22.4}\right) = 2.51\ \text{pF}$$

The equivalent resistance at the output of the amplifier is:

$$R_{out} = R_C \parallel R_L = \frac{R_C R_L}{R_C + R_L}$$

$$= \frac{(82\ \text{k}\Omega)(13\ \text{k}\Omega)}{82\ \text{k}\Omega + 13\ \text{k}\Omega} = 11.22\ \text{k}\Omega$$

So, the cut-off frequency in the output of the amplifier at high frequency is:

$$f_{cu(\text{output})} = \frac{1}{2\pi R_{out} C_{out(\text{Miller})}}$$

$$= \frac{1}{2\pi(11.22\ \text{k}\Omega)(2.51 \times 10^{-12})} = 5.65\ \text{MHz}$$