XMUT204 Electronic Design

Tutorial 6: BJT Frequency Response and Further Circuits (Solution)

A. Frequency Response of BJT Amplifiers

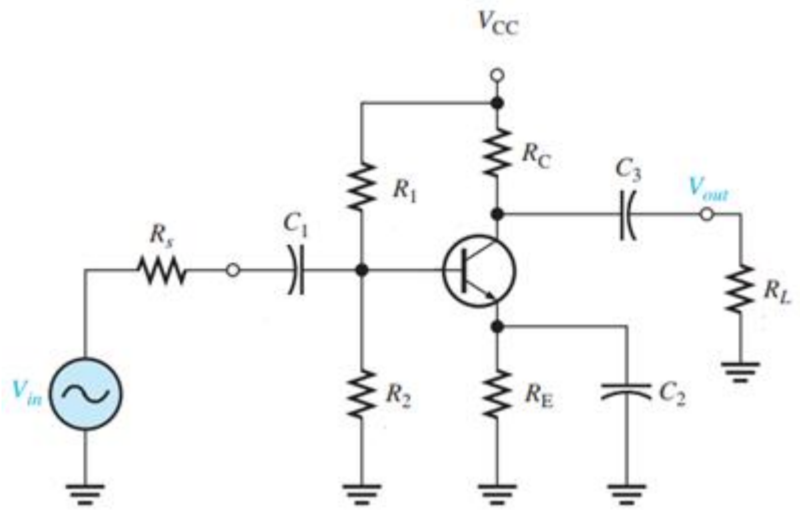
1. Bode plot is very handy for helping you to design the frequency response characteristics of the amplifier.

   a. Draw a composite Bode plot of a BJT amplifier response for three low-frequency RC circuits with different critical frequencies (e.g. $f_{cl(\text{bypass})} < f_{cl(\text{input})} < f_{cl(\text{output})}$). [4 marks]

   b. Draw a composite Bode plot of an amplifier response where all RC circuits have the same critical frequencies (f_{cl}). [4 marks]

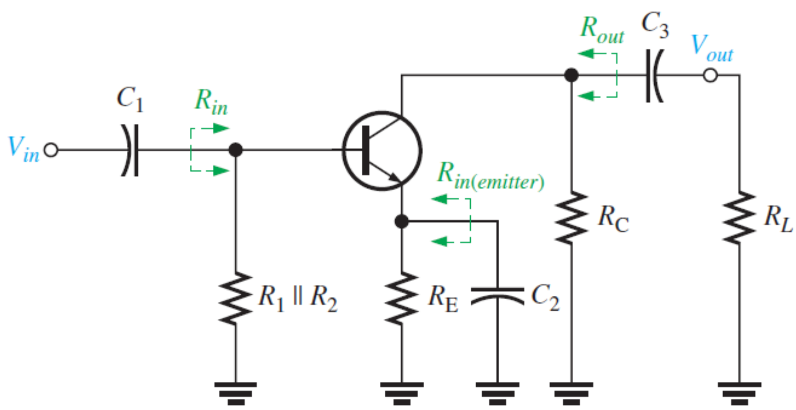
   c. The midrange voltage gain of a certain amplifier is 100. The input RC circuit has a lower critical frequency of 1 kHz. Determine the actual voltage gain at f_1 = 1 kHz, f_2 = 100 Hz, and f_3 = 10 Hz. [6 marks]

   As a reference for this question, an example of the BJT amplifier circuit is given in the following figure.

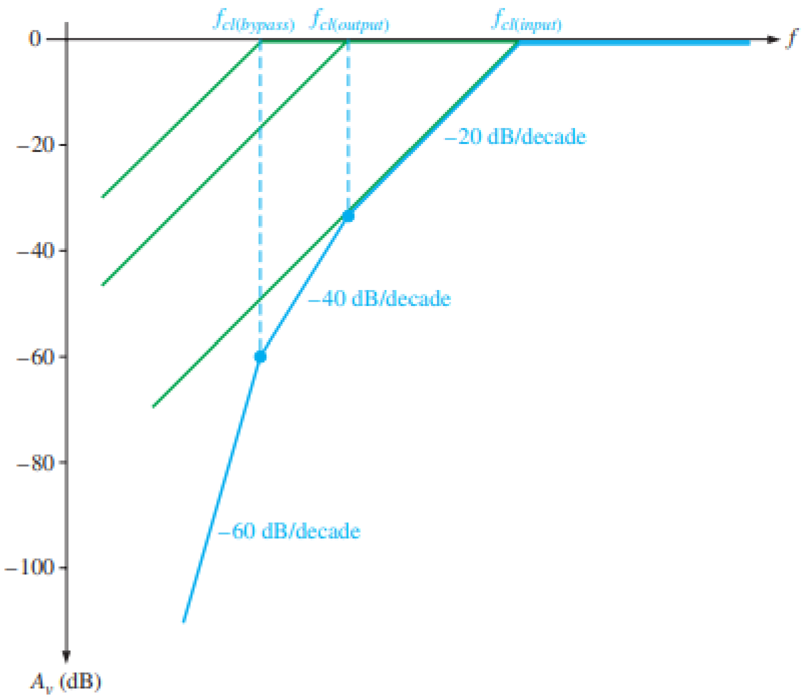


Solution

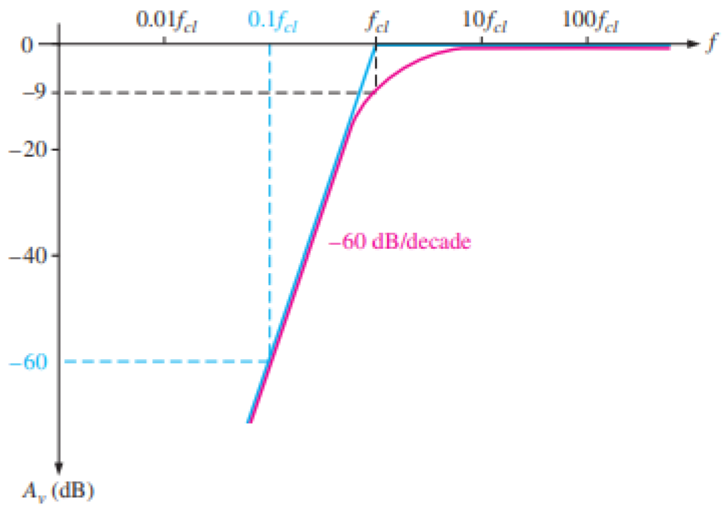
a. The low-frequency AC equivalent circuit of the amplifier circuit consist of three high-pass RC circuits e.g. input coupling capacitor with equivalent input resistance, output coupling capacitor with equivalent output resistance, and emitter bypass capacitor with equivalent resistance at the emitter.



Composite Bode plot of a BJT amplifier response for three low-frequency RC circuits with different critical frequencies. Total response is shown by the blue curve.



b. Composite Bode plot of an amplifier response where all RC circuits have the same f_{cl} (note: blue is ideal; red is actual).



c. When f_1 = 1 kHz, the voltage gain is 3 dB less than at midrange. At -3 dB, the voltage gain is reduced by a factor of 0.707.

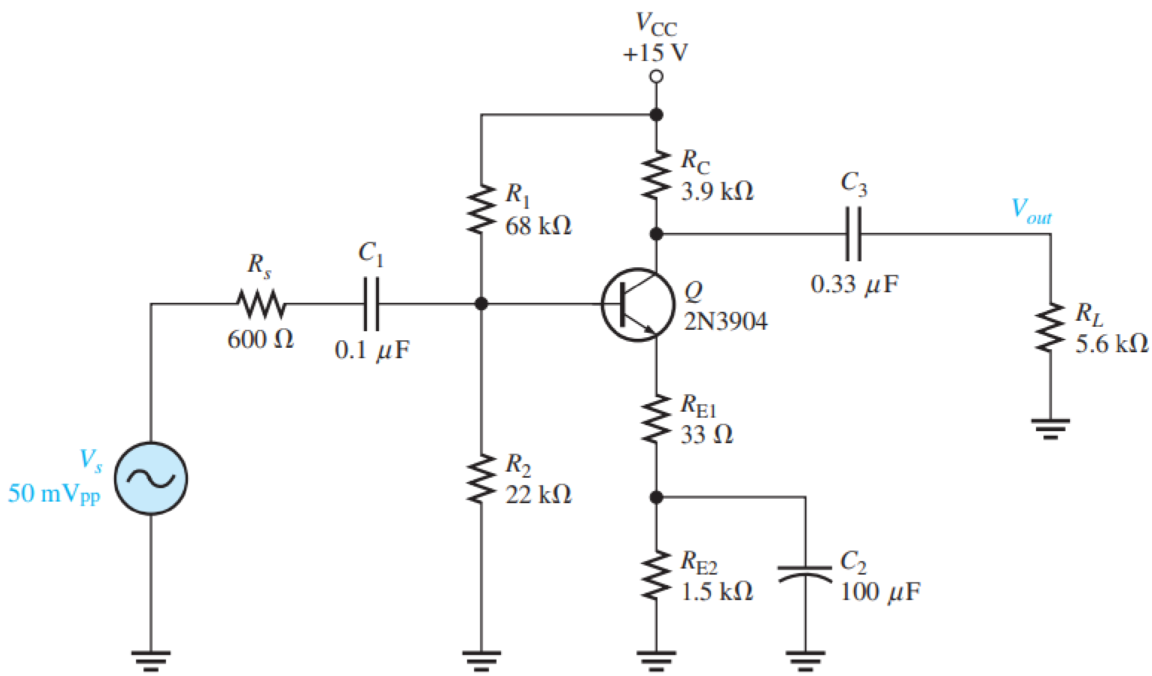
$$A_v = (0.707)(100) = 70.7$$

When f_2 = 100 Hz = $0.1f_c$, the voltage gain is 20 dB less than at f_c. The voltage gain at -20 dB is one-tenth of that at the midrange frequencies.

$$A_v = (0.01)(100) = 1$$

When f_3 = 10 Hz = $0.01f_c$, the voltage gain is 20 dB less than at f = $0.1f_c$ or -40 dB. The voltage gain at -40 dB is one-tenth of that at -20 dB or one-hundredth that at the midrange frequencies.

2. For the circuit in the figure given below, assuming r_e' = 9.6 Ω and β = 200, notice that a swamping resistor, R_{E1}, is used.



a. Calculate the lower critical frequency due to the input RC circuit. [4 marks]

b. Calculate the lower critical frequency due to the output RC circuit. [4 marks]

c. Calculate the lower critical frequency due to the bypass RC circuit. [4 marks]

d. Determine the mid-band gain in decibels and draw the Bode plot, showing each of the lower critical frequencies. [8 marks]

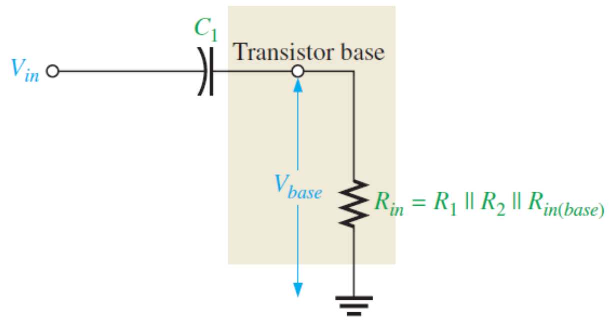
Solution

a. The equivalent resistance in the input RC circuit is:

$$R_{in} = R_1 \parallel R_2 \parallel \beta(r_e' + R_{E1}) = 68\text{ k}\Omega \parallel 22\text{ k}\Omega \parallel 200(9.6\ \Omega + 33\ \Omega) = 5.63\text{ k}\Omega$$

The lower critical frequency due to the input coupling capacitor is:

$$f_{cl(\text{input})} = \frac{1}{2\pi R_{in} C_1} = \frac{1}{2\pi(5.63\text{ k}\Omega)(0.1\ \mu\text{F})} = 282\text{ Hz}$$

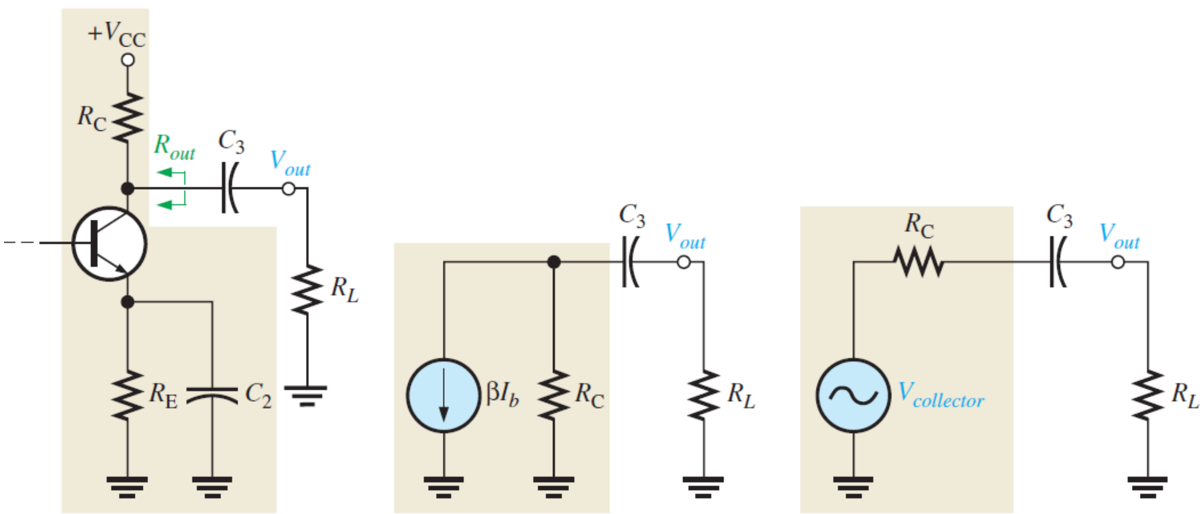


b. The equivalent resistance in the output RC circuit is:

$$R_C + R_L = 3.9\text{ k}\Omega + 5.6\text{ k}\Omega = 9.5\text{ k}\Omega$$

The lower critical frequency due to the output coupling capacitor is:

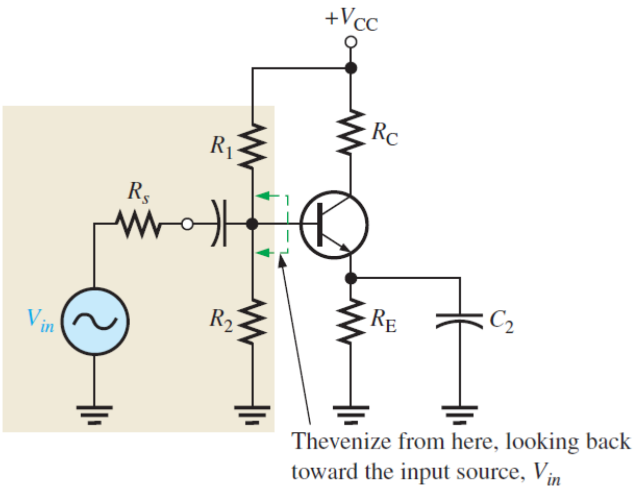
$$f_{cl(\text{output})} = \frac{1}{2\pi(R_C + R_L)C_3} = \frac{1}{2\pi(9.5\text{ k}\Omega)(0.33\ \mu\text{F})} = 50.8\text{ Hz}$$



c. The equivalent input resistance in the bypass emitter circuit is:

$$R_{in(\text{emitter})} = r_e' + R_{E1} + \frac{R_{th}}{\beta_{ac}}$$

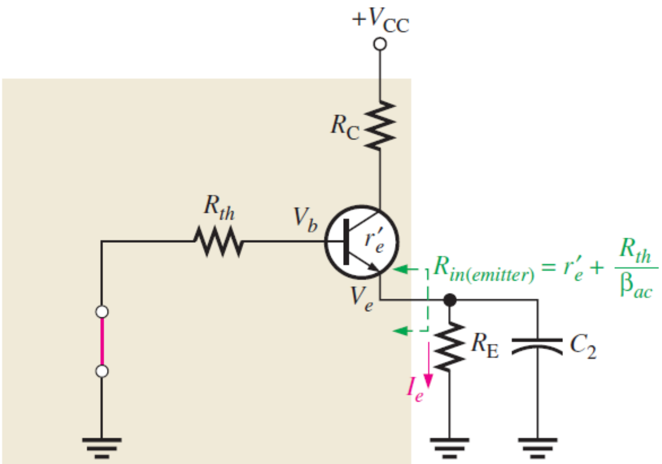
$$= 9.6\ \Omega + 33\ \Omega + \frac{(68\ \text{k}\Omega + 22\ \text{k}\Omega + 600\ \Omega)}{200} = 45.5\ \Omega$$



The lower critical frequency due to emitter bypass capacitor is:

$$f_{cl(\text{bypass})} = \frac{1}{2\pi(R_{in}(\text{emitter}) \parallel R_{E2})C_2}$$

$$= \frac{1}{2\pi(45.5\ \Omega \parallel 1.5\ \text{k}\Omega)(100\ \mu\text{F})} = 36\ \text{Hz}$$



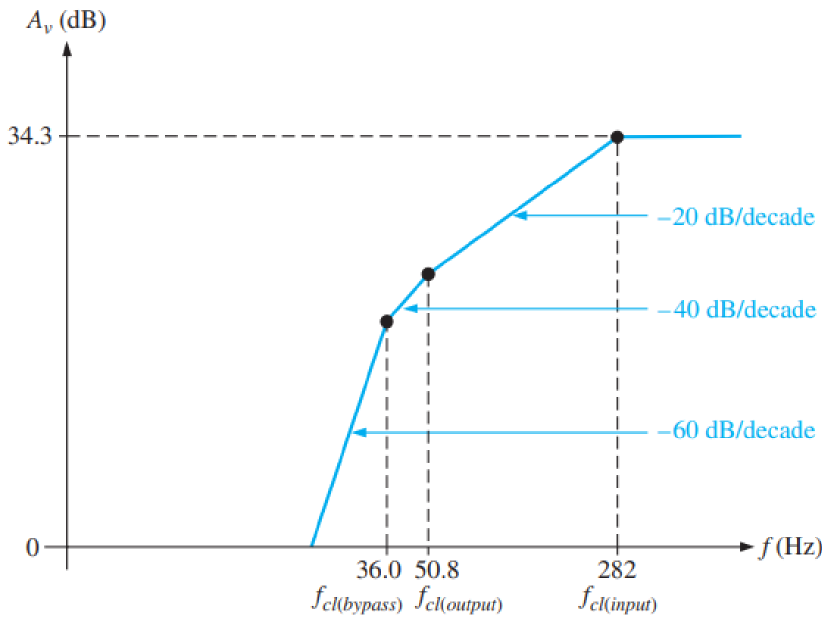
d. The mid-band gain of the frequency response of the amplifier circuit is:

$$A_v = \frac{R_C \parallel R_L}{r_e' + R_{E1}} = \frac{(3.9\ \text{k}\Omega) \parallel (5.6\ \text{k}\Omega)}{9.6\ \Omega + 33\ \Omega} = 54$$

In decibels,

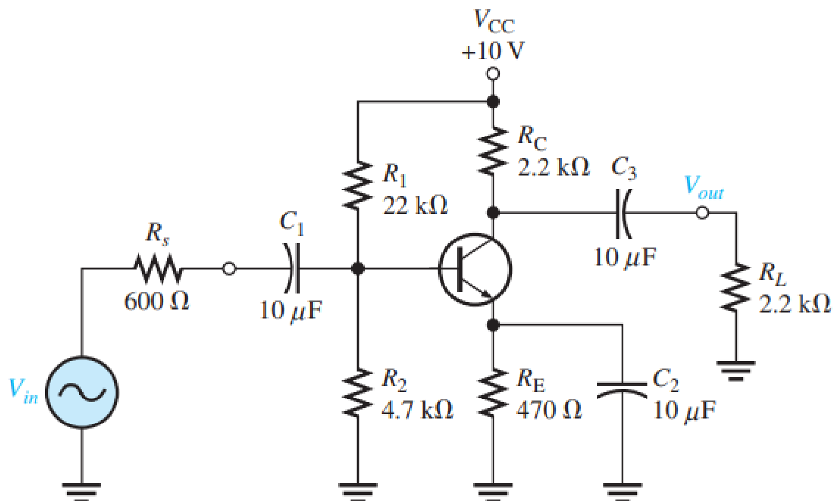
$$A_v = 20\log(54) = 34.3\ \text{dB}$$

The critical frequency for the input circuit was found in part (a) and is 282 Hz. The critical frequency for the output circuit was found in part (b) and is 50.8 Hz. The critical frequency for the emitter bypass circuit was found in part (c) and is 36.0 Hz.



The overall response is shown in the Bode plot of the figure given above. The lower critical frequency of the input circuit has the highest value and is therefore the overall or dominant critical frequency because the response first begins to roll off at this frequency.

3. For the following BJT amplifier, the transistor's datasheet provides the following: β_{ac} = 125, C_{be} = 20 pF, and C_{bc} = 2.4 pF.



a. Derive the equivalent high-frequency input RC circuit for the BJT. Use this to determine the upper critical frequency due to the input circuit. [20 marks]

b. Determine the upper critical frequency of the amplifier due to its output RC circuit. [8 marks]

Solution

a. The high frequency end of the frequency response of the amplifier is dominated by the cut-off frequencies due to parasitic capacitances in the BJT.

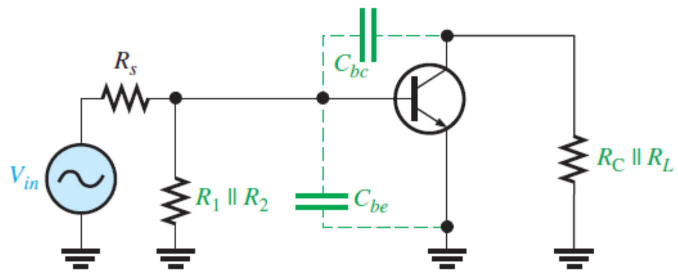
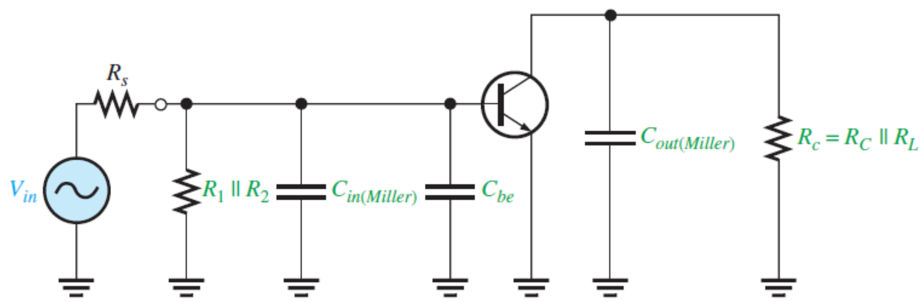
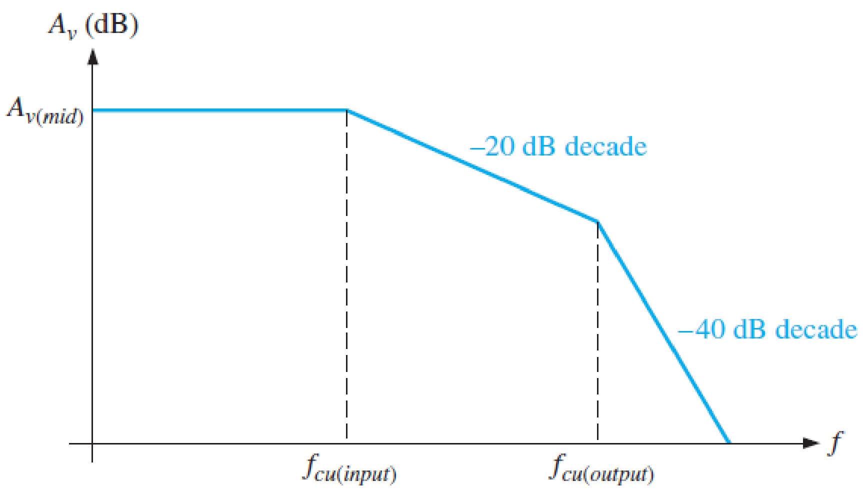


Figure below shows the high-frequency equivalent circuit after applying Miller's theorem.



The high-frequency end of the frequency response of the amplifier circuit due to the parasitic capacitances at the input and output of the BJT is as shown in the figure below.



First, find r_e' as follows:

$$V_B = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{4.7\ \text{k}\Omega}{22\ \text{k}\Omega + 4.7\ \text{k}\Omega}\right) \times 10\ \text{V} = 1.76\ \text{V}$$

$$V_E = V_B - V_{BE} = 1.76\ \text{V} - 0.7\ \text{V} = 1.06\ \text{V}$$

$$I_E = \frac{V_E}{R_E} = \frac{1.06\ \text{V}}{470\ \Omega} = 2.26\ \text{mA}$$

$$r_e' \cong \frac{25\ \text{mV}}{I_E} = \frac{25\ \text{mV}}{2.26\ \text{mA}} = 11.1\ \Omega$$

The total resistance of the input circuit is:

$$R_{in(\text{tot})} = R_s \parallel R_1 \parallel R_2 \parallel \beta_{ac} r'_e = 600\ \Omega \parallel 22\ \text{k}\Omega \parallel 4.7\ \text{k}\Omega \parallel 125(11.1\ \Omega) = 378\ \Omega$$

Next, to determine the capacitance, you must calculate the midrange gain of the amplifier, so that you can apply Miller's theorem.

$$A_{v(\text{mid})} = \frac{R_C}{r'_e} = \frac{R_C \parallel R_L}{r'_e} = \frac{1.1\ \text{k}\Omega}{11.1\ \Omega} = 99 \approx 40\ \text{dB}$$

Apply Miller's theorem to determine input internal capacitance of the transistor.

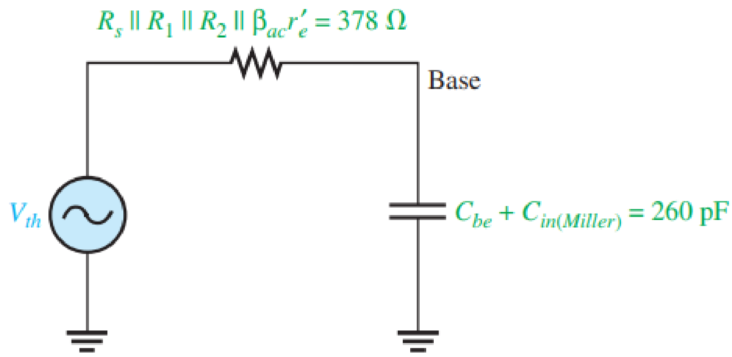
$$C_{in(\text{Miller})} = C_{bc}(A_{v(\text{mid})} + 1) = (2.4\ \text{pF})(99 + 1) = 240\ \text{pF}$$

The total input capacitance is $C_{in(\text{Miller})}$ in parallel with C_{be}.

$$C_{in(\text{tot})} = C_{in(\text{Miller})} + C_{be} = 240\ \text{pF} + 20\ \text{pF} = 260\ \text{pF}$$

The resulting high-frequency input RC circuit is shown in the figure below. The upper critical frequency is:

$$f_{cu(\text{input})} = \frac{1}{2\pi R_{in}(\text{tot}) C_{in(\text{tot})}} = \frac{1}{2\pi(378\ \Omega)(260\ \text{pF})} = 1.62\ \text{MHz}$$



b. Calculate the Miller output internal capacitance of the transistor:

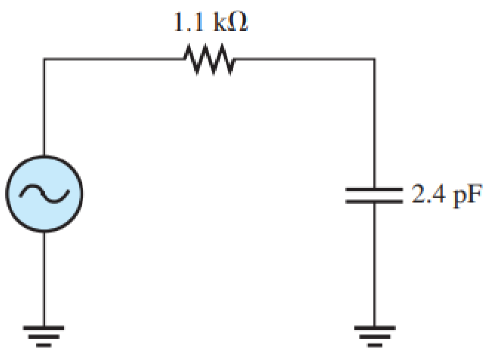
$$C_{out(\text{Miller})} = C_{bc}\left(\frac{A_v + 1}{A_v}\right) = (2.4\ \text{pF})\left(\frac{99 + 1}{99}\right) \cong 2.4\ \text{pF}$$

The equivalent resistance is:

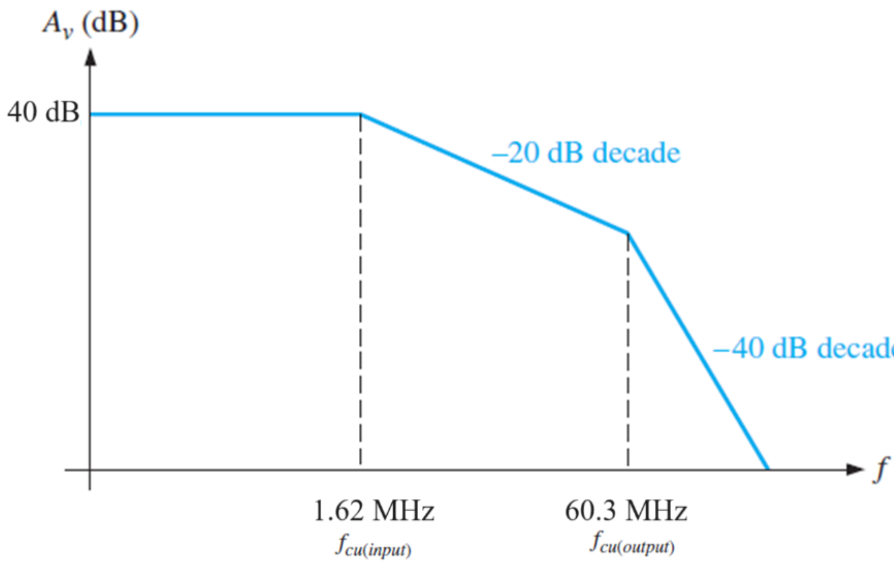
$$R_C = R_C \parallel R_L = 2.2\ \text{k}\Omega \parallel 2.2\ \text{k}\Omega = 1.1\ \text{k}\Omega$$

The equivalent output RC circuit is shown in the figure below. Determine the upper critical frequency as follows ($C_{out(\text{Miller})} \cong C_{bc}$).

$$f_{cu(\text{output})} = \frac{1}{2\pi R_c C_{bc}} = \frac{1}{2\pi(1.1\ \text{k}\Omega)(2.4\ \text{pF})} = 60.3\ \text{MHz}$$



The upper end of the frequency response of the amplifier is as shown in the figure below.

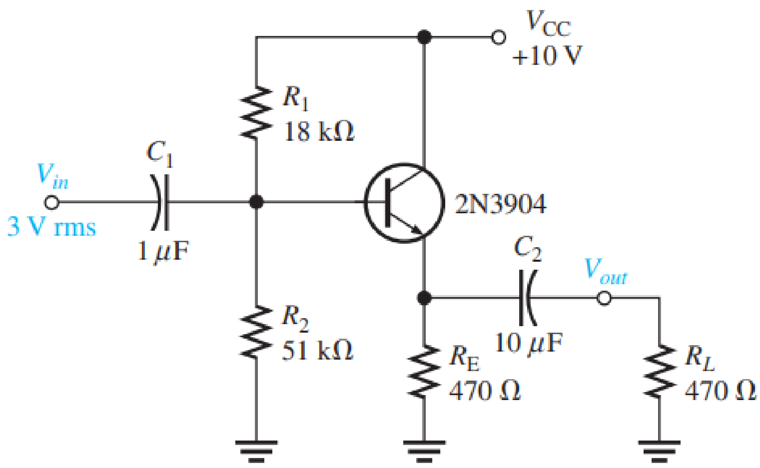


B. BJT Circuit Configurations

1. For the common collector (emitter-follower) in the figure given below, assume β_{AC} = 175 and that the capacitive reactance is negligible at the frequency of operation.

   a. Determine the total input resistance of the circuit. [6 marks]

   b. Also, find the voltage gain, current gain, and power gain in terms of power delivered to the load, R_L. [24 marks]



Solution

a. The AC emitter resistance external to the transistor is:

$$R_e = R_E \parallel R_L = 470\ \Omega \parallel 470\ \Omega = 235\ \Omega$$

The approximate resistance, looking in at the base, is:

$$R_{in(\text{base})} \cong \beta_{ac} R_e = (175)(235\ \Omega) = 41.1\ \text{k}\Omega$$

The total input resistance of the amplifier is:

$$R_{in(\text{tot})} = R_1 \parallel R_2 \parallel R_{in(\text{base})} = 18\ \text{k}\Omega \parallel 51\ \text{k}\Omega \parallel 41.1\ \text{k}\Omega = 10.1\ \text{k}\Omega$$

b. For a common collector, the voltage gain is:

$$A_v \cong 1$$

By using the AC emitter resistance, r_e', you can determine a more precise value of A_v if necessary

$$V_E = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} - V_{BE} = \left(\frac{51\ \text{k}\Omega}{18\ \text{k}\Omega + 51\ \text{k}\Omega}\right) \times 10\ \text{V} - 0.7\ \text{V} = 6.69\ \text{V}$$

Therefore

$$I_E = \frac{V_E}{R_E} = \frac{6.69\ \text{V}}{470\ \Omega} = 14.2\ \text{mA}$$

And

$$r_e' \cong \frac{25}{I_E} = \frac{25\ \text{mV}}{14.2\ \text{mA}} = 1.76\ \Omega$$

So

$$A_v = \frac{R_e}{r_e' + R_e} = \frac{235\ \Omega}{237\ \Omega} = 0.992$$

The small difference in A_v because of considering r_e' is insignificant in most cases.

The current gain of common collector circuit is calculated from:

$$A_i = \frac{I_e}{I_{in}}$$

The calculations of the current gain of the amplifier are as follows. The emitter current is calculated from:

$$I_e = \frac{V_e}{R_e} = \frac{A_v V_b}{R_e} = \frac{(0.992)(3\text{ V})}{235\text{ k}\Omega} = 12.7\text{ mA}$$

Since $V_e = A_v V_b$ for a common collector, the emitter current is:

$$I_e = \frac{A_v V_b}{R_e} = \frac{(0.992)(3\text{ V})}{235\text{ k}\Omega} = 12.7\text{ mA}$$

The input current is:

$$I_{in} = \frac{V_{in}}{R_{in(\text{tot})}} = \frac{3\text{ V}}{10.1\text{ k}\Omega} = 297\ \mu\text{A}$$

As a result, the current gain of the amplifier is:

$$A_i = \frac{I_e}{I_{in}} = \frac{12.7\text{ mA}}{297\ \mu\text{A}} = 42.8$$

The power gain of the amplifier is:

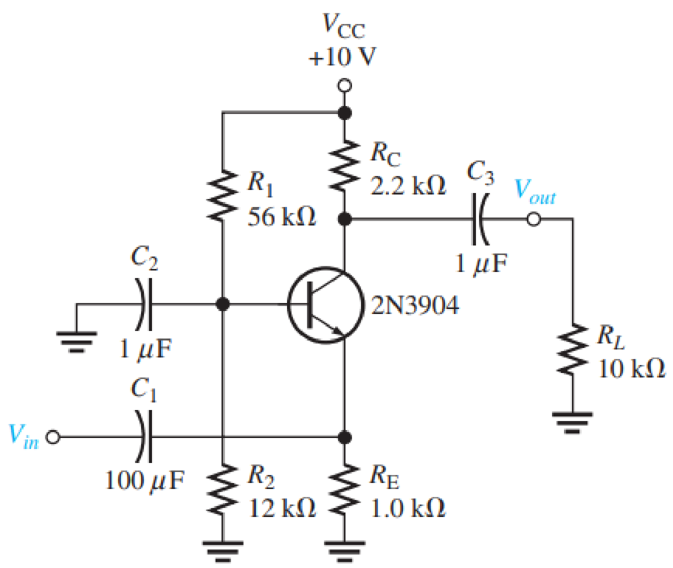
$$A_p \cong A_i = 42.8$$

Since $R_L = R_E$, one-half of the power is dissipated in R_E and one-half in R_L.

Therefore, in terms of power to the load, the power gain is:

$$A_{p(\text{load})} = \frac{A_p}{2} = \frac{42.8}{2} = 21.4$$

2. Find the input resistance, voltage gain, current gain, and power gain for the common-base amplifier as shown in the figure below. Consider that β_{DC} = 250. [18 marks]



Solution

First, find I_E so that you can determine r'_e. Then, the input resistance of the amplifier is:

$$R_{in} \cong r'_e$$

The Thevenin resistance at the input of the amplifier is:

$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(56\ \text{k}\Omega)(12\ \text{k}\Omega)}{56\ \text{k}\Omega + 12\ \text{k}\Omega} = 9.88\ \text{k}\Omega$$

The Thevenin voltage at the input of the amplifier is:

$$V_{Th} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{12\ \text{k}\Omega}{56\ \text{k}\Omega + 12\ \text{k}\Omega}\right) 10\ \text{V} = 1.76\ \text{V}$$

The current at the emitter is:

$$I_E = \frac{V_{Th} - V_{BE}}{R_E + (R_{Th}/\beta_{DC})} = \frac{1.76\ \text{V} - 0.7\ \text{V}}{1\ \text{k}\Omega + (9.88\ \text{k}\Omega/250)} = 1.02\ \text{mA}$$

Therefore, the input resistance of the amplifier is:

$$R_{in} \cong r'_e = \frac{25\ \text{mV}}{I_E} = \frac{25\ \text{mV}}{1.02\ \text{mA}} = 24.5\ \Omega$$

Calculate the voltage gain as follows

$$R_C = R_C \parallel R_L = 2.2\ \text{k}\Omega \parallel 10\ \text{k}\Omega = 1.8\ \text{k}\Omega$$

Thus, the voltage gain of the amplifier is:

$$A_v = \frac{R_C}{r'_e} = \frac{1.8\ \text{k}\Omega}{24.5\ \Omega} = 73.5$$

The current gain of the amplifier is:

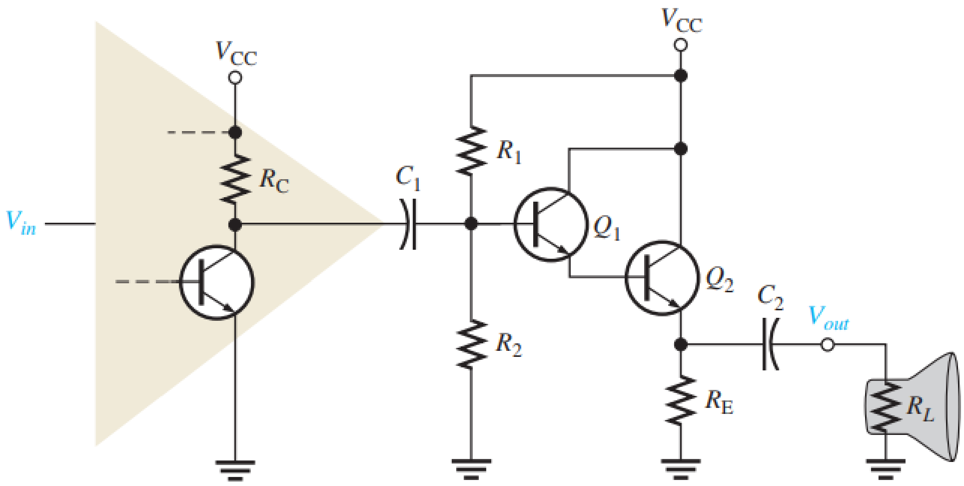
$$A_i \cong 1$$

The power gain of the amplifier is:

$$A_p = A_v = 76.3$$

C. Multistage BJT Amplifiers

1. In the figure below for the common-emitter amplifier, V_{CC} = 12 V, R_C = 1 kΩ and r'_e = 5 Ω. For the Darlington emitter-follower, R_1 = 10 kΩ, R_2 = 22 kΩ, R_E = 22 Ω, R_L = 8 Ω, V_{CC} = 12 V, and β_{dc} = β_{ac} = 100 for each transistor. Neglect $R_{in(\text{base})}$ of the Darlington.



a. Determine the voltage gain of the common-emitter amplifier. [18 marks]

b. Determine the voltage gain of the Darlington emitter-follower. [2 marks]

c. Determine the overall voltage gain and compare to the gain of the common-emitter amplifier driving the speaker directly without the Darlington emitter-follower. [2 marks]

Solution

a. To determine A_v for the common-emitter amplifier, first find r_e' for the Darlington emitter-follower

$$V_B = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{22\ \text{k}\Omega}{10\ \text{k}\Omega + 22\ \text{k}\Omega}\right) \times 12\ \text{V} = 8.25\ \text{V}$$

$$I_E = \frac{V_E}{R_E} = \frac{V_B - 2V_{BE}}{R_E} = \frac{8.25\ \text{V} - 1.4\ \text{V}}{22\ \Omega} = 311\ \text{mA}$$

$$r_e' = \frac{25\ \text{mV}}{I_E} = \frac{25\ \text{mV}}{311\ \text{mA}} = 80\ \text{m}\Omega$$

Note that R_E must dissipate a power of:

$$P_{R_E} = I_E^2 R_E = (311\ \text{mA})^2 (22\ \Omega) = 2.13\ \text{W}$$

and transistor Q_2 must dissipate

$$P_{Q_2} = (V_{CC} - V_E) I_E = (5.4\ \text{V})(3.11\ \text{mA}) = 1.68\ \text{W}$$

Next, the AC emitter resistance of the Darlington emitter-follower is:

$$R_e = R_E \parallel R_L = 22\ \Omega \parallel 8\ \Omega = 5.87\Omega$$

The total input resistance of the Darlington emitter-follower is:

$$R_{in(\text{tot})} = R_1 \parallel R_2 \parallel \beta_{ac}^2 (r_e' + R_e)$$

$$= 10\ \text{k}\Omega \parallel 22\ \text{k}\Omega \parallel (100)^2 (80\ \text{m}\Omega + 5.87\ \Omega) = 6.16\ \text{k}\Omega$$

The effective AC collector resistance of the common-emitter amplifier is:

$$R_C = R_C \parallel R_{in(\text{tot})} = 1\ \text{k}\Omega \parallel 6.16\ \text{k}\Omega = 860\ \Omega$$

The voltage gain of the common-emitter amplifier is:

$$A_{v(\text{CE})} = \frac{R_C}{r'_e} = \frac{860\ \Omega}{5\ \Omega} = 172$$

b. The effective AC emitter resistance was found in part (a) to be 80 mΩ. The voltage gain for the Darlington emitter-follower is:

$$A_{v(\text{EF})} = \frac{R_e}{r'_e + R_e} = \frac{5.87\ \Omega}{80\ \text{m}\Omega + 5.87\ \Omega} = 0.99$$

c. The overall voltage gain is:

$$A'_v = A_{v(\text{EF})}A_{v(\text{CE})} = (0.99)(172) = 170$$

If the common-emitter amplifier drives the speaker directly, the gain is 1.59 as we previously calculated.

2. Figure below shows a direct-coupled (that is, with no coupling capacitors between stages) two-stage amplifier. The DC bias of the first stage sets the DC bias of the second. Determine:

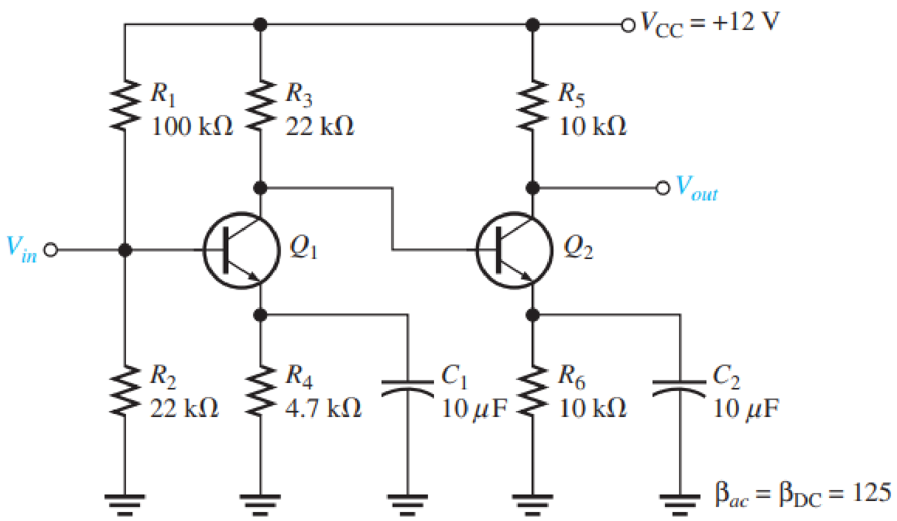
a. All DC voltages for the first stage. [8 marks]

b. All DC voltages for the second stage. [8 marks]

c. The AC voltage gain of the first stage. [4 marks]

d. The AC voltage gain of the second stage. [6 marks]

e. The overall AC voltage gain. [2 marks]



Solution

a. First, we work out the first stage of the amplifier and determine the voltage at the base from the following equation:

$$V_{B1} = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\frac{22\ \text{k}\Omega}{100\ \text{k}\Omega + 22\ \text{k}\Omega}\right) \times 12\ \text{V} = 2.16\ \text{V}$$

Then, the voltage at the emitter of the first stage is:

$$V_{E1} = V_{B1} - V_{BE} = 2.16\ \text{V} - 0.7\ \text{V} = 1.46\ \text{V}$$

The current that flows in the collector can be approximated from the following:

$$I_{C1} \cong I_{E1} = \frac{V_{E1}}{R_4} = \frac{1.46\ \text{V}}{4.7\ \text{k}\Omega} = 0.311\ \text{mA}$$

As a result, the voltage at the collector of the first stage is:

$$V_{C1} = V_{CC} - I_{C1}R_3 = 12\ \text{V} - (0.311\ \text{mA})(22\ \text{k}\Omega) = 5.16\ \text{V}$$

b. Then, we work out the second stage of the amplifier. Since it is a direct-coupled amplifier, as a result the voltage at the base of the second stage is equal to the voltage at the collector of the first stage.

$$V_{B2} = V_{C1} = 5.16\ \text{V}$$

The voltage at the emitter of the second stage is found from:

$$V_{E2} = V_{B2} - V_{BE} = 5.16\ \text{V} - 0.7\ \text{V} = 4.46\ \text{V}$$

As a result, the current that flows in the collector at the second stage we can approximate it from:

$$I_{C2} \cong I_{E2} = \frac{V_{E2}}{R_6} = \frac{4.46\ \text{V}}{10\ \text{k}\Omega} = 0.446\ \text{mA}$$

Then, the voltage at the collector of the second stage is:

$$\begin{aligned} V_{C2} &= V_{CC} - I_{C2}R_5 \\ &= 12\ \text{V} - (0.446\ \text{mA})(10\ \text{k}\Omega) = 7.54\ \text{V} \end{aligned}$$

c. With C_1 in parallel with R_3 at the output of the first stage, the approximate AC equivalent resistance of the first stage of the amplifier is found from:

$$r'_{e1} \cong \frac{V_T}{I_{E1}} = \frac{25\ \text{mV}}{0.311\ \text{mA}} = 80.4\ \Omega$$

The voltage gain of the first stage of the amplifier is:

$$A_{v1} = \frac{\left(R_3 \parallel R_{in(base2)}\right)}{r'_{e1}} = \frac{22\ \text{k}\Omega \parallel 7\ \text{k}\Omega}{80.4\ \text{k}\Omega} = 66$$

d. With C_2 in parallel with R_6 at the output of the second stage, the approximate AC equivalent resistance of the second stage of the amplifier is obtained from:

$$r'_{e2} \cong \frac{V_T}{I_{E2}} = \frac{25\ \text{mV}}{0.446\ \text{mA}} = 56\ \Omega$$

The resistance looking at the input of the second stage of the amplifier is:

$$R_{in(base2)} = \beta_{ac} r'_{e2} = (125)(56\ \Omega) = 7\ \text{k}\Omega$$

The voltage gain of the second stage of the amplifier is:

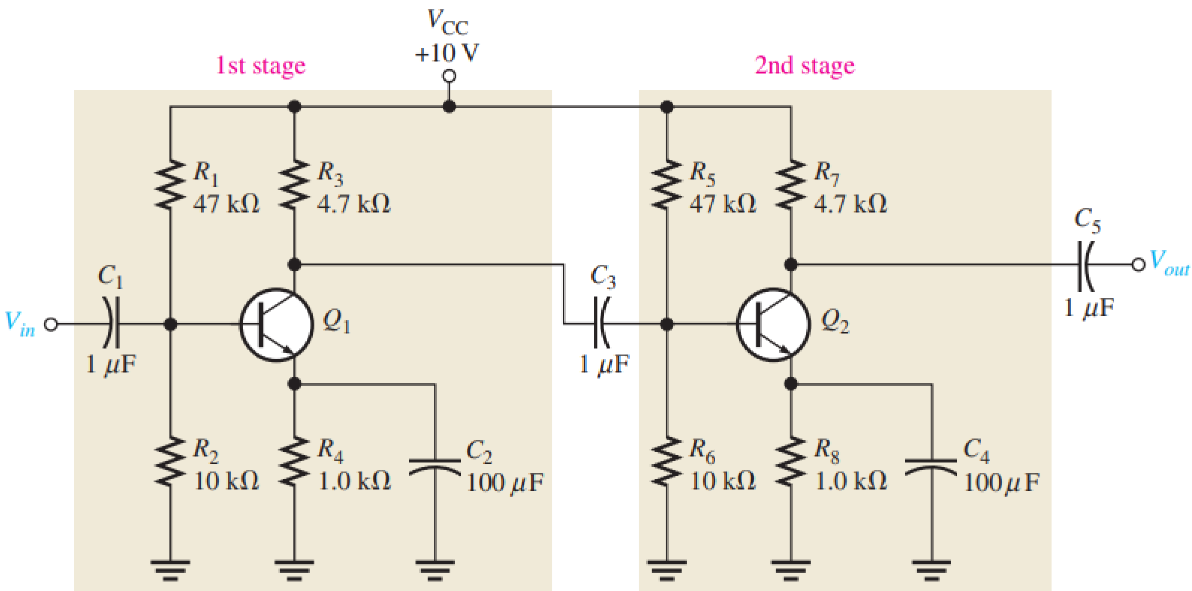
$$A_{v2} = \frac{R_5}{r'_{e2}} = \frac{10\ \text{k}\Omega}{56\ \text{k}\Omega} = 179$$

e. Finally, the overall voltage gain of the amplifier is:

$$A'_v = A_{v1} A_{v2} = (66)(179) = 11{,}814$$

3. For the following capacitively-coupled multistage NPN BJT transistor amplifier, answer the following questions.

a.	Describe the loading effect of the second stage of the amplifier.	[5 marks]
b.	Determine the voltage gain of the first stage.	[6 marks]
c.	Determine the voltage gain of the second stage.	[2 marks]
d.	Determine the overall voltage gain of the amplifier.	[6 marks]
e.	Determine the DC Voltages in the amplifier.	[10 marks]

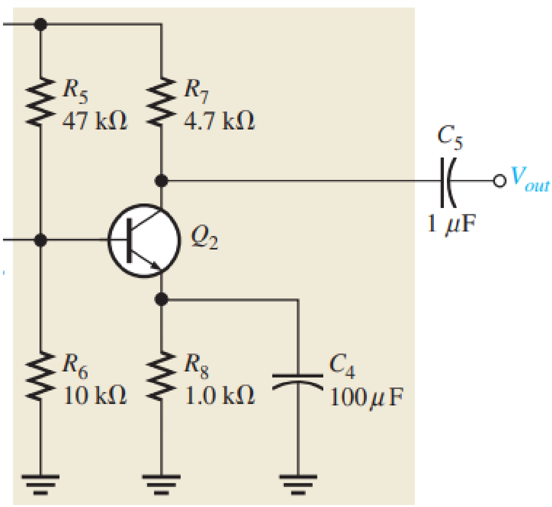


$\beta_{DC} = \beta_{ac} = 150$ for Q_1 and Q_2

Solution

a. Notice that the biasing DC voltage at the base of the second stage is:

$$V_{B2} = \left(\frac{R_6}{R_5 + R_6}\right)V_{CC} = \left(\frac{10\ \text{k}\Omega}{47\ \text{k}\Omega + 10\ \text{k}\Omega}\right)10\ \text{V} = 1.75\ \text{V}$$



The emitter current in the Q_2 is:

$$I_{E2} = \frac{V_{B2} - V_{BE2}}{R_8} = \frac{1.75\ \text{V} - 0.7\ \text{V}}{1\ \text{k}\Omega} = 1.05\ \text{mA}$$

Thus

$$r'_{e2} = \frac{V_T}{I_{E2}} = \frac{25\ \text{mV}}{1.05\ \text{mA}} = 23.8\ \Omega$$

The equivalent input resistance at the base of the second stage is:

$$R_{in(base2)} = \beta_{ac} r'_{e2} = (150)(23.8\ \Omega) = 3.57\ \text{k}\Omega$$

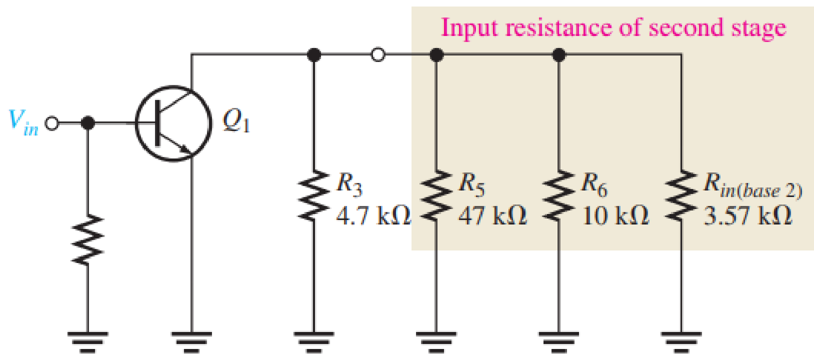
In determining the voltage gain of the first stage, you must consider the loading effect of the second stage.

Because the coupling capacitor C_3 effectively appears as a short at the signal frequency, the total input resistance of the second stage presents an AC load to the first stage.

Looking from the collector of Q_1, the two biasing resistors in the second stage, R_5 and R_6, appear in parallel with the input resistance at the base of Q_2.

In other words, the signal at the collector of Q_1 "sees" R_3, R_5, R_6, and $R_{in(\text{base2})}$ of the second stage all in parallel to AC ground.

Thus, the effective AC collector resistance of Q_1 is the total of all these resistances in parallel, as the figure given below illustrates.



The voltage gain of the first stage is reduced by the loading of the second stage because the effective AC collector resistance of the first stage is less than the actual value of its collector resistor, R_3.

Remember that without loading of the second stage:

$$A_v = \frac{R_c}{r_e'} = \frac{R_3}{r_{e1}'}$$

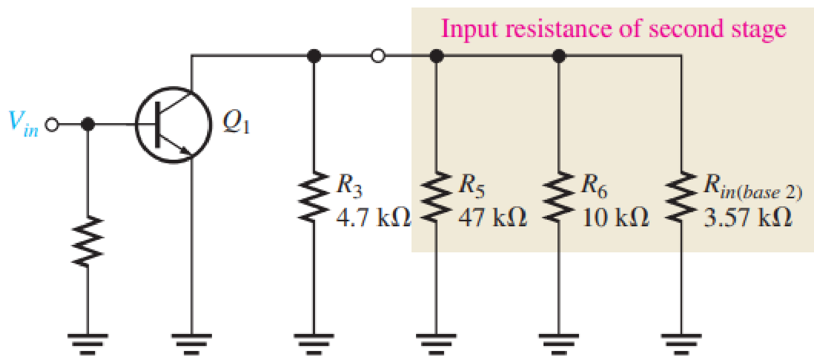
Thus, with loading of second stage:

$$A_v' = \frac{R_3 \parallel R_5 \parallel R_6 \parallel R_{in(base2)}}{r_{e2}'}$$

b. The AC collector resistance of the first stage is:

$$R_{c1} = R_3 \parallel R_5 \parallel R_6 \parallel R_{in(\text{base}2)}$$

Remember that lowercase italic subscripts denote AC quantities such as for R_c.



From part (a), we can verify that I_{E2} = 1.05 mA, r_{e2}' = 23.8 Ω, and $R_{in(\text{base}2)}$ = 3.57 kΩ. The effective AC collector resistance of the first stage is as follows:

$$R_{c1} = 4.7\text{ k}\Omega \parallel 47\text{ k}\Omega \parallel 10\text{ k}\Omega \parallel 3.57\text{ k}\Omega = 1.63\text{ k}\Omega$$

The biasing DC voltage at the base of the first stage is:

$$V_{B1} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{10\text{ k}\Omega}{47\text{ k}\Omega + 10\text{ k}\Omega}\right) 10\text{ V} = 1.75\text{ V}$$

The emitter current in the Q_1 is:

$$I_{E1} = \frac{V_{B1} - V_{BE1}}{R_4} = \frac{1.75\text{ V} - 0.7\text{ V}}{1\text{ k}\Omega} = 1.05\text{ mA}$$

Thus

$$r'_{e1} = \frac{V_T}{I_{E1}} = \frac{25\text{ mV}}{1.05\text{ mA}} = 23.8\ \Omega$$

Therefore, the base-to-collector voltage gain of the first stage is

$$A_{v1} = \frac{R_{c1}}{r'_{e1}} = \frac{1.63\text{ k}\Omega}{23.8\ \Omega} = 68.5$$

c. The second stage has no load resistor, so the AC collector resistance is R_7, and the gain is:

$$A_{v2} = \frac{R_7}{r'_{e2}} = \frac{4.7\text{ k}\Omega}{23.8\ \Omega} = 197$$

Compare this to the gain of the first stage and notice how much the loading from the second stage reduced the gain.

d. The overall amplifier gain with no load on the output is:

$$A'_v = A_{v1}A_{v2} = (68.5)(197) \cong 13{,}495$$

If an input signal of 100 µV, for example, is applied to the first stage and if there is no attenuation in the input base circuit due to the source resistance, an output from the second stage of:

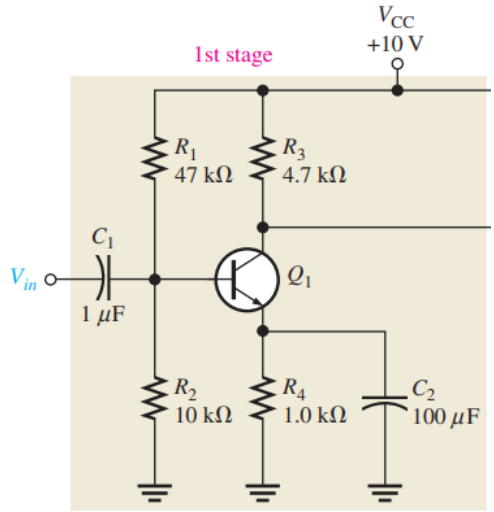
$$V_{out} = V_{in}A'_v = (100\ \mu\text{V})(13{,}495) \cong 1.35\text{ V}$$

The overall voltage gain can be expressed in dB as follows:

$$A'_v(\text{dB}) = 20\log(13{,}495) = 82.6\text{ dB}$$

e. Since both stages in the cascaded amplifier are identical, the DC voltages for Q_1 and Q_2 are the same. Since $\beta_{DC}R_4 \gg R_2$ and $\beta_{DC}R_8R_6$, the DC base voltage for Q_1 and Q_2 is:

$$V_{B1} = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\frac{10\text{ k}\Omega}{47\text{ k}\Omega + 10\text{ k}\Omega}\right) \times 10\text{ V} = 1.75\text{ V}$$



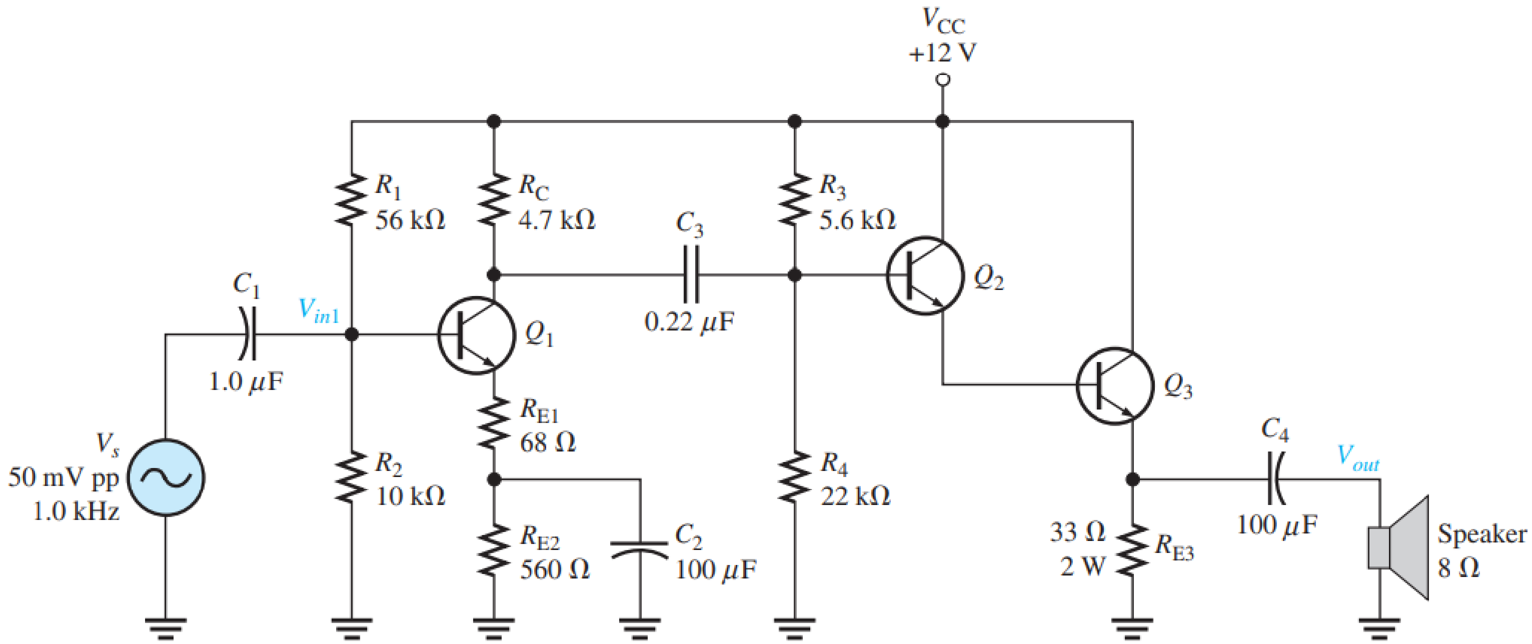
The DC emitter and collector voltages are as follows:

$$V_{E1} = V_{B1} - V_{BE1} = 1.75 - 0.7 = 1.05 \text{ V}$$

$$I_{C1} \cong I_{E1} = \frac{V_{E1}}{R_4} = \frac{1.05 \text{ V}}{1 \text{ k}\Omega} = 1.05 \text{ mA}$$

$$V_{C1} = V_{CC} - I_{C1}R_3 = 10 \text{ V} - (1.05 \text{ mA})(4.7 \text{ k}\Omega) = 5.07 \text{ V}$$

4. Determine the voltage gain and the power gain of the Class A power amplifier in the figure below. Assume β_{ac} = 200 for all transistors. [16 marks]


V_CC
+12 V
R_1
56 kΩ
R_C
4.7 kΩ
C_3
R_3
5.6 kΩ
Q_2
C_1
V_in1
Q_1
0.22 μF
1.0 μF
Q_3
R_E1
68 Ω
C_4
V_out
V_s
50 mV pp
1.0 kHz
R_2
10 kΩ
R_4
22 kΩ
R_E2
560 Ω
C_2
100 μF
33 Ω
2 W
R_E3
100 μF
Speaker
8 Ω

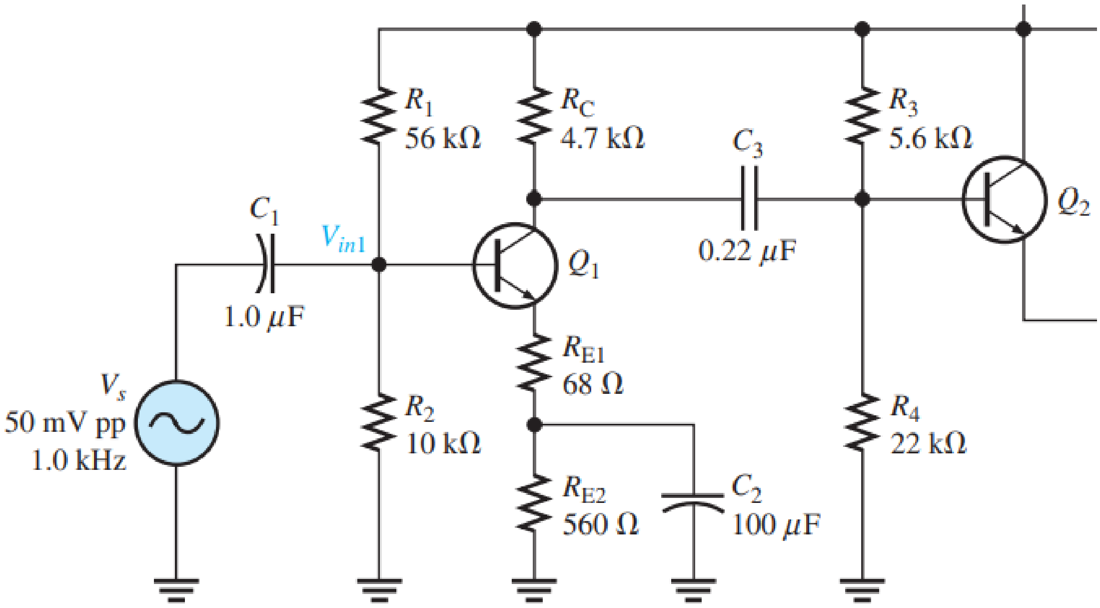
Solution

Notice that the first stage (Q_1) is a voltage-divider biased common-emitter with a swamping resistor (R_{E1}). The second stage (Q_2 and Q_3) is a Darlington voltage follower configuration. The speaker is the load.

First stage:

The AC collector resistance of the first stage is R_C in parallel with the input resistance to the second stage.

$$R_{c1} \cong R_C \parallel (R_3 \parallel R_4) = 4.7\text{ k}\Omega \parallel 5.6\text{ k}\Omega \parallel 22\text{ k}\Omega = 2.29\text{ k}\Omega$$



The voltage gain of the first stage is the AC collector resistance, R_{c1}, divided by the AC emitter resistance, which is the sum of $R_{E1} + r'_{e1}$. The approximate value of r'_{e1} is determined by first finding I_{E1}.

$$V_{B1} \cong \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{10\text{ k}\Omega}{56\text{ k}\Omega + 10\text{ k}\Omega}\right) \times 12\text{ V} = 1.82\text{ V}$$

$$I_{E1} = \frac{V_{B1} - V_{BE1}}{R_{E1} + R_{E2}} = \frac{1.82\text{ V} - 0.7\text{ V}}{560\ \Omega + 68\ \Omega} = 1.78\text{ mA}$$

$$r'_{e1} = \frac{V_T}{I_{E1}} = \frac{25\text{ mV}}{1.78\text{ mA}} = 14\ \Omega$$

Using the value of r'_e, determine the voltage gain of the first stage with the loading of the second stage considered:

$$A_{v1} = -\left(\frac{R_{c1}}{R_{E1} + r'_{e1}}\right) = -\left(\frac{2.29\text{ k}\Omega}{68\ \Omega + 14\ \Omega}\right) = -27.9$$

Notice that the negative sign is for inversion.

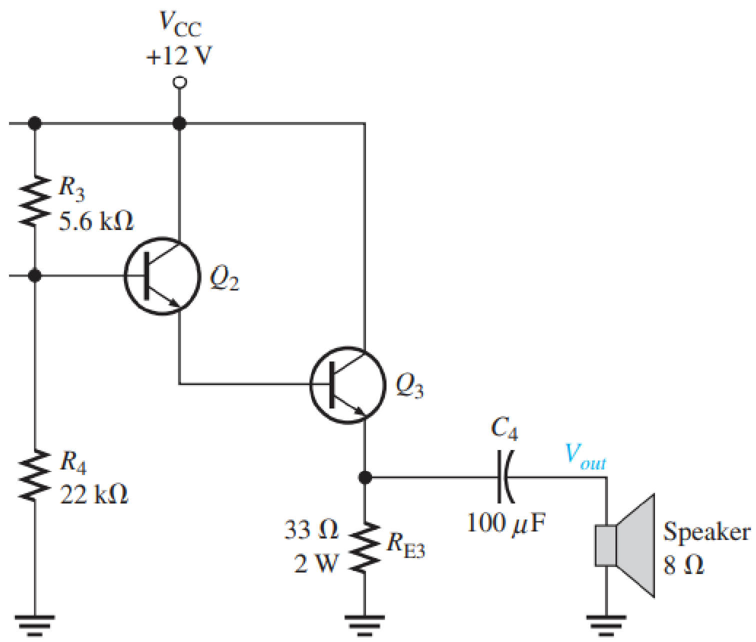
The total input resistance of the first stage is equal to the bias resistors in parallel with the AC input resistance at the base of Q_1.

$$R_{in(\text{tot})1} = R_1 \parallel R_2 \parallel \beta_{ac1}(R_{E1} + r'_{e1}) = 56\text{ k}\Omega \parallel 10\text{ k}\Omega \parallel 200(68\ \Omega + 14\ \Omega) = 8.4\text{ k}\Omega$$

Second stage:

The voltage gain of the Darlington emitter-follower is approximately equal to 1.

$$A_{v2} \cong 1$$



Overall amplifier:

The overall voltage gain is the product of the first and second stage voltage gains. Since the second stage has a gain of approximately 1, the overall gain is approximately equal to the gain of the first stage

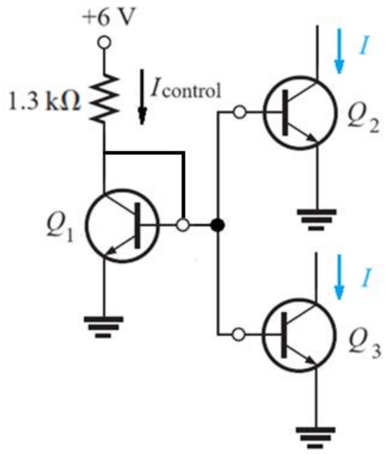
$$A_{v(\text{tot})} = A_{v1}A_{v2} = (-27.9)(1) = -27.9$$

Power gain: The power gain of the amplifier can be calculated using the equation given below:

$$A_p = A^2_{v(\text{tot})}\left(\frac{R_{in(\text{tot})1}}{R_L}\right) = (-27.9)^2\left(\frac{8.4\text{ k}\Omega}{8\ \Omega}\right) = 817{,}330$$

D. Other BJT Circuits

1. Calculate the current I through each of the transistor Q_2 and Q_3 in the current mirror circuit given below. Assume the V_{BE} of the BJT is 0.7 V. [10 marks]



Solution

Since the three NPN transistors are identical, thus $V_{BE1} = V_{BE2} = V_{BE3}$. Then $I_{B1} = I_{B2} = I_{B3}$. Substituting

$$I_{B1} = \frac{I_{control}}{\beta_1} \qquad I_{B2} = \frac{I_{C2}}{\beta_2} \qquad I_{B3} = \frac{I_{C3}}{\beta_3}$$

Knowing that $\beta_1 = \beta_2 = \beta_3$, we have:

$$\frac{I_{control}}{\beta_1} = \frac{I_{C2}}{\beta_2} = \frac{I_{C3}}{\beta_3}$$

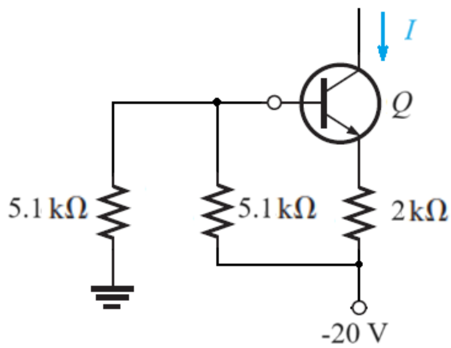
So, for a current mirror circuit, I must equal $I_{control}$:

$$I = I_{control}$$

And

$$I_{control} = \frac{V_{CC} - V_{BE}}{R} = \frac{6\text{ V} - 0.7\text{ V}}{1.3\text{ k}\Omega} = 4.08\text{ mA}$$

2. Calculate the constant current I in the constant-current source circuit as shown in the figure below. Assume the V_{BE} of the BJT is 0.7 V. [7.5 marks]



Solution

Assume for the given transistor that $I_C = I_E$ due to large gain of the capacitor and V_{BE} = 0.7 V. Considering the circuit given above, the voltage at the base is:

$$V_B = \left(\frac{R_1}{R_1 + R_2}\right)(-V_{EE}) = \left(\frac{5.1\ \text{k}\Omega}{5.1\ \text{k}\Omega + 5.1\ \text{k}\Omega}\right)(-20\ \text{V}) = -10\ \text{V}$$

The voltage at the emitter is:

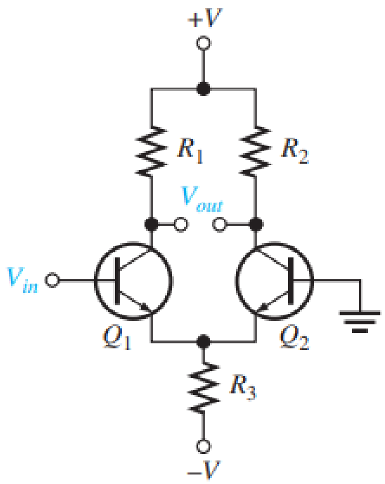
$$V_E = V_B - V_{BE} = -10\ \text{V} - 0.7\ \text{V} = -10.7\ \text{V}$$

The current I is:

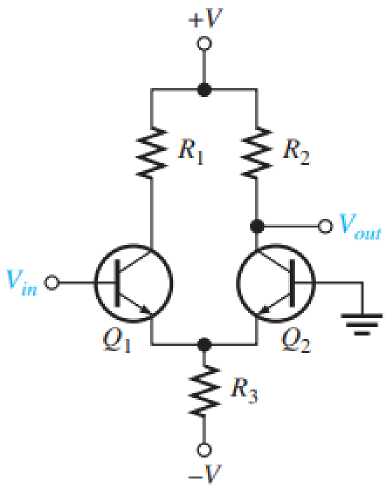
$$I = I_E = \frac{V_E - (-V + V_{EE})}{R_E} = \frac{-10.7\ \text{V} - (-20\ \text{V})}{2\ \text{k}\Omega} = 4.65\ \text{mA}$$

3. Differential amplifier is an amplifier that produces outputs that are a function of the difference between two input voltages. The differential amplifier has two basic modes of operation: differential (in which the two inputs are different) and common mode (in which the two inputs are the same).

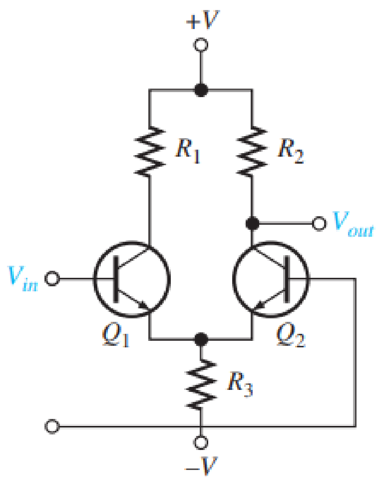
   a. Identify the type of input and output configuration for each basic differential amplifier given below. [2 marks]



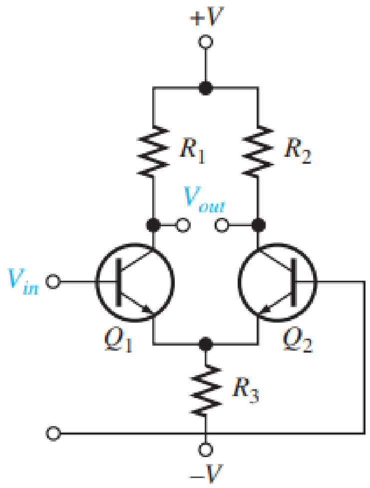
Differential amplifier (i)



Differential amplifier (ii)

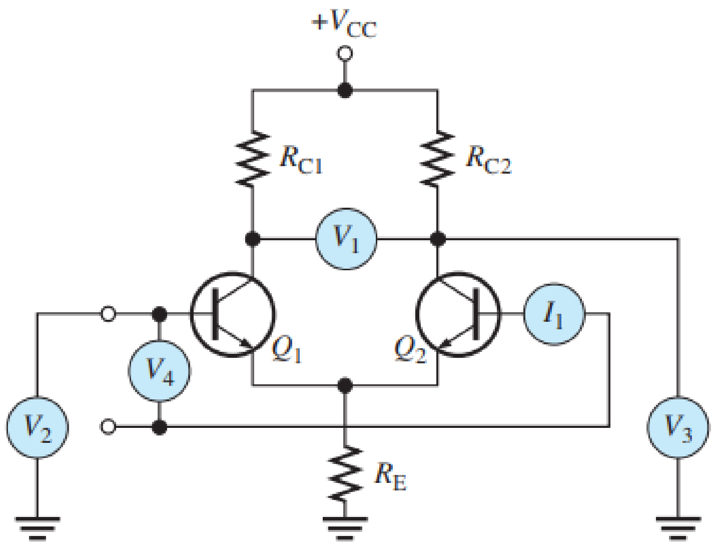


Differential amplifier (iii)



Differential amplifier (iv)

b. Identify the quantity measured by each meter in the differential amplifier given in the figure below. [2.5 marks]



Solution

a. For the given differential amplifiers:

(i) Single-ended differential input; differential output.

(ii) Single-ended differential input; single-ended output.

(iii) Double-ended differential input; single-ended output.

(iv) Double-ended differential input; differential output.

b. For the given differential amplifier:

V_1 = differential output voltage.

V_2 = noninverting input voltage.

V_3 = single-ended output voltage.

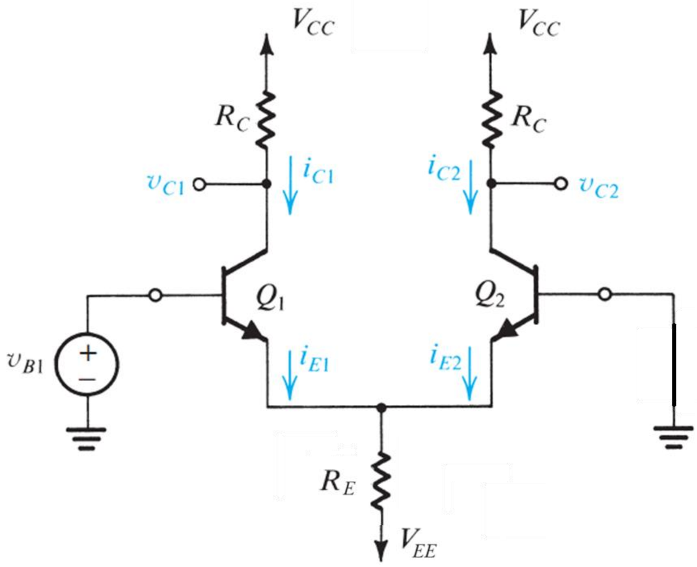
V_4 = differential input voltage.

I_1 = bias current.

4. For the differential amplifier circuit given below, V_{CC} = +15 V, R_C = 5 kΩ, V_{B1} = 1 mV, R_E = 7.5 kΩ, and V_{EE} = -15 V.

a. What are the tail currents (I_E) and collector voltage (V_C)? [6 marks]

b. What is the AC output voltage? If β = 300, what is the input impedance of the difference amplifier. [8 marks]



Solution

a. Assume identical components are used for the collector resistors and BJTs. Also assume that $I_C = I_E$ due to the large gain of the capacitor and V_{BE} = 0.7 V.

For the given differential amplifier circuit, analysing the circuit at Q_2, the tail current is:

$$I_T = \frac{V_{EE} - V_{BE}}{R_E} = \frac{-15\text{ V} - 0.7\text{ V}}{7.5\text{ k}\Omega} \approx 2\text{ mA}$$

Each emitter current is half of the tail current which is:

$$I_E = \frac{I_T}{2} = \frac{2\text{ mA}}{2} = 1\text{ mA}$$

Each collector has a quiescent voltage of approximately:

$$V_C = V_{CC} - I_C R_C = 15\text{ V} - (1\text{ mA})(5\text{ k}\Omega) = 10\text{ V}$$

b. From part (a), the emitter current (I_E) is 1 mA. Thus, we calculate the AC emitter resistance of the differential amplifier:

$$r_e' = \frac{25\text{ mV}}{I_E} = \frac{25\text{ mV}}{1\text{ mA}} = 25\ \Omega$$

The voltage gain is:

$$A_v = \frac{R_C}{r_e'} = \frac{5\text{ k}\Omega}{25\ \Omega} = 200$$

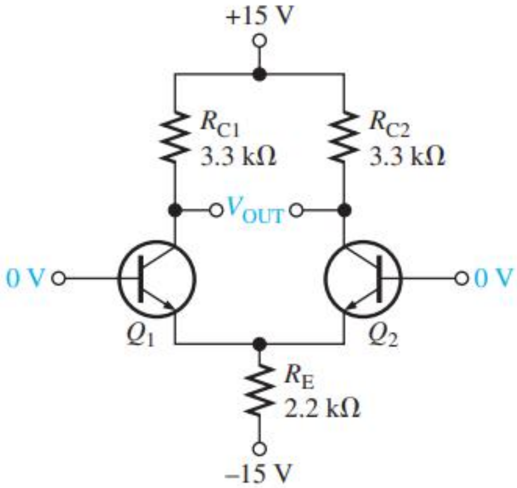
The AC output voltage is:

$$V_{out} = A_v V_{B1} = 200(1\text{ mV}) = 200\text{ mV}$$

Considering the differential inputs of the differential amplifier, the input impedance of the differential amplifier is:

$$Z_{in(\text{base})} = 2\beta r_e' = 2(300)(25\ \Omega) = 15\text{ k}\Omega$$

5. The DC base voltages in the differential amplifier in the figure below are zero. Using your knowledge of transistor analysis, determine the DC differential output voltage as a function of emitter current. Assume that Q_1 has an α = 0.980 and Q_2 has an α = 0.975. [18 marks]



Solution

Voltage at the emitter of differential amplifier is found from the following equation by applying KVL in the circuit at the emitter:

$$V_E = V_{BE} = -0.7\text{ V}$$

As a result, the current that flows in the emitter is equal to:

$$I_E = \frac{V_{EE} - V_{BE}}{R_E} = \frac{-15 - (-0.7)}{2.2\text{ k}\Omega} = 6.5\text{ mA}$$

The current at the emitter is found from the following equation:

$$I_E = I_{E1} + I_{E2} = 6.5\text{ mA}$$

So, rearranging the equation above:

$$I_{E2} = 6.5\text{ mA} - I_{E1}$$

Voltage at the collector of transistor Q_1 is found from applying KVL in the circuit at the left-hand side:

$$V_{C1} = V_{CC} - I_{C1}R_C$$

Voltage at the collector of transistor Q_2 is found from applying KVL in the circuit at the right-hand side:

$$V_{C2} = V_{CC} - I_{C2}R_C$$

The output voltage of the differential amplifier is given as:

$$V_{out} = V_{C1} - V_{C2} = (V_{CC} - I_{C1}R_C) - (V_{CC} - I_{C2}R_C) = (I_{C2} - I_{C1})R_C$$

Knowing that: $I_{C1} = \alpha_{DC1}I_{E1}$ and $I_{C2} = \alpha_{DC2}I_{E2}$, the voltages at the collectors of the differential amplifier becomes:

$$V_{out} = (\alpha_{DC2}I_{E2} - \alpha_{DC1}I_{E1})R_C$$

Knowing that $I_{E2} = 6.5\ \mathrm{mA} - I_{E1}$, the equation above becomes after applying all values:

$$V_{out} = [0.975 \times (6.5\ \mathrm{mA} - I_{E1}) - 0.980 \times I_{E1}] \times 3.3\ \mathrm{k\Omega} = 20.9\ \mathrm{k\Omega} - 6.45 I_{E1}$$