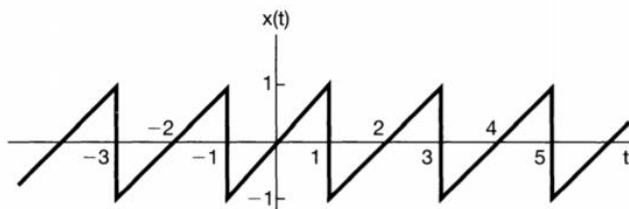


ECEN220 (XMUT) Homework Questions: Fourier Series

From textbook Oppenheim:

(Each whole, or sub-question, is worth 1 mark; total of 30 marks)

3.22 Determine the Fourier series representation of the following signal:



1 mark

3.24. Let

$$x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2 - t, & 1 \leq t \leq 2 \end{cases}$$

3 marks

be a periodic signal with fundamental period $T = 2$ and Fourier coefficients a_k .

- (a) Determine the value of a_0 .
- (b) Determine the Fourier series representation of $dx(t)/dt$.
- (c) Use the result of part (b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of $x(t)$.

3.25. Consider the following three continuous-time signals with a fundamental period of $T = 1/2$:

$$\begin{aligned} x(t) &= \cos(4\pi t), \\ y(t) &= \sin(4\pi t), \\ z(t) &= x(t)y(t). \end{aligned}$$

4 marks

- (a) Determine the Fourier series coefficients of $x(t)$.
- (b) Determine the Fourier series coefficients of $y(t)$.
- (c) Use the results of parts (a) and (b), along with the multiplication property of the continuous-time Fourier series, to determine the Fourier series coefficients of $z(t) = x(t)y(t)$.
- (d) Determine the Fourier series coefficients of $z(t)$ through direct expansion of $z(t)$ in trigonometric form, and compare your result with that of part (c).

3.26. Let $x(t)$ be a periodic signal whose Fourier series coefficients are

$$a_k = \begin{cases} 2, & k = 0 \\ j(\frac{1}{2})^{|k|}, & \text{otherwise} \end{cases}$$

3 marks

Use Fourier series properties to answer the following questions:

- (a) Is $x(t)$ real?
- (b) Is $x(t)$ even?
- (c) Is $dx(t)/dt$ even?

- 3.27.** A discrete-time periodic signal $x[n]$ is real valued and has a fundamental period $N = 5$. The nonzero Fourier series coefficients for $x[n]$ are

$$a_0 = 2, a_2 = a_{-2}^* = 2e^{j\pi/6}, \quad a_4 = a_{-4}^* = e^{j\pi/3}.$$

Express $x[n]$ in the form

1 mark

$$x[n] = A_0 + \sum_{k=1}^{\infty} A_k \sin(\omega_k n + \phi_k).$$

- 3.28.** Determine the Fourier series coefficients for each of the following discrete-time periodic signals. ~~Plot the magnitude and phase of each set of coefficients.~~

(a) Each $x[n]$ depicted in Figure P3.28(a)–(c)

~~$x[n] = \sin(2\pi n/3) \cos(\pi n/2)$~~

~~(X)~~ $x[n]$ periodic with period 4 and

$$x[n] = 1 - \sin \frac{\pi n}{4} \quad \text{for } 0 \leq n \leq 3$$

~~(d)~~ $x[n]$ periodic with period 12 and

$$x[n] = 1 - \sin \frac{\pi n}{4} \quad \text{for } 0 \leq n \leq 11$$

3 marks

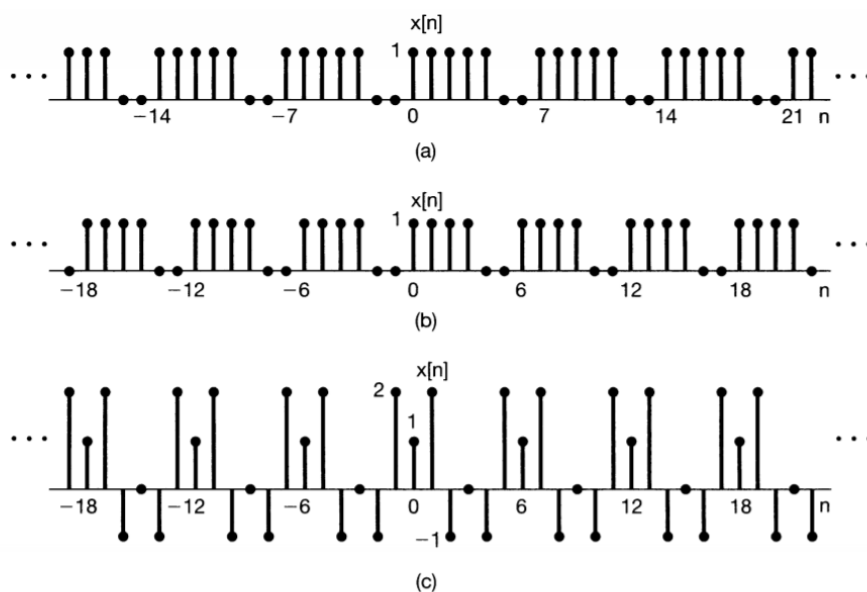


Figure P3.28

4.21. Compute the Fourier transform of each of the following signals:

10 marks

- (a) $[e^{-\alpha t} \cos \omega_0 t] u(t), \alpha > 0$ (b) $e^{-3|t|} \sin 2t$
(c) $x(t) = \begin{cases} 1 + \cos \pi t, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$ (d) $\sum_{k=0}^{\infty} \alpha^k \delta(t - kT), |\alpha| < 1$
(e) $[te^{-2t} \sin 4t] u(t)$ (f) $\left[\frac{\sin \pi t}{\pi t} \right] \left[\frac{\sin 2\pi(t-1)}{\pi(t-1)} \right]$
(g) $x(t)$ as shown in Figure P4.21(a) (h) $x(t)$ as shown in Figure P4.21(b)
(i) $x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$ (j) $\sum_{n=-\infty}^{+\infty} e^{-|t-2n|}$

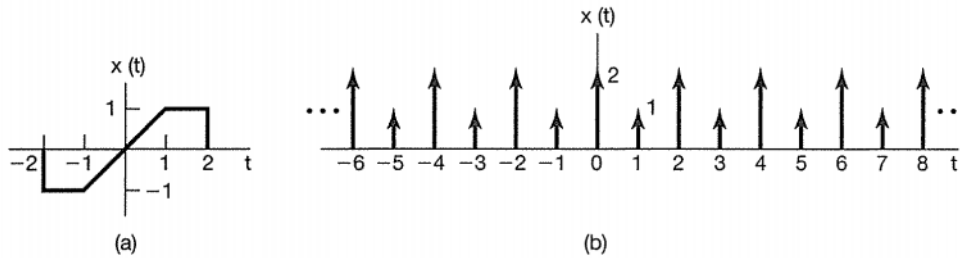


Figure P4.21

4.22. Determine the continuous-time signal corresponding to each of the following transforms.

5 marks

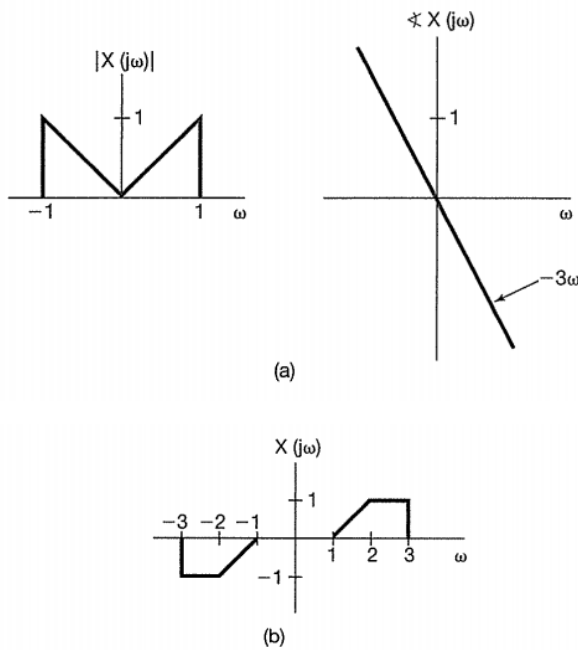


Figure P4.22

- (a) $X(j\omega) = \frac{2 \sin[3(\omega - 2\pi)]}{(\omega - 2\pi)}$
(b) $X(j\omega) = \cos(4\omega + \pi/3)$
(c) $X(j\omega)$ as given by the magnitude and phase plots of Figure P4.22(a)
(d) $X(j\omega) = 2[\delta(\omega - 1) - \delta(\omega + 1)] + 3[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$
(e) $X(j\omega)$ as in Figure P4.22(b)