XMUT220 Signals and Systems

Homework 4: Sampling and Reconstruction

Due Date: Sunday, 27th December 2026 (online submission to VUW XMUT220 website)

1. Determine the Nyquist frequency and sampling rate of the following signals: [15 marks]
   a. $x(t) = 5\sin(t/100)$.
   b. $x(t) = 2\cos(\pi t/10) + \sin(5\pi t)$.
   c. $x(t) = \cos(t)\,[1 + \cos(2\pi t)]$.
   d. $x(t) = 1 + \cos(2000\pi t) + \sin(4000\pi t)$.
   e. $x(t) = \sin(4000\pi t)/\pi t$.
   f. $x(t) = (\sin(4000\pi t)/\pi t)^2$.

2. You are given a task to perform sampling of a signal of frequency f_0 as follows with a sampling frequency $F = 6$ Hz.

$$x(t) = \cos(2\pi f_0 t)$$

   Considering the sampling effects in time domain as well as in frequency domain, derive equations for the sampled signal for three case studies of different frequency of the signal given below. Describe whether the signal is experiencing aliasing or not, what are its alias(es), and derive its equation. Sketch frequency spectrum of the signal and all possible alias that exists in the process.
   a. f_0 = 4 Hz. [15 marks]
   b. f_0 = 8 Hz. [15 marks]
   c. f_0 = 12 Hz. [15 marks]

3. Given a compound signal below consisted of two frequency components:

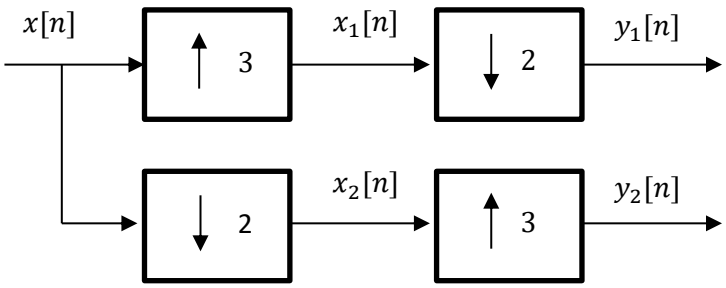
$$x(t) = 2\cos(2\pi 5t) + \cos(2\pi 2t)$$

a. Derive the equation for the discrete signal after sampling when the sampling frequency is $F = 10$ samples/second and the sampling period is $t = 0.1$ seconds. Is the signal experiencing aliasing or not? If it is, derive the equation for its alias. Simulate the frequency spectra of the signal and any of its possible alias in MATLAB. [10 marks]

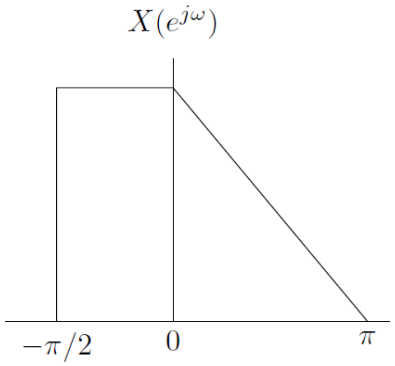
b. Like part (a), if the first frequency component is increased to $f_1 = 8$ Hz, while the sampling frequency is still $F = 10$. [10 marks]

c. Comment on the results of parts (a) and (b) by explaining on how increasing frequency component of a sampled signal while maintaining the same sampling frequency could influence the sampling and aliasing. [5 marks]

4. Consider the two discrete-time sampling systems given in the figure below that implement up sampling and down sampling methods.



a. For $X(e^{j\omega})$ as given in the figure below, sketch $X_1(e^{j\omega})$ and $Y_1(e^{j\omega})$. [5 marks]



b. Like part (a), sketch $X_2(e^{j\omega})$ and $Y_2(e^{j\omega})$. [5 marks]

c. Derive the equations that show the relationship between $Y_1(e^{j\omega})$ and $Y_2(e^{j\omega})$. [5 marks]

Marking Schedule

Student ID : _____

Student Name : _____

No	Question	Mark	Your Mark	Remarks
1a-f	Nyquist frequency and sampling rate of the signals.	15		
2a	The equations for the sampled signal and alias and sketch of frequency spectrum at f_0 = 4 Hz.	15		
2b	The equations for the sampled signal and alias and sketch of frequency spectrum at f_0 = 8 Hz.	15		
2c	The equations for the sampled signal and alias and sketch of frequency spectrum at f_0 = 12 Hz.	15		
3a	The equations for the discrete signal and alias after sampling and their MATLAB simulation.	10		
3b	Like 3(a), but $f_1 = 8$ Hz, while the sampling frequency is still $F = 10$.	10		
3c	Comment on the results of parts (a) and (b).	5		
4a	Sketch $X_1(e^{j\omega})$ and $Y_1(e^{j\omega})$.	5		
4b	Sketch $X_2(e^{j\omega})$ and $Y_2(e^{j\omega})$.	5		
4c	The equations that show the relationship between $Y_1(e^{j\omega})$ and $Y_2(e^{j\omega})$	5		
	Total	100		

Comment: